A Multi Objective Control approach to Online Dual Arm Manipulation¹

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Abstract: In this paper, we propose a new way to exploit the redundancy of dual arm mobile manipulators when performing inherently bi-manual tasks using online controllers.

Bi-manual tasks are tasks that require motion of both arms in order to be carried out efficiently, such as holding and cleaning an object, or moving an object from one hand to the other. These tasks are often associated with several constraints, such as singularity- and collision avoidance, but also a high degree of redundancy, as the relative positions of the two grippers is far more important than the absolute positions, when for example handing an object from one arm to the other.

By applying a modular multi objective control framework, inspired by earlier work on sub-task control, we exploit this redundancy to form a subset of the joint space that is feasible, i.e. not violating any of the constraints. Earlier approacher added the additional tasks in terms of equality constraints, thereby reducing the dimension of the feasible subset until it was a single point. Here however, we add the additional tasks in terms of inequalities, removing parts of the feasible set rather than collapsing its dimensionality. Thus, we are able to handle an arbitrary number of constraints, instead of a number corresponding to the dimension of the feasible set (degree of redundancy). Finally, inside the feasible set we choose controls stay in the set, while simultaneously minimizing some given objective.

The proposed approach is illustrated by several simulation examples.

Keywords: Robotic manipulators, Redundant Manipulators, Robot control, Robot kinematics, Autonomous mobile robots

1. INTRODUCTION

Dual arm mobile manipulation is an important research area that has received an increasing amount of attention in the recent years. This development is driven by reduced hardware costs as well as growing expectations on the future capabilities of dual arm systems.

The potential benefits of endowing robots, such as the one in Figure 1, with two arms fall into four main categories, first, teleoperation is easier if the robot is similar to the operator (Jau, 1988; Yoon et al., 1999; Kron and Schmidt, 2004; Buss et al., 2006; Taylor and Seward, 2010), second, using tools and workflows designed for humans is easier if the robot is similar to a human (Kemp et al., 2007; Fuchs et al., 2009; Bloss, 2010; Kr uger et al., 2011), third, the two arms can either provide additional strength and precision by cooperating as a parallel manipulator, or provide flexibility and speed by doing two separate tasks simultaneously (Lee and Kim, 1991), fourth, the two arms can perform task that are inherently bi-manual (Chiacchio and Chiaverini, 1998; Caccavale et al., 2000). i.e., tasks that require motion of both arms to be carried out efficiently. In this paper we will focus on such tasks, and examples include the following.



- Fig. 1. Semi-anthropomorphic robot at CAS/KTH. Future work includes adapting and implementing the proposed approach to this dual arm manipulator.
 - Moving an object from one hand to the other

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- Opening a fridge door with one hand and removing an object from inside of fridge door with the other hand.
- Manual dish washing, e.g., holding a frying pan and scrubbing it with a cleaning utensil
- Holding an object with one hand and painting/cleaning it with the other hand (similar to dish washing)
- Self cleaning, i.e. the robot dusting itself off with a cloth. This involves extensive motion of both arms in order to reach every part of own body and clean it.

While doing these things, one would want the robot to simultaneously carry out the following sub tasks: avoid singularities, avoid internal collisions, avoid external collisions, prefer arm motion to base motion, and finally keep end effector and manipulated objects in sensor field of view.

In general, the approaches to solving the problems above can be divided into two categories, offline (global) and online (local). In the offline approaches one plans joint space trajectories that achieve the desired objectives while taking all other constraints into account. These methods (Patel et al., 2005) can provide very efficient solutions, e.g. by solving minimum time problems, but are computationally expensive and often require structured environments with non-moving obstacles. Online approaches on the other hand can be less computationally expensive, can handle unstructured environments with moving obstacles, such as domestic environments with humans nearby, but might produce less efficient solutions and might even fail to solve problems that are really difficult, such as maze like motion planning problems. We believe that these complementary qualities can be exploited by a combined algorithm, using an online approach to do most of the tasks, and invoking an offline algorithm as a fallback, when the previous one is stuck in e.g. a local minima. The offline algorithm then computes a joint space trajectory that is used as input to the online one, while adding the online qualities of collision avoidance with respect to moving, or previously unknown, obstacles. In this paper, we will focus on the online (local) part of the problem.

The ideas presented here are closely related to the additional tasks of Seraji (1989), the user defined objective functions of Peng and Adachi (1993), and the sub-tasks of Tatlicioglu et al. (2008). However, all of the above assume that the main objective is given by a desired, possibly time varying, position and orientation of the end effector. Hence, all additional tasks must be addressed using socalled self motion, in the null space of the end-effector Jacobian. In the present paper, we generalize these ideas by letting the main task, as well as the sub tasks, be given in terms of *scalar inequalties* that must be satisfied. We believe that this is suitable for many, but not all applications. For example, task such as cleaning, or moving an object from one hand to the other, can be formulated using inequalities in terms of bounds on precision, whereas task such as welding might be best formulated using equalities. implying best possible accuracy. Furthermore, instead of exploiting the null-space of different Jacobians we apply a Linear Programming (LP) approach to find controls that do not violate the relevant constraints. Finally, redundancy exploitation becomes even more important in a dual arm setting, as for example, moving an object from

one arm to the other can be carried out in many different parts of the workspace, whereas picking up a stationary object obviously has to be done at the position of that object.

The concept of using scalar inequalities to realize modular multi objective control was earlier applied to mobile robot obstacle avoidance in Ögren (2008) and surveillance UAV control in Ögren and Robinson (2011). The contribution of the present work lies in the combination of those ideas with the work of Seraji (1989), Peng and Adachi (1993) and Tatlicioglu et al. (2008), in a dual arm mobile manipulation setting.

The outline of the paper is as follows. In Section 2 we give a brief background on sub-task control and modular multi objective control. Section 3 then states the dual arm manipulation problem we are trying to solve. The proposed solution is given in Section 4, followed by simulation results in Section 5. Finally, conclusions are drawn in Section 6.

2. BACKGROUND AND RELATED WORK

In this section we will first review the concepts of modular multi objective control and then discuss how it relates to the work of Seraji (1989).

Modular multi objective control was described in Ögren (2008) and Ögren and Robinson (2011). Here we will just use the basic ideas, in the form presented below. Given a time interval $[t_0, t_f]$, initial state $q(t_0) = q_0$ and a control system

$$\dot{q} = h(q, u)$$

where $q \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. Let the control objective be formulated in terms of a set of functions $f_i : \mathbb{R}^n \to \mathbb{R}$ and bounds $b_i \in \mathbb{R}, i \in I \subset \mathbb{N}$ as follows

$$\min_{u(\cdot)} f_j(q(t_f)), \ j \in I \tag{1}$$

(while)
$$f_i(q(t)) \le b_i, \ \forall i \in I, \ t > t_0$$
 (2)

where we assume that the bounds are satisfied at t_0 , i.e. $f_i(q_0) \leq b_i$ for all i.

Now, assuming that the above optimal control problem can not be solved, either due to uncertainties or available computational resources, we instead apply the following online (local) controller to find a new control each time step

$$\min_{u} \dot{f}_{j}(q(t), u), \ j \in I$$
(3)

s.t.)
$$\dot{f}_i(q,u) \leq -k(f_i(q) - b_i), \ \forall i \in I,$$
 (4)

where k is a positive scalar. It is clear that as long as Equation (4) is satisfied, so will Equation (2) be. Furthermore, in the worst case, if we have equality in Equation (4) then the bounds of Equation (2) will be approached, but not violated, exponentially, with time constant 1/k, see Ögren and Robinson (2011).

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Finally, we note that the presentations in Ögren (2008) and Ögren and Robinson (2011) are somewhat more complex than the above, to allow for changes in the index \hat{j} and different bounds b_i in different parts of the state space.

We will now relate the ideas above to the work of Seraji. In Seraji (1989), the *basic task* to be accomplished is encoded in the desired orientation and position of the gripper $Y_d(t)$. Then, a set of r additional tasks are added in terms of scalar functions $\phi_i(q)$ with corresponding desired values $\phi_{di}(t)$, where the number of tasks r is given by the degree of redundancy. The additional tasks are then combined with the basic task to form a complete set of configuration variables, $X = (Y, \phi)$. The controller is then designed, given the set of time varying desired values for these configuration variables. In Section IID of Seraji (1989), it is also discussed how the approach can be modified to include inequalities in the additional tasks, by setting the corresponding error to zero if the inequality is satisfied. Seraji furthermore notes that inequalities are the natural choice when expressing constraints such as collision avoidance, singularity avoidance and joint limits.

We believe that the observations of Seraji provide a good argument for investigating the use of inequalities in dual arm manipulation problems. By incorporating the inequalities at a higher level, i.e., Equations (3)-(4), instead of switching on and off errors at the lowest control layer, we achieve the following. As long as the set of feasible joint states, satisfying Equation (2), is not empty, any number of constraints can be added and simultaneously taken into account, not just r, the degree of redundancy. This is important, since the set of conceivable constraints include joint limits, singularity avoidance, collision avoidance with respect to multiple obstacles and multiple parts of the robot, sensor-object occlusion avoidance, etc. The parameter k also makes it possible to vary the softness of the constraint boundary, i.e., how fast the robot is allowed to approach the bound, as noted above.

3. PROBLEM FORMULATION

In this section, we first state the informal Dual Arm Manipulation (DAM) problem, we then adapt the multi objective control approach described in Section 2 above, to state a formalized version of the problem.

Problem 1. (Informal DAM-problem). The DAM-problem includes the following list of objectives and constraints.

- Achieve the desired main objective in workspace (wash frying pan)
- Avoid singularities
- Avoid internal collisions
- Avoid external collisions
- Keep end effector in sensor field of view

We will now formalize the problem described above. First we write down the *kinematic* equations of motion for our mobile dual arm manipulator, Figure 1, assuming that the low level controllers take care of the system dynamics.

$$\dot{q}_0 = u_1,\tag{5}$$

$$\dot{q}_1 = u_2 \cos q_0,\tag{6}$$

$$\dot{q}_2 = u_2 \sin q_0,\tag{7}$$

$$\dot{q}_j = u_j, \ j \in \{3, \dots 16\}$$
(8)

where Equations (5)-(7), a unicycle (de Wit et al., 1996) models the mobile base, with the orientation $q_0 \in \mathbb{R}$,

position $(q_1, q_2) \in \mathbb{R}^2$, angular velocity $|u_1| \leq U_1$ and translational velocity $|u_2| \leq U_2$. The two manipulator arms are modelled by Equation (8), with $|u_j| \leq U_j$, $|q_j| \leq Q_j$, where q_j are the joint angles, u_j are the joint velocities and U_j and Q_j are bounds on velocities and angles respectively. Thus, as the two arms of our dual arm manipulator each have 7 joints, and the base can be described by a position in \mathbb{R}^2 and an orientation, the combined state space of the robot is a subset of \mathbb{R}^{17} .

In order to apply the ideas reviewed in Section 2 above, we now write the objectives in terms of scalar functions $f_i(q), f_i : \mathbb{R}^{17} \to \mathbb{R}$. We then formulate the constraints in terms of bounds on these functions $f_i(q) \leq b_i$.

The desired end effector motion in workspace between the frying pan and the cleaning utensil can be modeled as follows.

$$f_1(q) = \frac{1}{2} ||p_1(q) - p_2(q) - d(t, y_1, z_1)||^2 + x_1^T x_2 \le b_1,$$
(9)

where $p_1, p_2 \in \mathbb{R}^3$ is the positions of the frying pan and cleaning utensil respectively, $d(t, y_1, z_1)$ is an offset determining what part of the frying pan to clean, se below, and $R_i \in SO(3)$ the orientation matrices, $R_i = (x_i, y_i, z_i)$. Note that $f_1 = -1$ corresponds to the tip of the cleaning utensil p_2 touching the center of the frying pan at $p_1 - d(t, y_1, z_1)$ with opposite directions (x_2 being the main axis of the cleaning utensil and x_1 being the surface normal of the frying pan) but random rotation round those axes. The offset $d(t, y_1, z_1)$ is chosen to provide a circular cleaning pattern or radius 0.1m in the frying pan $d(t, y_1, z_1) = 0.1(y_1 \cos(t) + z_1 \sin(t))$, where y_1, z_1 are part of the frying pan frame, as noted above.

Avoiding singularities can be expressed using a manipulability index (Siciliano et al., 2009) as follows:

$$f_{2i}(q) = \frac{-1}{2} det(J_i^T J_i) \le b_2 < 0, \quad i \in \{1, 2\}$$
(10)

where $J_i \triangleq \begin{bmatrix} J_{pi}^T & J_{\omega i}^T \end{bmatrix}^T$ is the manipulator Jacobian which consists of Jacobians J_{pi} and $J_{\omega i}$ related to the translational and rotational motion of the end-effector respectively. Avoiding collisions can be formulated in terms of the minimal distance as

$$f_3(q) = -\min_{x_r \in X_r, x_o \in X_o} ||x_r - x_o|| \le b_3 < 0,$$
(11)

where X_r is the subset of the workspace that is occupied by the robot, and X_o is the subset of the workspace that is occupied by obstacles. Depending on the needed accuracy, one could either use simple conservative obstacle representations, such as spheres, or more elaborate computations of the minimal distance, e.g. using the critical points and directions of Patel et al. (2005).

We can now formalize Problem 1 using Equations (1)-(2) as follows.

Problem 2. (Formal DAM-problem).

$$\min_{u(\cdot)} f_j(q(t_f)), \ j \in I$$
(12)

(while)
$$f_i(q) \le b_i, \ \forall i \in I, \ t > t_0$$
 (13)

where $I \subset \{1, \ldots, 3\}$, $\hat{j} \in I$, and f_i are given by Equations (9), (10) and (11).

4. PROPOSED SOLUTION

Following the ideas reviewed in Section 2, Equations (3)-(4), instead of solving Problem 2 to optimality we hope to find a feasible *good enough* solution by applying the following controller

$$\min f_j(q(t), u), \ j \in I \tag{14}$$

(s.t.)
$$f_i(q, u) \leq -k(f_i(q) - b_i), \ \forall i \in I.$$
 (15)

We will now use the equations of motion (5)-(8), to rewrite the controller. We have the following

$$\dot{f}_i(q(t),u) = \frac{df_i}{dt} = \frac{df_i}{dq}^T \frac{dq}{dt} = \frac{df_i}{dq}^T u$$

which implies the following structure of the controller

$$\min_{u} \frac{df_{j}}{dq}^{T} u, \ j = \hat{j} \in I$$

s.t.
$$\frac{df_{i}}{dq}^{T} u \leq k(b_{i} - f_{i}), \ \forall i \in I$$

which is equivalent to a Linear Programming problem (LP)

$$\min_u c^T u \tag{16}$$

s.t.
$$Au \le k(b-f)$$
 (17)

where $c = \frac{df_j}{dq}$, and each row of A, b, f contains the corresponding $\frac{df_i}{dq}, b_i, f_i$ respectively. Such LPs can be solved very efficiently, even in quite high dimensions.

We furthermore note that the values of $\frac{df_i}{dq}$ can be given either in closed form, or as numerical estimates. For instance, f_1 in Equation (9) can be analytically differentiated to give

$$\frac{df_1}{dq} = x_2^T (-S(x_1)J_{\omega 1}) + (p_1 - p_2)^T J_{p1} + x_1^T (-S(x_2)J_{\omega 2}) - (p_1 - p_2)^T J_{p2}.$$

where S(a) denotes a skew-symmetric matrix which is used in order to produce the outer product of a vector $a \in \mathbb{R}^3$ with some vector $b \in \mathbb{R}^3$ i.e. $S(a)b = a \times b$. To conclude, we thus propose to provide good enough, feasible, solutions to Problem 1, by Iteratively solving the LP in Equations (16)-(17).

Remark 1. In order to only move the base when necessary, we can try to solve the "arm part" of the LP above first (i.e. removing the bottom part of the u vector and corresponding parts of the other matrices. If the LP turns out to be infeasible, we add the bottom part and solve the original LP. Thus the base will only be moved when it needs to be, and in most situations the arms alone will move to carry out the task.

5. SIMULATIONS

The simulations in this section is performed using the Matlab Robotics Toolbox of Corke (1996), where the dual arm manipulator is realized by two Puma 560 robots



Fig. 2. The dual arm manipulator of the simulations is composed of two Puma 560 manipulators sharing the same workspace. An obstacle in form of a table is introduced at height z = -0.6.

sharing the same workspace, see Figure 2. Note that they have 6 degrees of freedom, instead of the 7 of the robot in Figure 1, but redundancy is still present in the inherently bi-manual tasks we use to illustrate the approach. Also note that we are using only the arm part of the robot, as discussed in Remark 1 above.

In all examples, we let $\hat{j} = 1$ and $I = \{1, 2, 3\}$. We have a very simple obstacle, in form of a table surface at height z = -0.6, see Figure 2, and we only check obstackle avoidance with the gripper positions, i.e., $X_r = \{p_1, p_2\}$ and $X_o = \{x \in \mathbb{R}^3 : x_z = -0.6\}$, in Equation (11). Furthermore, we let the bound on f_1 in Equation (9) be $b_1 = 1$, but f_1 will be much less most of the time, since we are optimizing over it.

Three different simulations are presented. Figures 3 and 6 show the results of all three, for comparison, while Figures 4 and 5 only show the first, for clarity.

In the first simulation we set the bound on the singularity measure in (10) to $b_2 = -0.002$ and the inverse time constant k = 0.5. The resulting motion of the cleaning utensil in the frying pan can be seen in Figure 3 (blue solid) and component wise in Figure 4. As can be seen, the cleaning utensil approaches the pan and traces out a circular pattern corresponding to the desired offset d in Equation (9). The joint angles are plotted in Figure 5, and we can see that the task is carried out in a truly bi-manual fashion, where both arms are contributing to carrying out the task efficiently. The functions f_i are shown in Figure 6 (blue solid). We can see that the task measure f_1 is decreasing, and that the bounds b_2, b_3 on singularities and obstacle avoidance margins f_2, f_3 are satisfied.

In the second simulation, we keep the bounds b_i unchanged, and decrease the inverse time constant k, i.e. $k = 0.1, b_2 = -0.002$. The results are shown in Figures 3 and 6. As can be seen, the path (red dashed) of the cleaning utensil in the frying pan frame is similar but not



Fig. 3. The motion of the cleaning utensil in the frying pan, projected onto the frying pan plane. The results of three simulations are shown, (blue solid: $k = 0.5, b_2 = -0.002$), (red dashed: $k = 0.1, b_2 = -0.002$), (green dash-dotted: $k = 0.5, b_2 = -0.005$).



Fig. 4. The motion of the cleaning utensil in the frying pan coordinate frame for the first simulation ($k = 0.5, b_2 = -0.002$). The x_1 coordinate is drawn in blue (solid), the y_1 in red (dashed) and the z_1 in green (dash-dotted).

identical, Figure 3. In Figure 6 we see the results of the changed constant k. In all the bottom three plots, the different bounds $b_2 = -0.002, b_3 = -0.3$ are approached less aggressively. A close up of the middle plot of Figure 6 is shown in Figure 7. Here we have chosen an instant with a worst case convergence towards the bound b_2 , and we can see the exponential shape of the two curves, as noted in Section 2.

In the third simulation we use the same k as the first simulation, but instead change the bound b_3 concerning singularity avoidance. Thus we have $k = 0.5, b_2 = -0.005$. This gives a completely different initial path of the cleaning utensil, see Figure 3 (green dash-dotted), and a slighty worse final cleaning pattern in the frying pan. Looking at Figure 6 (green dash-dotted) we see that the curves are quite different from the earlier two simulations, but that the required bounds are satisfied. Note in particular the



Fig. 5. All 12 joint angles as a function of time for the first simulation $(k = 0.5, b_2 = -0.002)$.



Fig. 6. Time evolution of the functions $f_i(q)$. Note how the scalar inequalities $f_i \leq b_i$ are satisfied for the different settings (blue solid: $k = 0.5, b_2 = -0.002$), (red dashed: $k = 0.1, b_2 = -0.002$), (green dashdotted : $k = 0.5, b_2 = -0.005$).

plateu touching the new lower bound $b_2 = -0.005$ around t = 50 in the middle plot.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have discussed a new way of doing online dual arm manipulation tasks that are inherently bimanual. Earlier work on sub task control applied equality constraints in order to reduce the system redundancy. Thus, each additional constraint reduced the dimension of the degree of redundancy, until a single point in joint space remained, corresponding to the new desired state. By applying inequality constraints instead of equality



Fig. 7. Parts of the middle plot of Figure 6. Note how the bound $b_2 = -0.002$ is approached exponentially in a worst case scenario where this produces the best decrease in f_1 . Then, slightly after t = 57 the, optimization produces a rapid improvement in the singularity measure. Note also how a smaller k = 0.1produces a softer convergens (red dashed) than the larger k = 0.5 (blue solid).

constraints, each additional constraint is instead a new boundary surface in joint space. Thereby, the degree of redundancy is no longer coupled to the number of constraints, and often, a subset of joint space remain as the feasible set. We then formulate an optimization problem in order to move the system to the most attractive part of that subset.

Future work includes extending and implementing the proposed approach in the dual arm manipulator shown in Figure 1. We are also adding equality constraints and explicit time dependencies to the approach in a way that is completely analogous to the inequalities of this paper.

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