

HYPER-REALITY in Physics

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www.bodysoulmath.org, www.fenics.org, www.icarusmath.com

THE CLOCK AND THE ARROW

- A BRIEF HISTORY OF TIME
- SCIENCE-FICTION
- DIALOG: PHIL MAT
- SCIENCE
- PHILOSOPHY
- ARTS
- LITERATURE

WHY IS TIME MOVING FORWARD?



- What do you get every day but cannot keep?
- What do you have but cannot touch?

BODY&SOUL: www.bodysoulmath.org

Solve

$$\dot{u} = f(u)$$

- Derivative, integral, lin alg, Gauss, Stokes
- Compute general ODE
- Compute general PDE: Poisson, heat, wave, convection, Maxwell
- Compute Euler/Navier-Stokes, Schrödinger...
- Turbulence...Computer Games...

BOOKS

- Vol 1: Derivatives and Geometry in \mathbb{R}^3 2003
- Vol 2: Integrals and Geometry in \mathbb{R}^n 2003
- Vol 3: Calculus in Several Dimensions 2003
- Vol 4: Comp Turbulent Incompress Flow 2007
- Vol 5: Computational Thermodynamics 2008
- Vol 6. THE CLOCK and the ARROW 2008
- Vol 7: Many-Minds Relativity 2008
- Vol 8: Computational Quantum Mechanics 2009
- Vol 9: ...

FENICS: AUTOMATION of CC

- G2 General PDE
- FEniCS Form Compiler
- Adaptivity–Duality
- A Posteriori Error Control
- Optimization- Control
- www.fenics.org

ICARUS: www.icarusmath.com

- Web version of BODY&SOUL
- Computer Game = CC
- DEMO: Crash Course Thermodynamics

COMPUTER GAMES

- $\dot{u} = f(u)$
- Interactive Model
- Input: Data
- Output: Solution
- CONTROL
- Stimulate Students: Active Learning

SUMMARY

- COMPUTATIONAL CALCULUS
- COMPLEX MODELING: TURBULENCE
- HYPERREALITY: SIMULATION
- KNOWLEDGE SOCIETY
- REFORM? WHEN?

THE ARROW: DIRECTION of TIME

- WHY IS TIME MOVING FORWARD?
- WHY NO REWIND BUTTON?
- NEW ANSWER:
- FINITE PRECISION COMPUTATION
- WORLD: CLOCK of FINITE PRECISION
- WORLD: ANALOG COMPUTER

EQUATIONS WITHOUT SOLUTION

- EULER-LAGRANGE-SCHRÖDINGER
- EXACT SOLUTIONS DO NOT EXIST
- COMPUTATIONAL SOLUTIONS DO EXIST
- APPROX OF NON-EXISTING EXACT SOL
- SIMULATIONS OF NON-EXISTING REALITY

■ HYPERREALITY

EULER EQUATIONS

- AIR/WATER
- VOLUME Ω
- TIME INTERVAL I
- SMALL VISC HEAT CONDUCTIVITY
- DENSITY ρ
- MOMENTUM $m = \rho u$
- VELOCITY $u = (u_1, u_2, u_3)$
- INTERNAL ENERGY e

CONS: MASS, MOM, INT ENERGY

Find ρ , m and e such that in $\Omega \times I$

$$\dot{\rho} + \nabla \cdot (\rho u) = 0$$

$$\dot{m} + \nabla \cdot (mu) + \nabla p = \text{force}$$

$$\dot{e} + \nabla \cdot (eu) + \gamma e \nabla \cdot u = \text{heatsource}$$

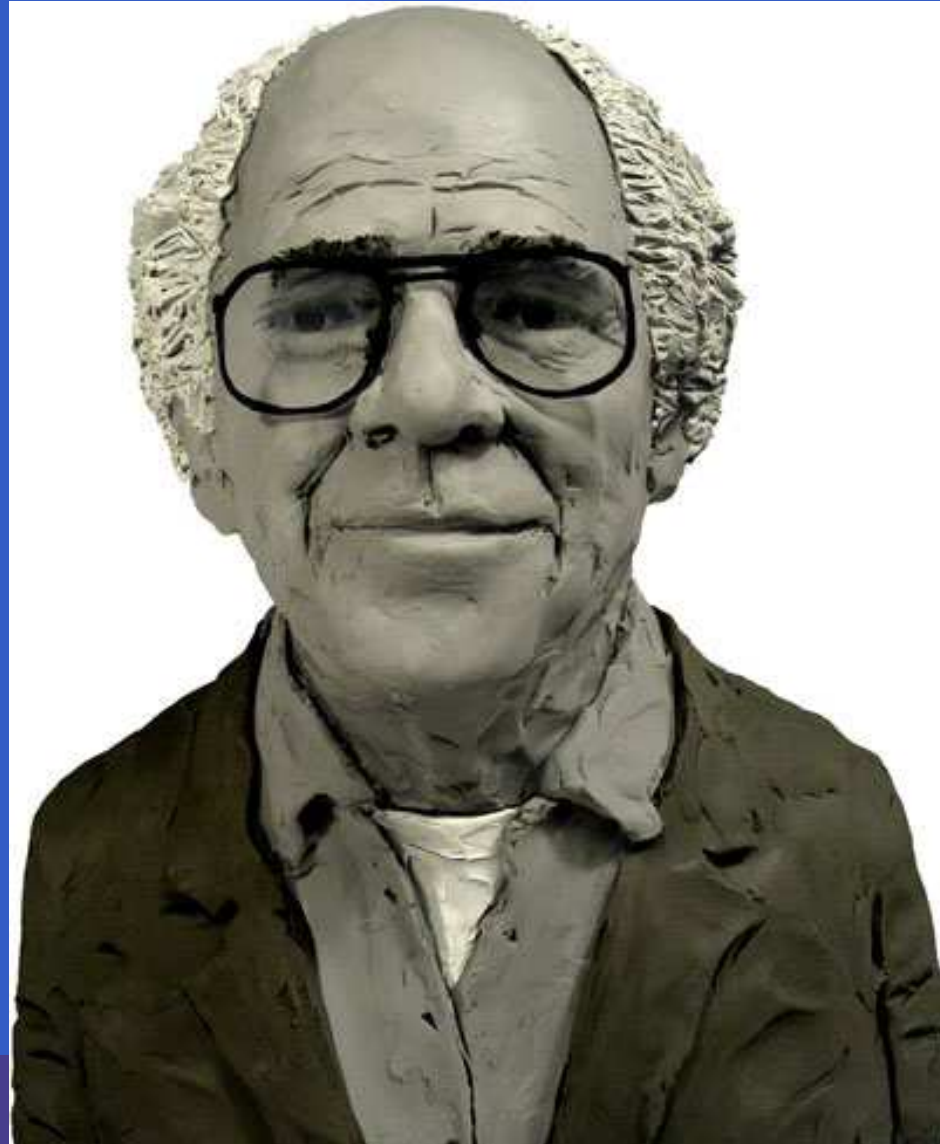
initial/boundary condition

- $\dot{v} = \frac{\partial v}{\partial t}$ TIME DERIV
- $\gamma > 0$ GAS CONSTANT

BAUDRILLARD (1929-2007)

- REAL = what can be reproduced
- HYPER-REAL = what is already reproduced
- SIMULATION of NON-EXIST REALITY
- MODELS of REAL without REAL ORIGIN
- MASKS NON-EXIST of REAL REALITY

SIMULATION of BAUDRILLARD



1ST-2ND ORDER SIMULATION

- BORGES
- EXACTITUDE in SCIENCE
- MAP COVERS TERRITORY

3RD ORDER SIM: HYPERREAL

- MAP REPLACES TERRITORY
- OUTSIDE REALM of GOOD and EVIL
- ONLY PERFORMATIVITY COUNTS
- CYBERNETICS CONTROL
- ALEATORY CHANCE

BANK ROBBERY: GOOD-EVIL

- REAL: PUNISHED for BEING REAL
- SIMULATED: NOT PUNISHED for being SIMULATION
- SIMULATED: PUNISHED for UPSETTING JUDICIARY SYSTEM

DISNEYLAND

- IMAGE of
- AMERICAN SOC NEVER EXISTING
- MASKS NON-EXIST of REAL REALITY
- REPLACES REAL
- MODELS of WANTED REALITY

WATERGATE PROCESS

- MASKS NON-EXISTENCE of
- NON-CORRUPT AMERICAN SOC
- SCANDAL:
- NOT BREAK-IN
- NOT COVER-UP of BREAK-IN
- RETURN to ORDER: FORD replaces NIXON
- ILLUSION of NON-CORRUPT SYSTEM

MAGRITTE



BARBIE DOLL



The GULF WAR did not take place

- CNN REPORTERS WATCH CNN NEWS
- CNNs WAR DID NOT HAPPEN
- NEWS GENERATED by NEWS

BAUDRILLARD

- REAL:
- Why is there SOMETHING, rather than NOTHING?
- THE REAL NO LONGER EXISTS
- HYPERREAL:
- Why is there NOTHING, rather than SOMETHING?

DELEUZE (1925-1995)

- HYPER-REALITY:
- the ONLY REALITY there is

REFLECTIONS of DELEUZE



MODERN vs POSTMODERN

- MODERN: OBJ EXIST REAL WORLD
- POST-MODERN:
- HYPERREAL SIMULACRA of
- NON-EXIST REAL WORLD

SECOND LIFE

- COMPUTER GAMES
- DOCU-SOAP
- PORNOGRAPHY

BIBLE

- God created man in His own image, in the image of God He created him; male and female He created them.
- EXISTENCE of GOD?
- HUMAN BEING HYPERREAL?

DIJKSTRA

- Originally I viewed it as the function of the abstract machine to provide a truthful picture of the physical reality. Later, however, I learned to consider the abstract machine as the *true* one, because that is the only one we can *think*; it is the physical machine's purpose to supply a *working model*, a (hopefully) sufficiently accurate physical simulation of the true, abstract machine.

HYPERREAL PHYSICS

- SPACE-TIME
- STATISTICAL MECHANICS
- QUANTUM MECHANICS

HYPERREAL PHYSICS

- SIMULATION of
- NON-EXISTING PHYSICS
- APPROXIMATIONS of
- NON-EXISTING EXACT SOLUTIONS

HYPERREAL SOCIETY

- APPROXIMATION of
- NON-EXISTING PERFECT SOCIETY



CLAY INST \$1 MILLION PRIZE

- EXISTENCE of
- EXACT SOLUTIONS of
- EULER NAVIER-STOKES EQUATIONS?

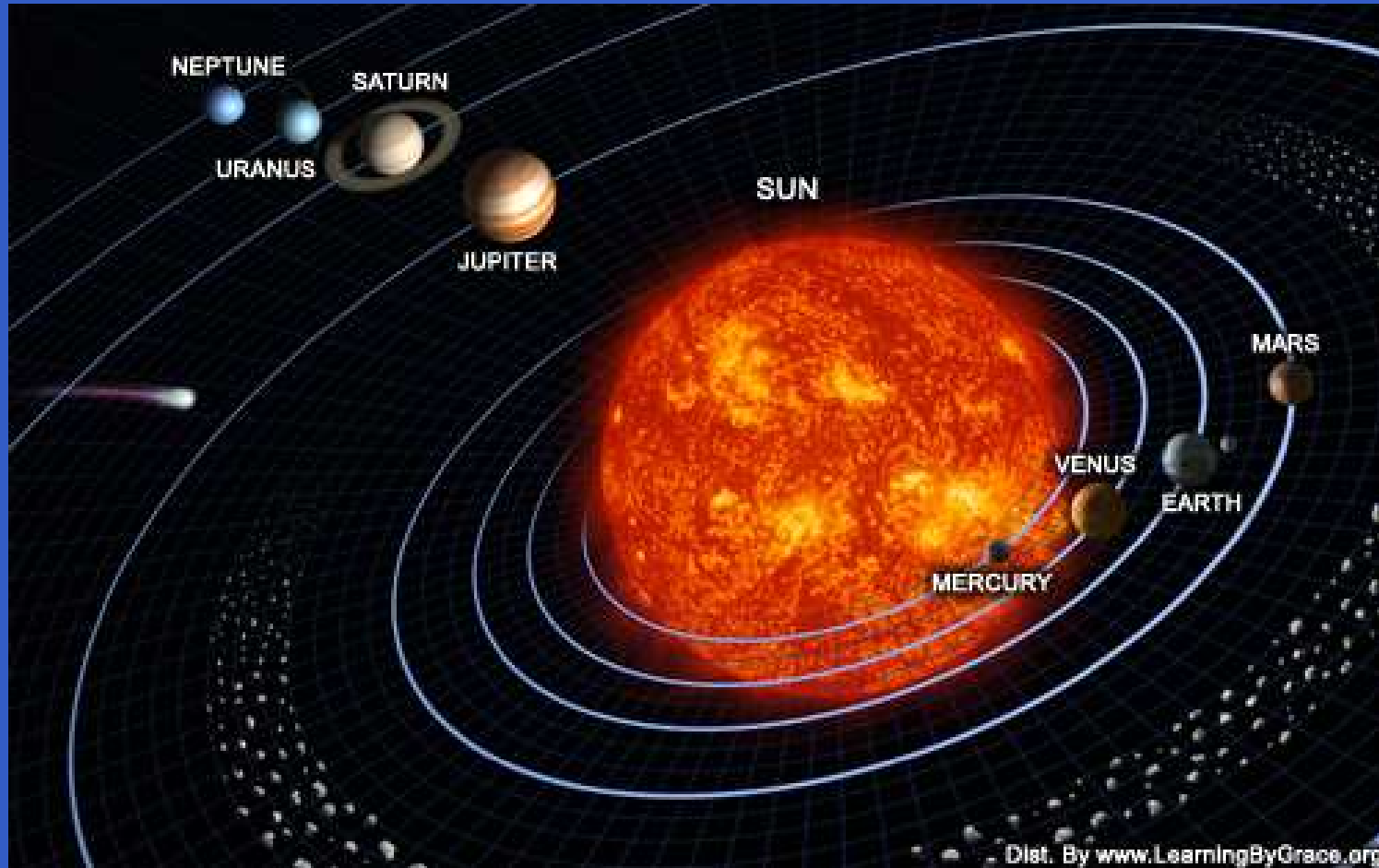
PERSPECTIVE: Three Periods

- CLASSICAL 1600-1900
- MODERN 1900-2000
- POST-MODERN 2000-

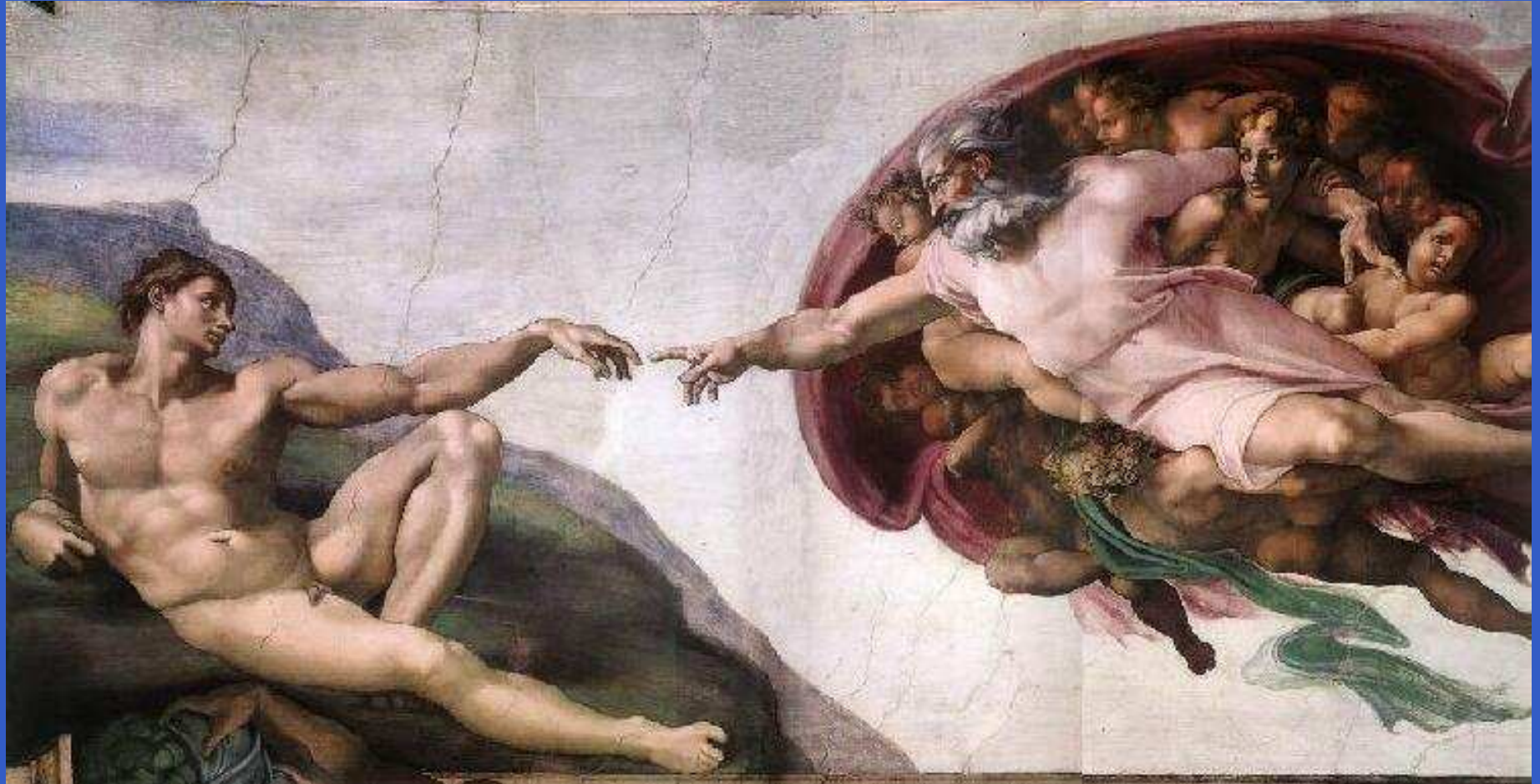
Classical 1600-1900

- Mathematics: Calculus: Analytical Solution
- Physics: Newtonian Mechanics
- Industrial Society: Mass Production
- Leibniz Newton Euler Lagrange Laplace...
- Main Challenges:
- N-Body, Heat, Wave, ElectroMagnetism

Solar System



Local Interaction: Aristotle



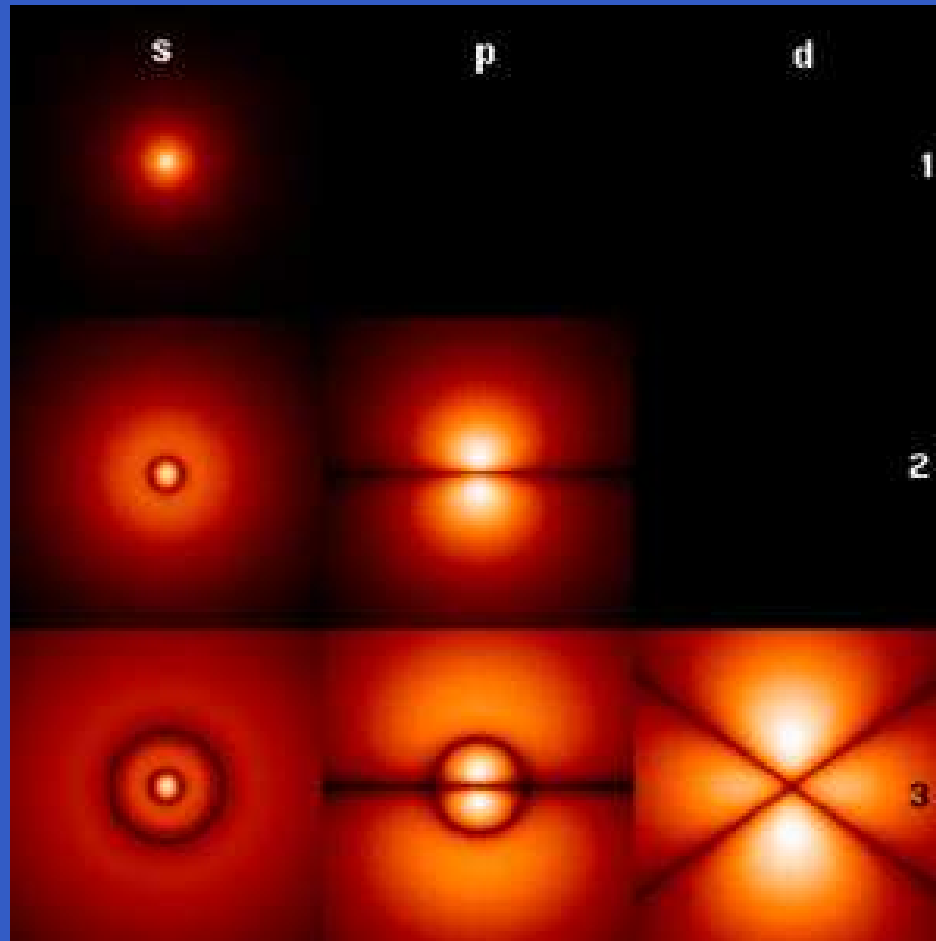
Action at Distance: Newton



Modern 1900-2000

- Mathematics: Calculus: Existence of Solution
- Physics: Quantum Mechanics Relativity
- Service Society
- Hilbert Courant von Neumann Lions Lax...
- Main Challenges: Turbulence, Quantum Mech
- Why is there something (rather than nothing)?
- BIG BANG?

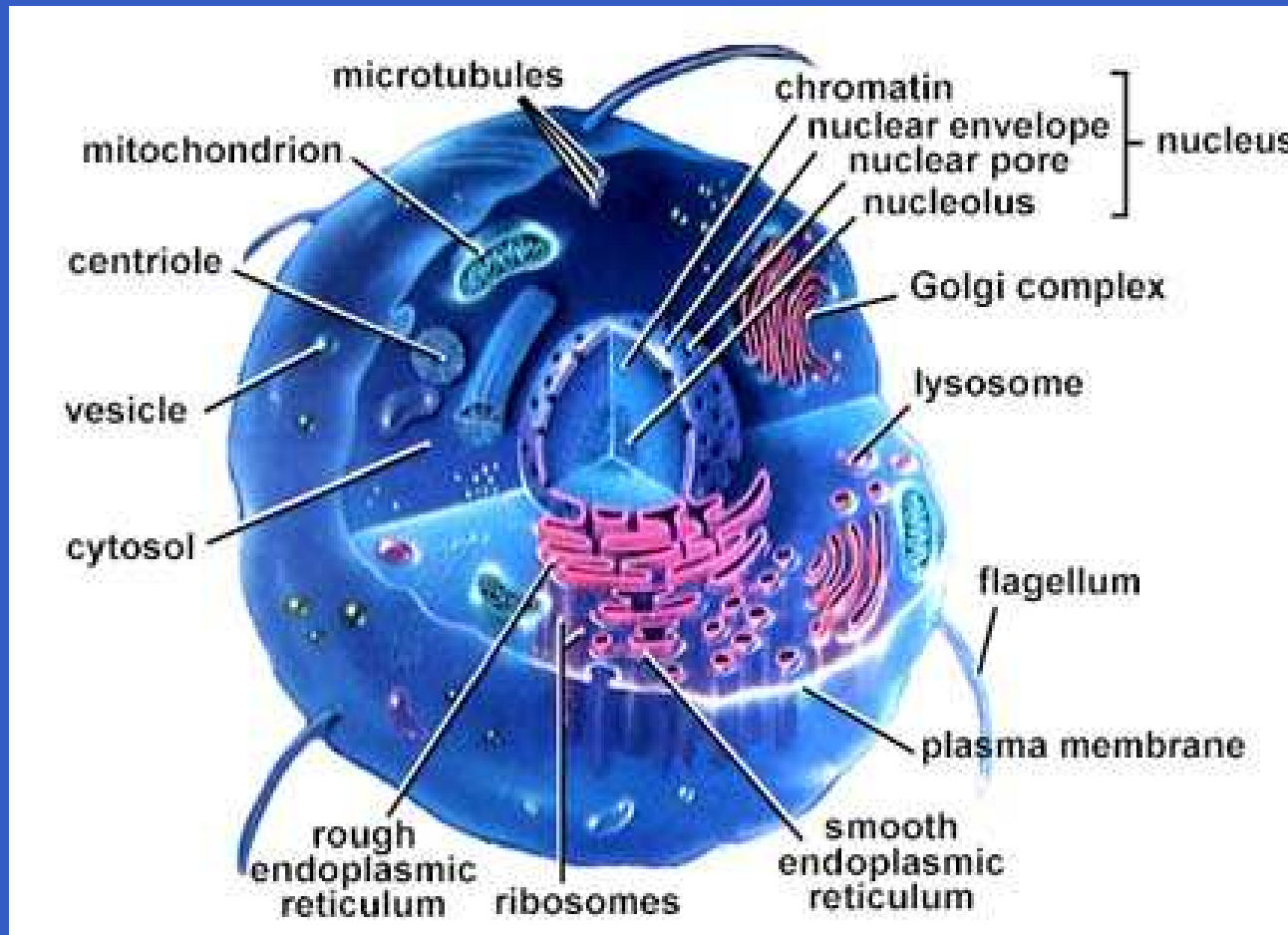
Schrödinger Equation: Electron Density



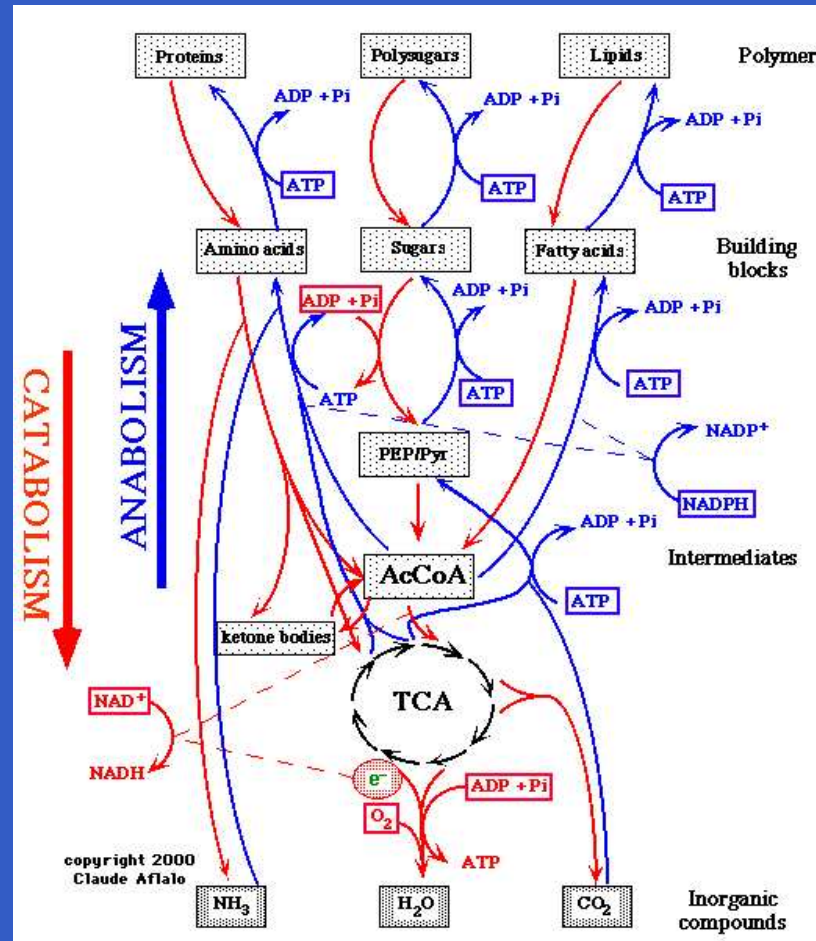
Post-Modern 2000–

- Mathematics: COMPUTATIONAL CALCULUS
- COMPUTATIONAL SOLUTION
- Physics: Nano-Micro-Bio—Cosmology
- INFORMATION SOCIETY:
- SIMULATION—VIRTUAL REALITY
- Why is there nothing rather than something?

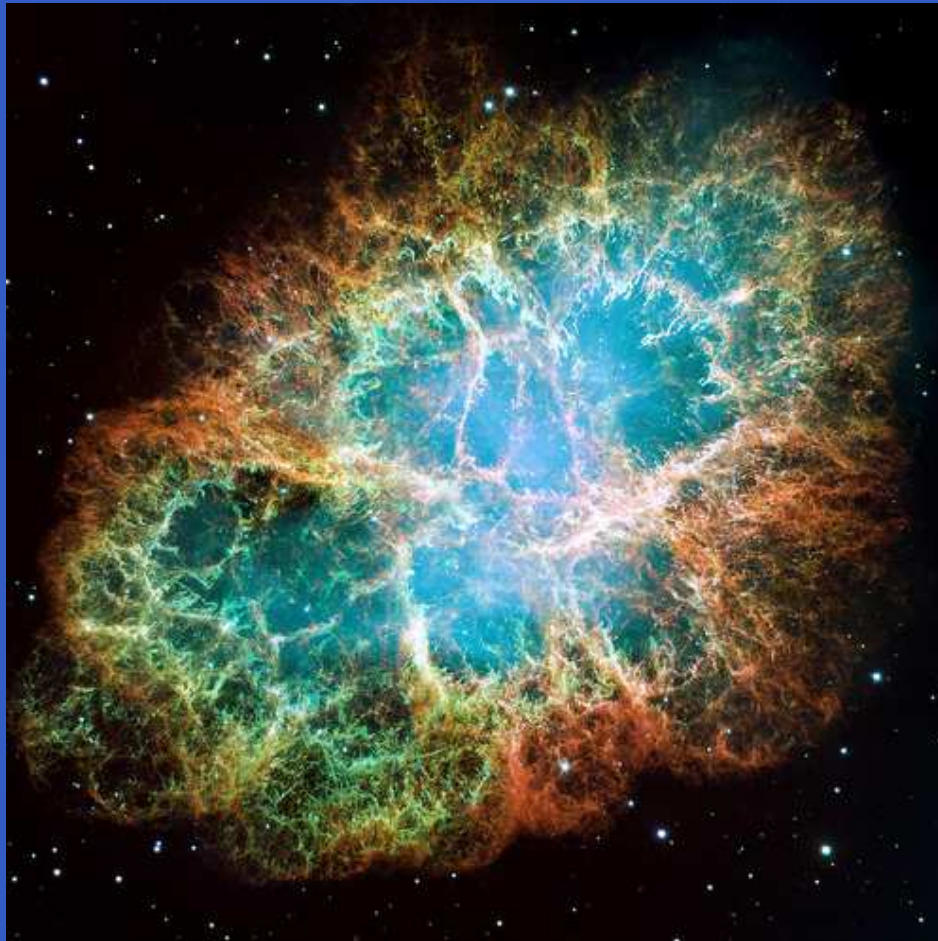
CHALLENGE: LIFE



CHALLENGE: LIFE



CHALLENGE: BIG BANG



MODERN PHYSICS

- The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy. (Stephen Weinberg)
- What exactly is the meaning of time and its directionality – the “arrow of time”? Has it something to do with quantum theory, or does it arise at some other level? (David Peat in *Superstrings and the SeaRch for the Theory of Everything*, 1988)

TRAGEDIES of MODERN PHYSICS

- ENTROPY/STATISTICAL MECHANICS
- SUPERPOSITION of QM
- COPENHAGEN INTERPRET of QM
- SPACE-TIME of RELATIVITY

STATISTICAL MECHANICS

- ATOMS PLAY ROULETTE
- MICROSCOPIC GAMES of ROULETTE
- PSEUDO-SCIENCE
- IMPOSSIBLE TO DISPROVE/UNDERSTAND
- MICROSCOPICS of MICROSCOPICS

QUANTUM MECHANICS

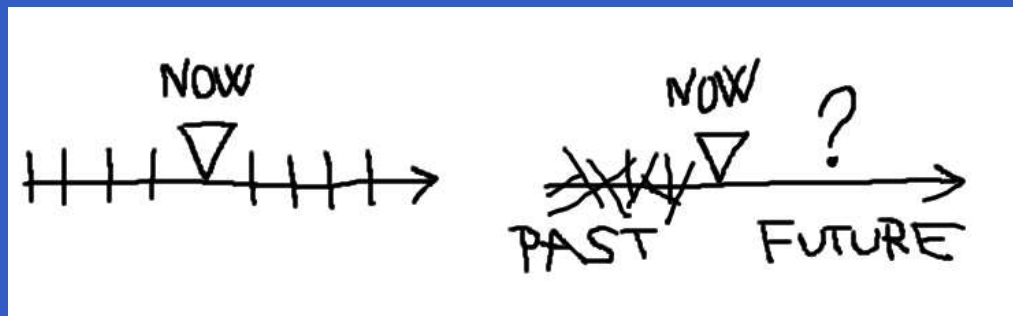
- WAVE FUNCTION $\Psi(t, r_1, r_2, \dots, r_n)$
- N ELECTRONS
- $3N$ SPACE DIM
- SCHRÖDINGER EQ: $i\frac{\partial\Psi}{\partial t} = H\psi$
- H HAMILTONIAN
- LINEAR-SUPERPOSITION

WAVE FUNCTION DOES NOT EXIST

- WALTER KOHN
- NOBEL PRIZE CHEMISTRY 1998
- Ψ DOES NOT EXIST if $N \geq 100$
- Ψ MONSTER
- INSTEAD ELECTRON DENSITY 3-DIM
- SUPERPOSITION FAILS
- SCHRÖDINGERS CAT DEAD OR ALIVE
- NOT DEAD/ALIVE SUPERPOSITION

TIME

- TIME-LINE 0 1 2 3 4 5 6 ...
- LINEAR TIME
- ORDERED
- PAST-PRESENT-FUTURE
- VIDEO-FILM



ROBINSON CRUSOE



RELATIVITY SPACE-TIME

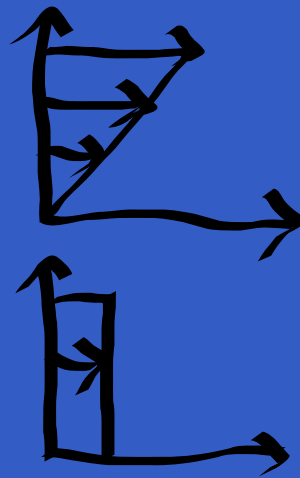
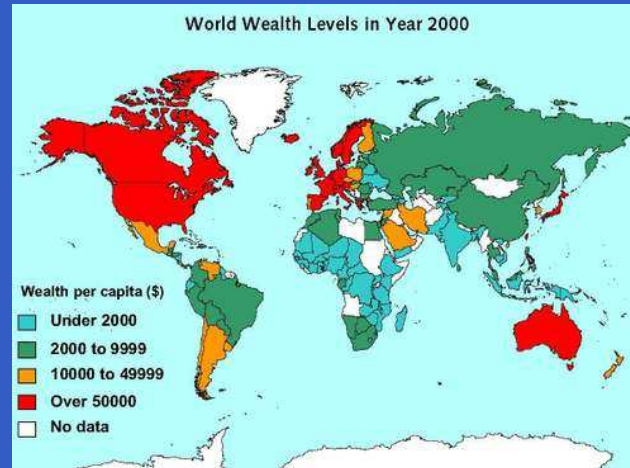
- TIME DIM LIKE SPACE DIM
- ARROW of TIME?
- DIRECTION of TIME?



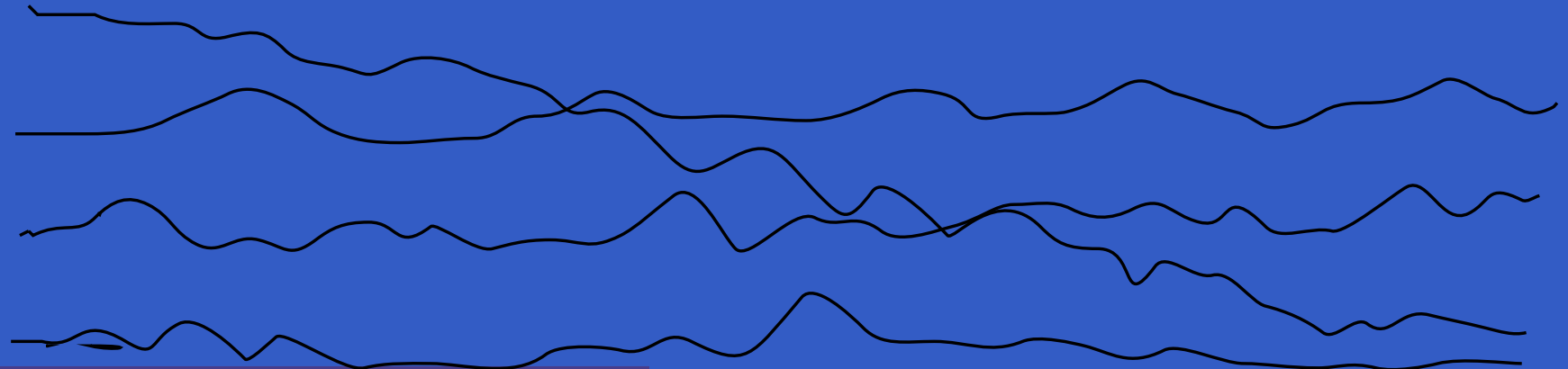
CAPITALISTIC SYSTEM

- INVISIBLE HAND
- SHARPEN DIFFERENCE
- UNSHARPEN DIFFERENCE
- TURBULENCE-CHAOS

CAPITALISTIC SYSTEM



MIGRATION



HYPERREALITY IN PHYSICS

- THE ARROW
- DIRECTION OF TIME
- IRREVERSIBILITY

CLOCK I: INFINITE PRECISION

- LAPLACE: THE WORLD as a CLOCK
- INITIAL VALUE PROBLEM
- MATHEMATICS MECHANICS
- EXACT SOLUTIONS
- DETERMINISM
- NO FREE WILL
- REVERSIBLE

CLOCK II: FINITE PRECISION

- DIGITAL COMP: FINITE PRECISION
- THE WORLD as an ANALOG COMPUTER
- FINITE PRECISION COMPUTATION
- FREE WILL POSSIBLE
- NO STATISTICS
- IRREVERSIBLE

REAL-HYPERREAL

- REAL: II APPROX OF I
- HYPERREAL: II REPLACES NON-EXIST I

REVERS HAMILT SYST I

- PARTICLE SYSTEM
- HARMONIC OSCILLATOR: $\ddot{u} + u = 0$
- QUANTUM MEECHANIC
- TIME REVERSIBLE
- Invariant: $t \rightarrow -t, u \rightarrow -u$
- IRREVERSIBILITY FROM WHERE?

FEYNMAN



FEYNMAN LECTURE NOTES

- *Where does irreversibility come from? It does not come from Newton's laws. Obviously there must be some law, some obscure but fundamental equation. perhaps in electricity, maybe in neutrino physics, in which it does matter which way time goes.*

2ND LAW of THERMODYNAMICS

- ENTROPY CANNOT DECREASE
- INCREASING ENTROPY–IRREVERSIBILITY
- INCREASING TIME
- ARROW
- HEAT DEATH: ORDER \rightarrow DISORDER
- WHAT IS ENTROPY? ORDER? DISORDER?

THERMODYNAMICS



■ KINETIC ENERGY → HEAT ENERGY

MATH vs THERMODYNAMICS

- Every mathematician knows it is impossible to understand an elementary course in thermodynamics. (V. Arnold)
- ...no one knows what entropy is, so if you in a debate use this concept, you will always have an advantage. (von Neumann to Shannon)
- As anyone who has taken a course in thermodynamics is well aware, the mathematics used in proving the 2nd Law is of a very special kind, having only the most tenuous relation to that known to mathematicians. (S. Brush, *The Kind of Motion we call Heat*)

THERMODYNAMICS

- Thermodynamics is a funny subject. The first time you go through it, you don't understand it at all. The second time you go through it, you think you understand it, except for one or two small points. The third time you go through it, you know you don't understand it, but by that time you are so used to it, it doesn't bother you any more. (Sommerfeld)

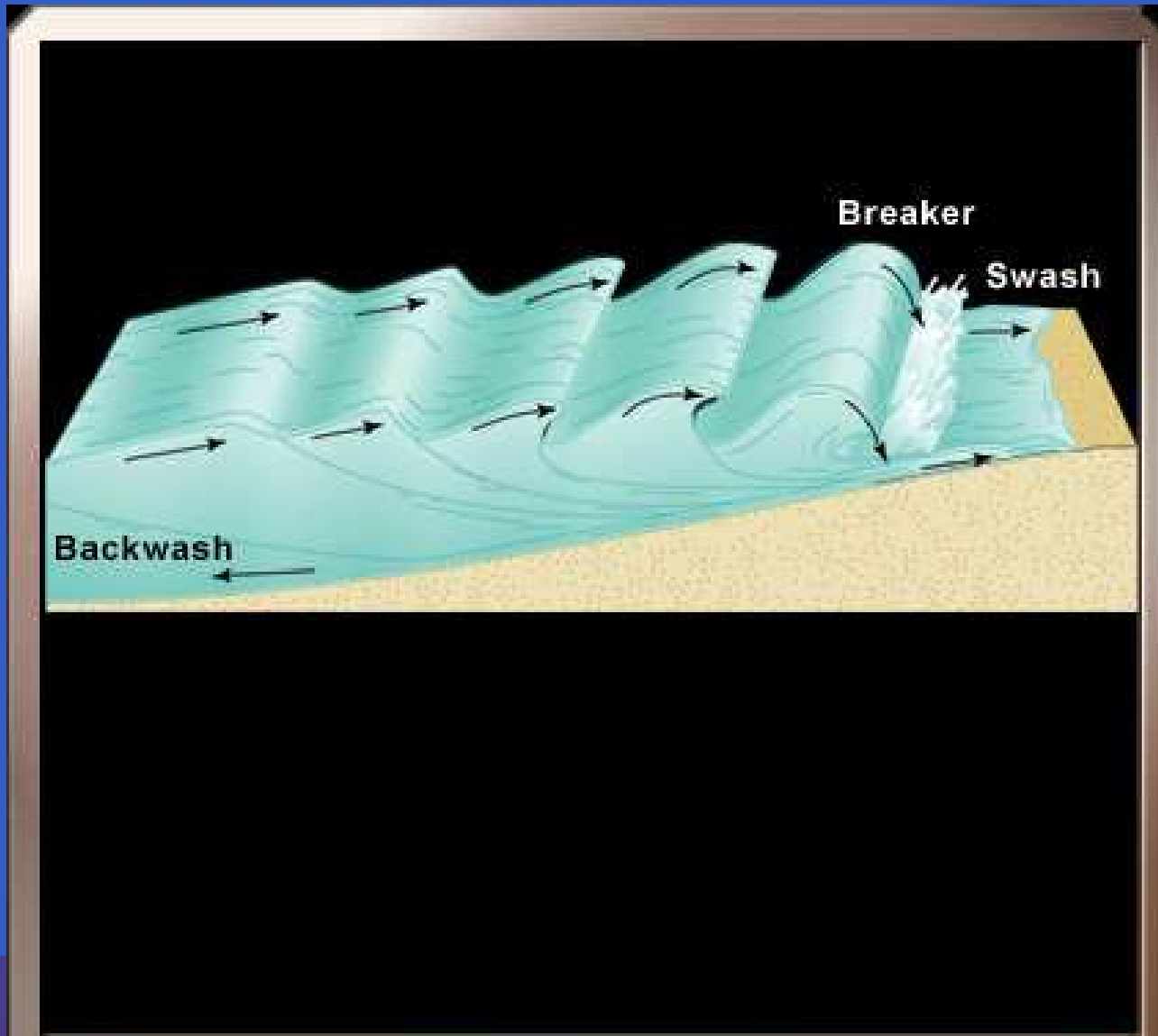
ENIGMA

- IRREVERSIBILITY in REVERSIBLE SYST
- NEWTON'S EQ REVERSIBLE
- PENDULUM REVERSIBLE
- CLOCK REVERSIBLE
- QUANTUM MECHANICS REVERSIBLE
- IRREVERSIBILITY FROM WHERE?
- HYPERREALITY??

BREAKING WAVE



BREAKING WAVE



CLASSICAL 2ND LAW

- CARNOT 1824:
- EFF of HEAT ENGINE $\leq 1 - T_{cold}/T_{hot}$
- CLAUSIUS 1850:
- Heat cannot by itself flow from cold to hot
- CLAUSIUS 1865:
- ENTROPY CANNOT DECREASE
- WHY? ENTROPY?
- MAXWELL, GIBBS, BOLTZMANN

STATISTICAL MECHANICS

- BOLTZMANN'S ASSUMPTION:
- MOLECULAR CHAOS:
- Velocities independent BEFORE COLLISION
- H-THEOREM: ENTROPY cannot decrease
- Irreversibility BY ASSUMPTION
- Loschmidt: ASSUMES what is to be proved
- *To derive 2ND LAW from stat mech has so far eluded the deepest thinkers (Lieb 1999)*

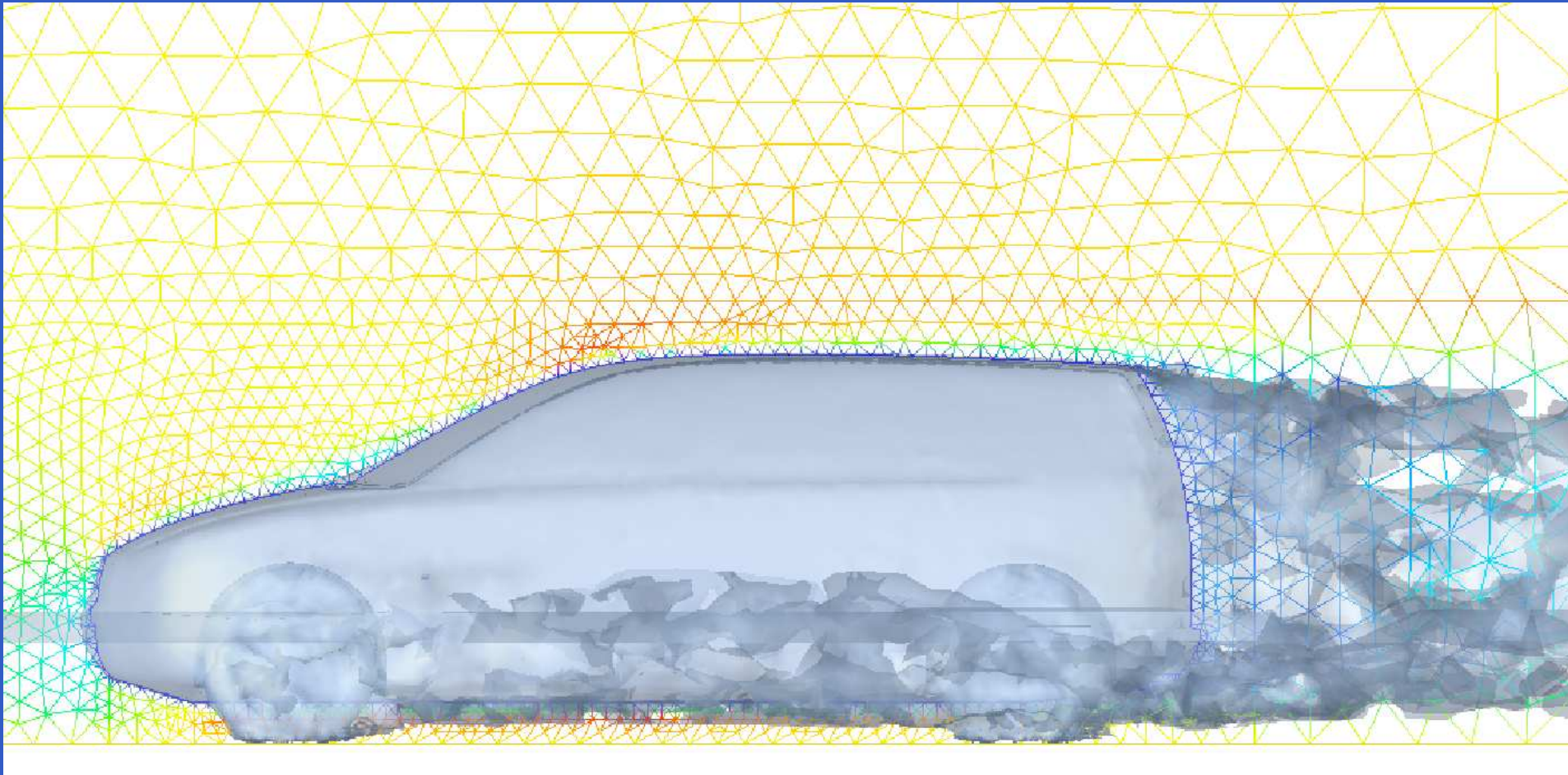
OBJECTIVE

- Thermodynamics WITHOUT ENTROPY!!
- Thermodynamics WITHOUT STATISTICS!!
- 2nd Law WITHOUT ENTROPY!!
- ARROW of TIME WITHOUT ENTROPY!!
- COMPUTATION instead of STATISTICS
- COMPUTATIONAL THERMODYNAMICS
- 1ST LAW + FINITE PREC → 2ND LAW

FINITE PRECISION + STABILITY

- NON-EXISTENCE of POINTWISE SOL
- TURBULENCE SHOCKS
- EDGE STABILITY
- EXISTENCE of COMPUTATIONAL SOL
- EXIST of IRREV COMPLEX WORLD
- NON-EXISTENCE of REV SIMPLE WORLD

TURBULENCE: ARROW



COMP TURB INCOMP FLOW 4

- MYSTERIES-PARADOXES:
- d'Alembert, Loschmidt, Sommerfeld, Gibbs
- SECRETS: flying, sailing, ball sports,...
- TURBULENCE
- EULER EQUATIONS
- www.bodysoulmath.org Books: Vol 4

COMP THERMODYNAMICS 5

- NEW FOUNDATION:
- COMPUTATIONAL
- DETERMINISTIC
- TURBULENCE/SHOCKS:
- NONEXISTENCE POINTWISE SOLUTIONS
- EULER EQUATIONS

COMP BLACKBODY RADIATION 6

- HIGH FREQUENCY IN
- LOW FREQUENCY OUT
- FINITE PRECISION COMPUTATION

COMP QUANTUM MECHANICS 7

- COMP SOL of SCHRÖDINGER
- HARTREE MANY-ELECTRON
- NON-EXISTENCE of
- COMPLETE WAVEFUNCTION

THE CLOCK AND THE ARROW

- A BRIEF HISTORY OF TIME
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PRINCIPE PERFEITO

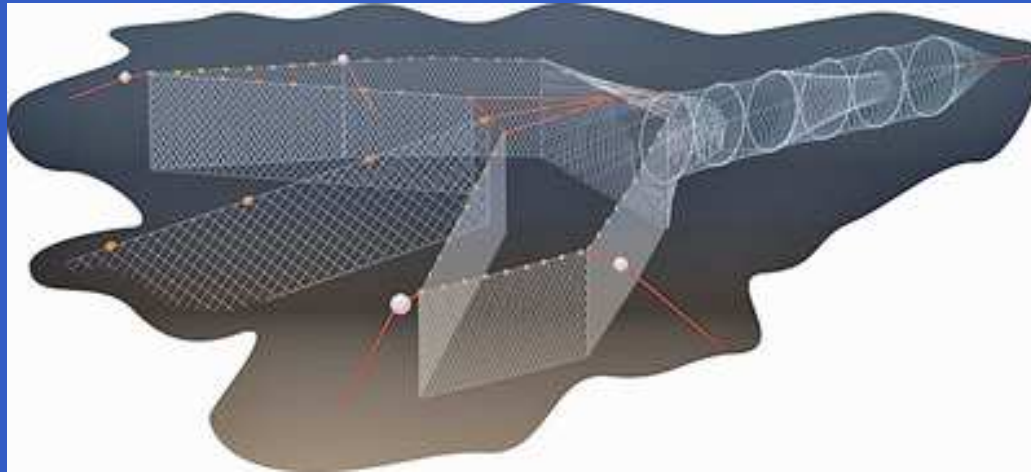
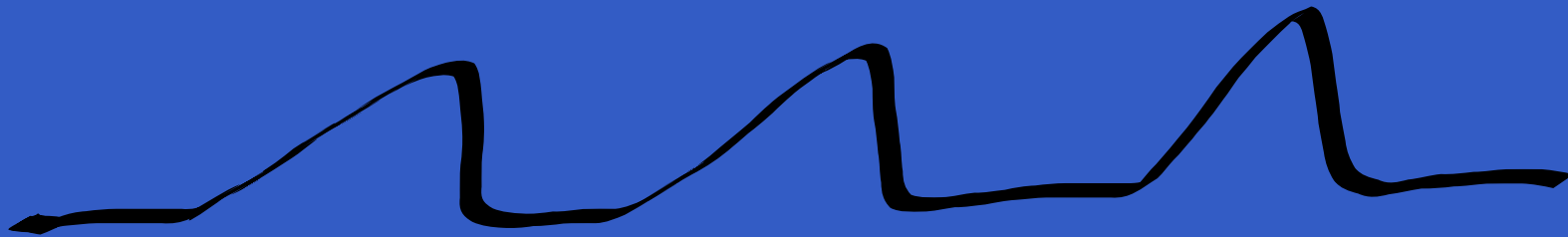
- FINITE PREC + EDGE STAB
- TURBULENCE SHOCKS
- SHARP GRADIENTS: EDGE STAB
- TURBULENT DISSIPATION: FINITE PREC
- CAPITALISM: INCOME DIFF + TAX

PRINCIPE PERFEITO



PRINCIPE PERFEITO

OK → ← NOT OK



PRINCIPE PERFEITO

- MIXING: FAST/LOW PREC
- UNMIXING: SLOW/HIGH PREC
- IMPRECISE SEP: EASY/FAST
- PRECISE SEP: DIFFICULT/SLOW
- ANABOLISM: SLOW PRECISE
- CATABOLISM: FAST IMPRECISE

PRINCIPE PERFEITO

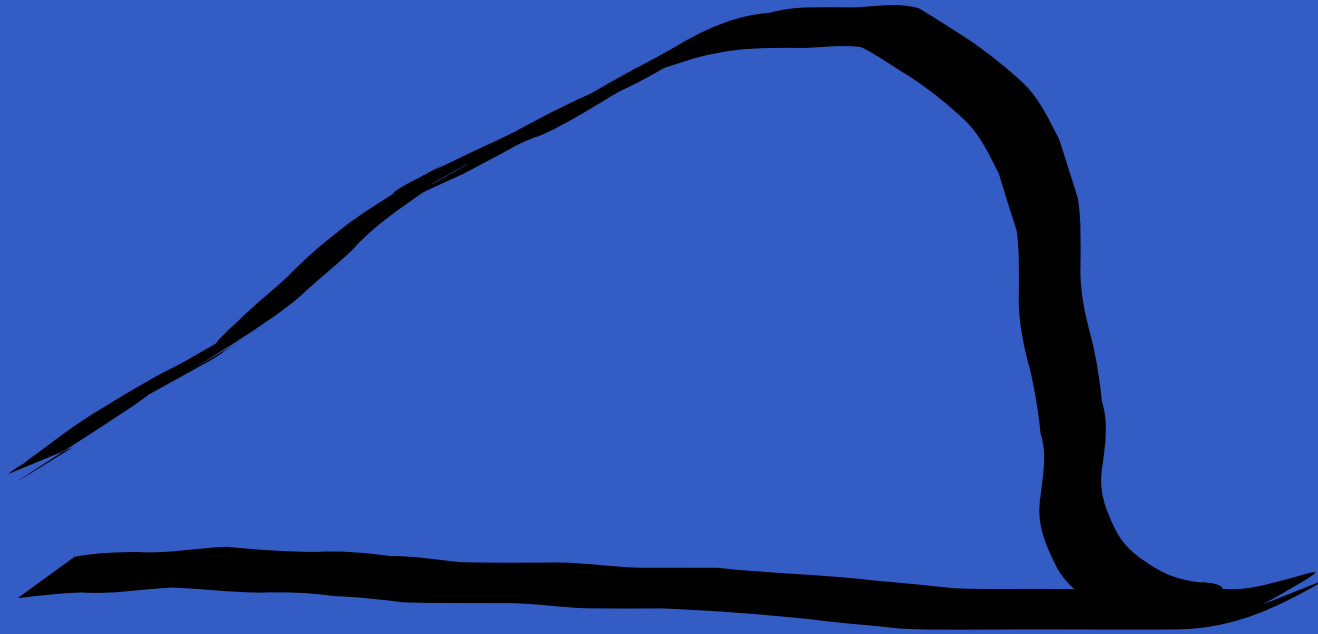


■ ANABOLISM powered by CATABOLISM

SAWTOOTH DYNAMICS: TENSION

- SLOW INCREASE – FAST RELEASE
- tragedy, detective story,
- sexual act,
- music,
- telling a funny story,
- winning the Nobel Prize
- life-death
- ARROW!

SAWTOOTH



■ SLOW BUILD-UP QUICK DECAY

SAWTOOTH POLITICS



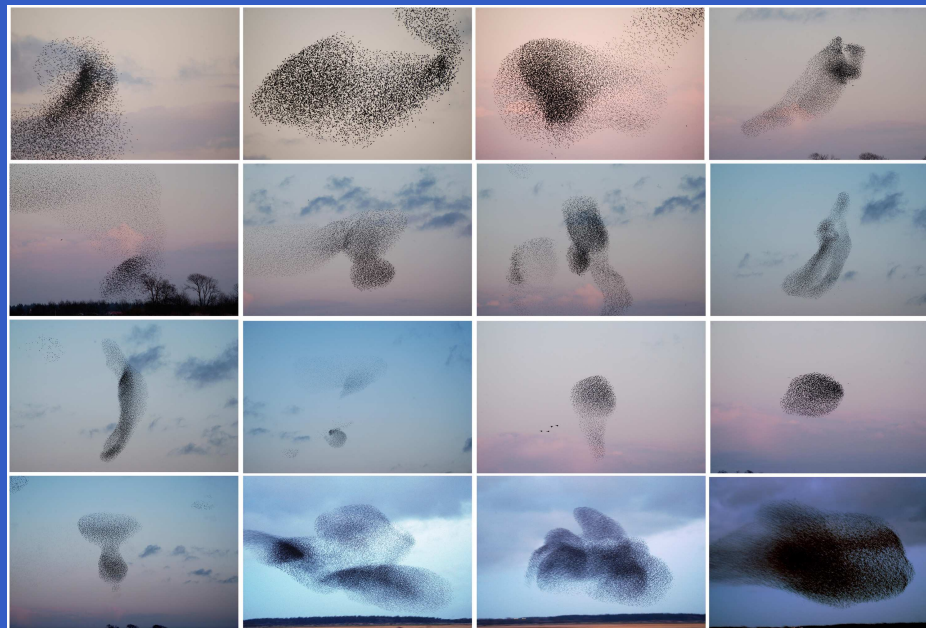
- INCREASING TENSION – REVOLUTION
- REVERSAL of TROTSKY

COMPLEXITY

- SIMPLE SYSTEM: REVERSIBLE
- COMPLEX SYSTEM: IRREVERSIBLE
- LAMINAR FLOW: REVERSIBLE
- TURBULENT FLOW: IRREVERSIBLE
- EDGE STABILITY →
- TRANSITION TO TURBULENCE
- IRREVERSIBILITY IMPOSSIBLE TO AVOID
- PERPETUM MOBILE IMPOSSIBLE

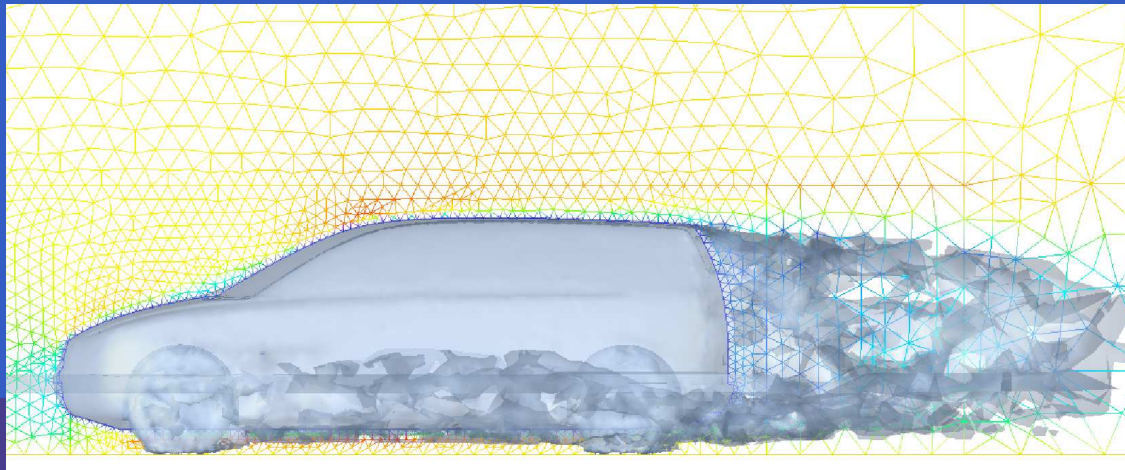
EMERGENCE Laughlin:

- simple rules \rightarrow complex structures
- mean-values stable
- point-values unpredictable



d'ALEMBERT'S PARADOX

- ZERO DRAG OF POTENTIAL FLOW
- NON-ZERO DRAG OF REAL FLOW
- RESOLUTION: Wikipedia (Vol 4)
- Potential sol UNSTABLE
- Turbulent sol develops with non-zero drag



NEW 2ND LAW without ENTROPY

- EULER EQ IDEAL PERFECT GAS
- 1ST LAW:
- CONSERVATION of
- MASS MOMENTUM ENERGY

NEW 2ND LAW without ENTROPY

$$\dot{K} - W = -D, \quad \dot{E} + W = D$$

- K Kinetic energy, E Internal/heat energy
- W Work $> 0 / < 0$ in expansion/compression
- $D > 0$ turbulent/shock dissipation
- Transfer $K \rightarrow E$ “Internal = Lost”
- Irreversibility: Arrow of Time
- K grows by expansion ONLY
- E grows by compression and turbulence
- 1ST LAW + FINITE PREC. \rightarrow 2ND LAW

NEW 2ND LAW without ENTROPY

$$\dot{K} - W = -D, \quad \dot{E} + W = D$$

- 1ST LAW + FINITE PREC \rightarrow 2ND LAW
- MULT of MOM EQ BY VEL \rightarrow KIN ENERGY
- VIOL CONS of MOM \rightarrow TURB DISS D
- TRANSFER $K \rightarrow E$
- NONEXISTENCE of EXACT SOL \rightarrow IRREV

TRANSFER $K \rightarrow E$

- Large scale kinetic energy $K \rightarrow$
- small scale kinetic energy = E = heat energy
- Change of \$100 Bill \rightarrow Coins: Possible
- Coins \rightarrow Bill: Impossible:
- Finite Precision Coordination: Impossible
- Drop a stone \rightarrow heats up
- Reverse: Lift itself by cooling off: Impossible

CLASS vs NEW 2ND Law

New 2nd Law $\dot{E} + W = D$:

$$dE + pdV = D \geq 0, \quad D \text{ turbulent dissipation}$$

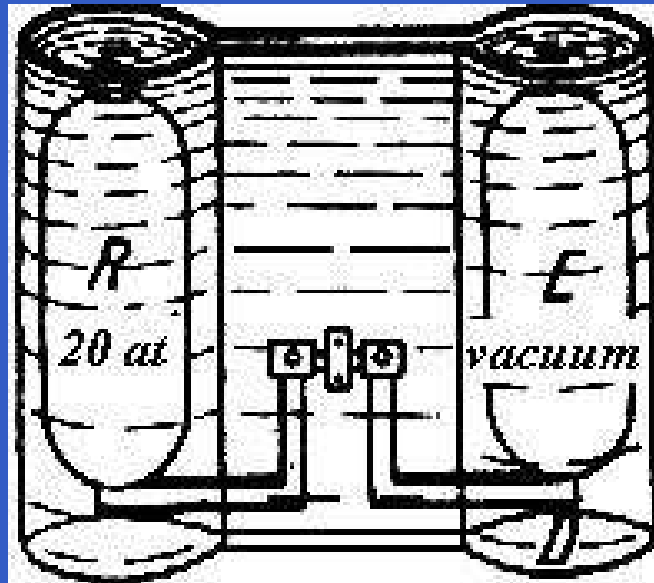
Classical 2nd Law:

$$TdS = dE + pdV = D, \quad dS = \frac{D}{T}, \quad dS \geq 0$$

New 2nd ($TdS \geq 0$) = Class 2nd ($dS \geq 0$)

WITHOUT introducing ENTROPY!!

JOULE'S EXPERIMENT 1845

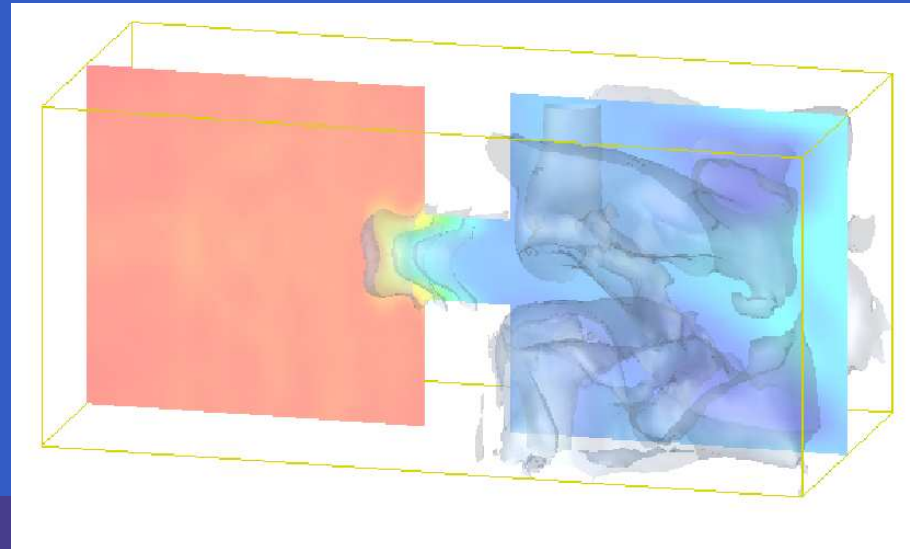
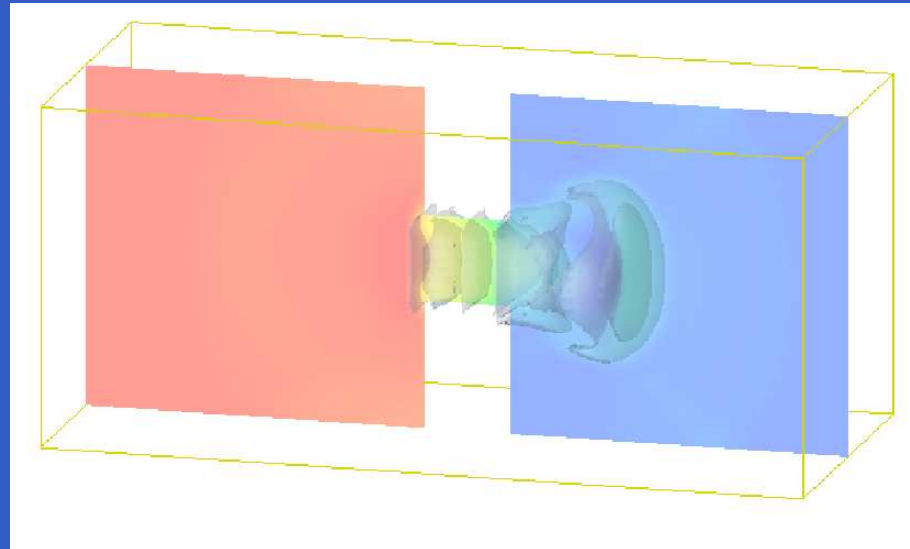


**Fig. 358 Concerning overflowing experiment of Joule (Scientific Papers).
R contains at first air compressed to 20 atm, *E* is initially a vacuum, *D* the tube**

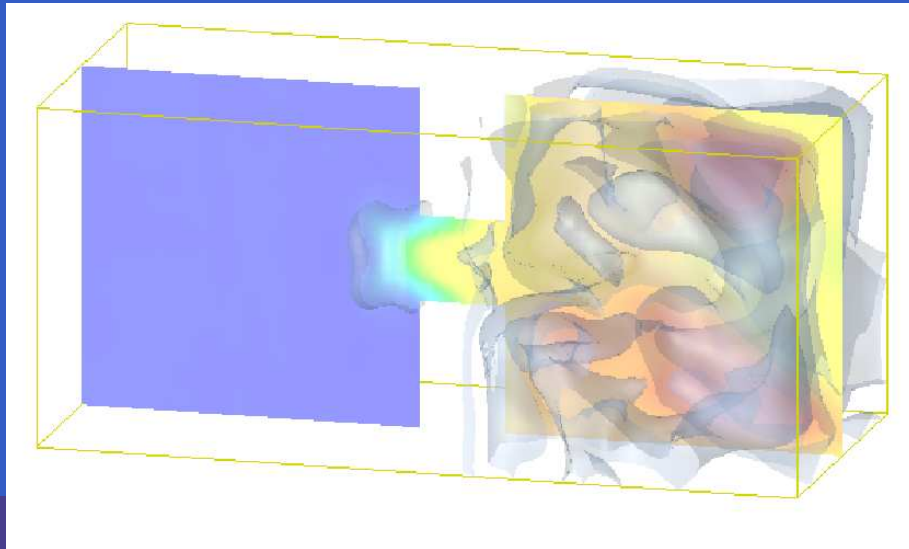
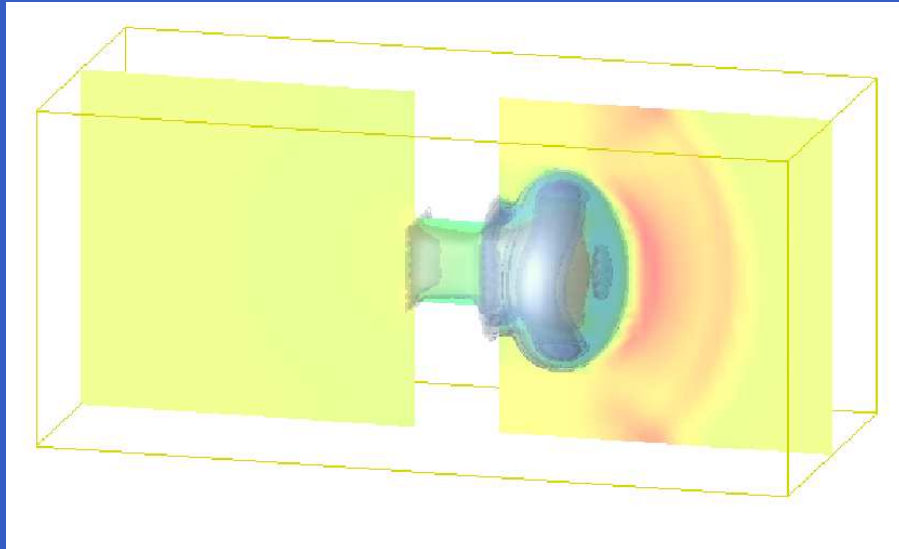
JOULE'S EXPECTATION

- $T = 1$ in both chambers
- High Pressure/Density in 1
- Gas expands from 1 into 2.
- Kinetic energy K increases
- Temperature T drops < 1
- Finally $T = ?$

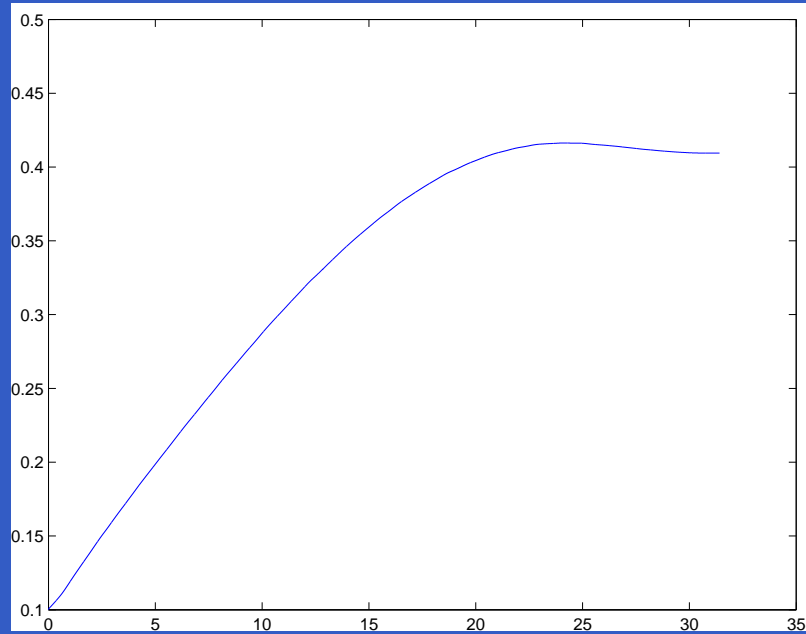
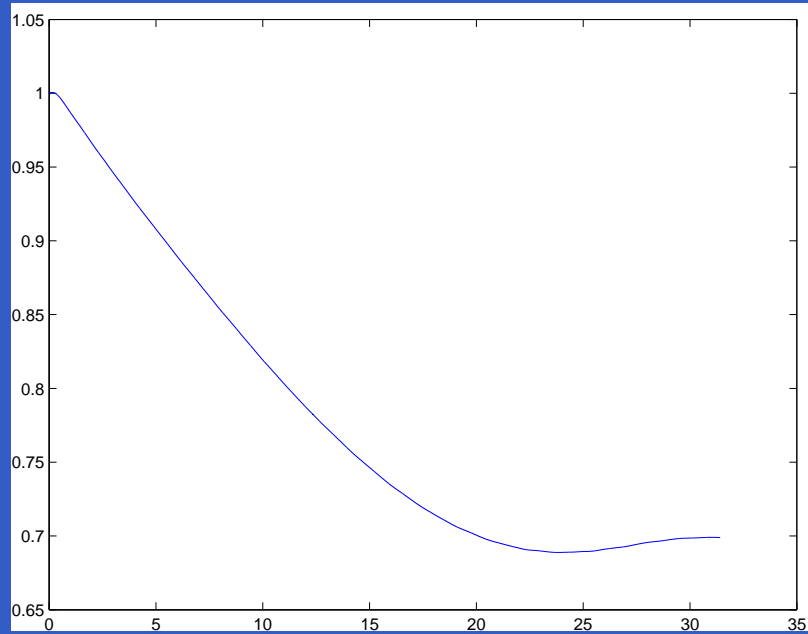
Density at two times



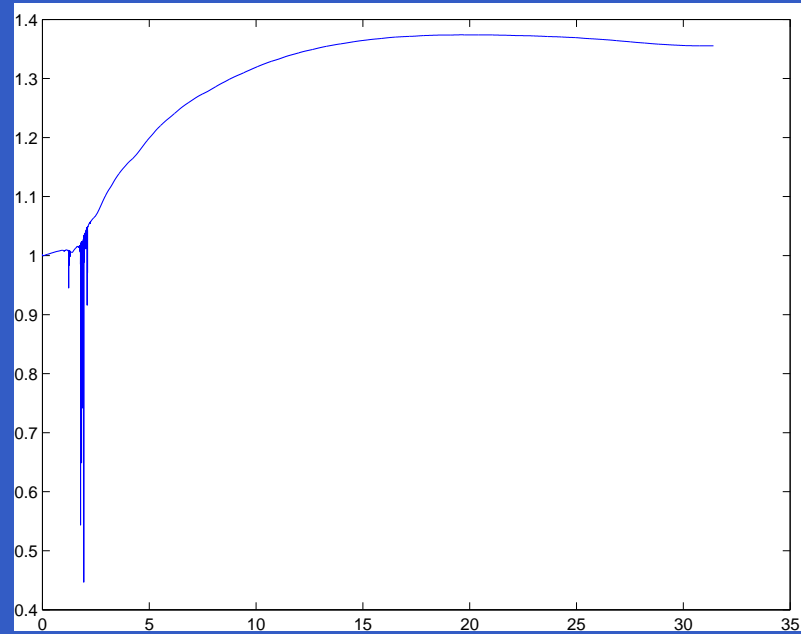
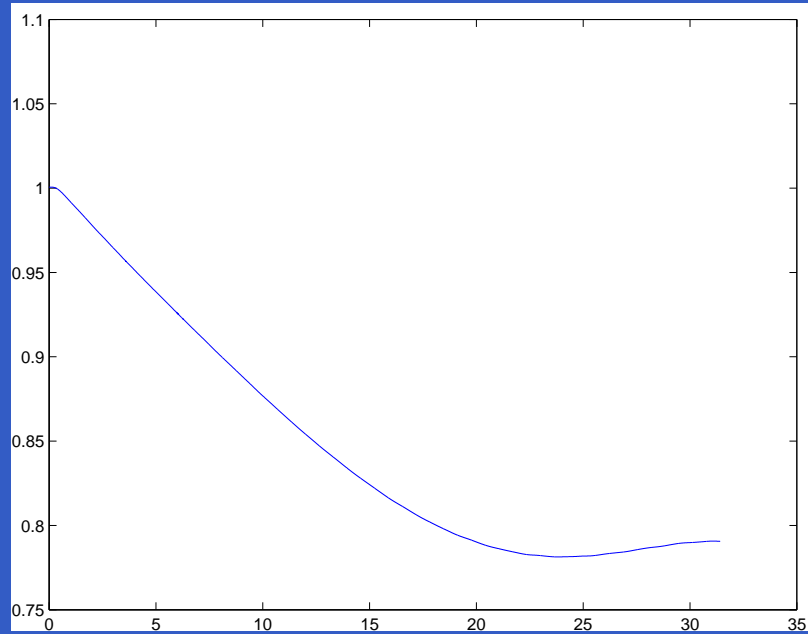
Temperature at two times



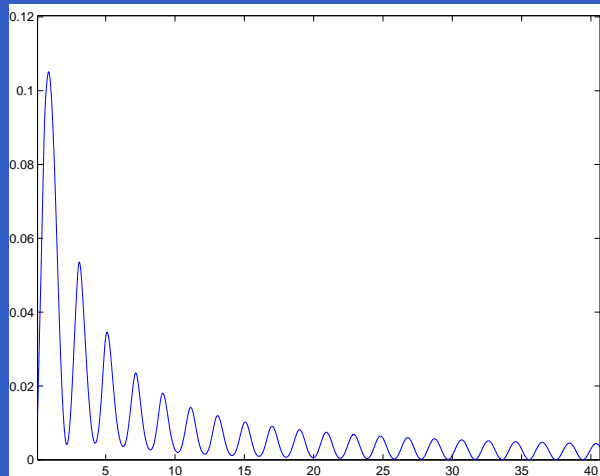
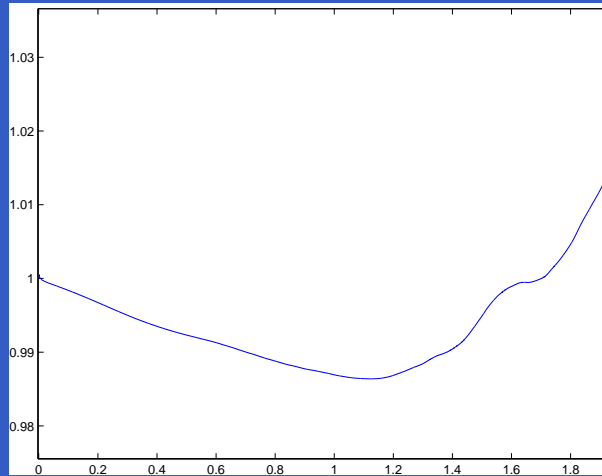
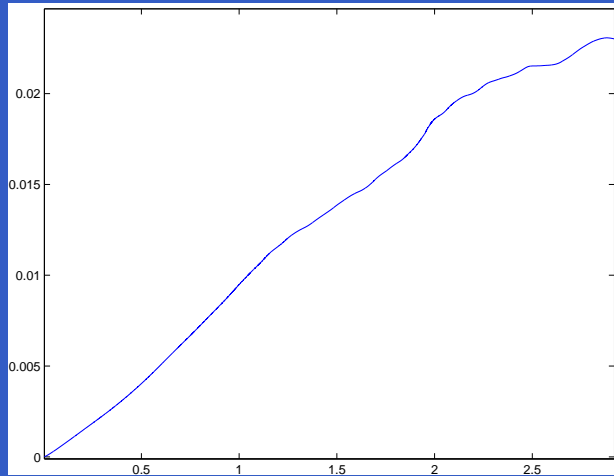
Average Density in Left/Right Chamber



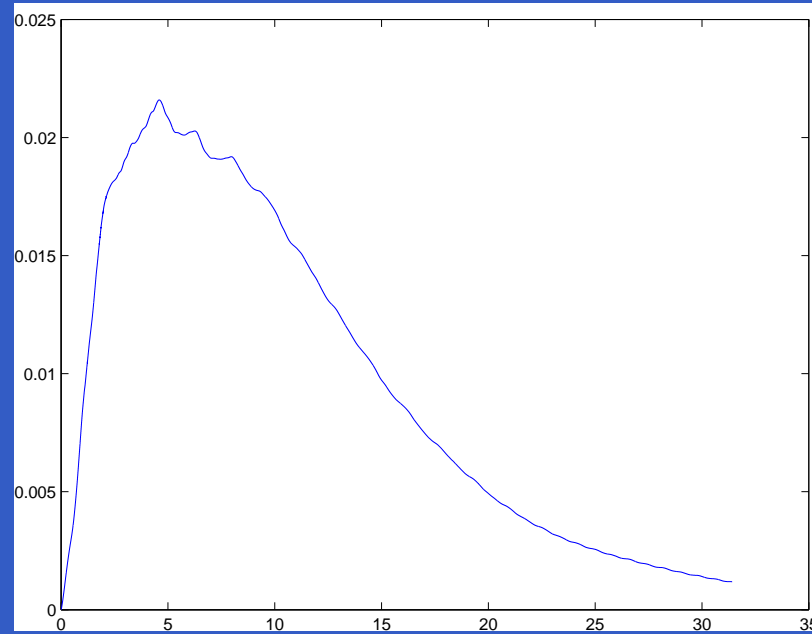
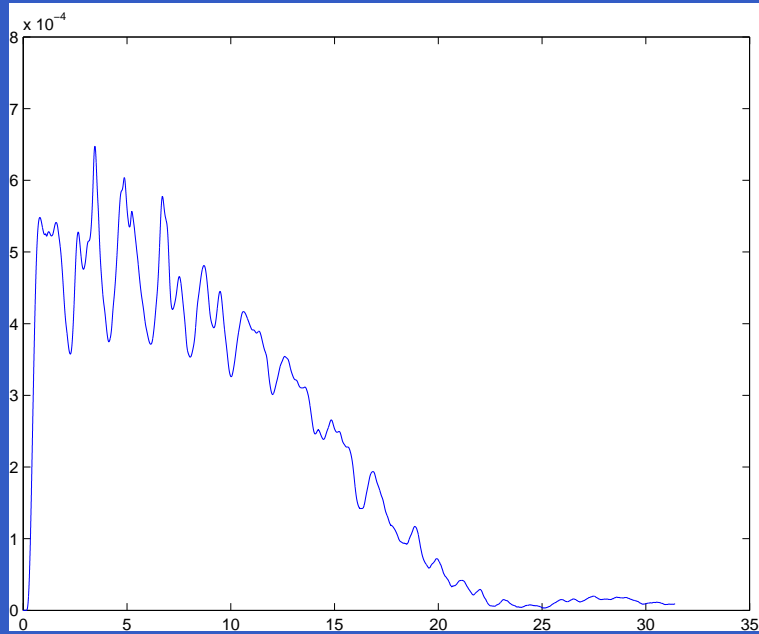
Average Temp Left/Right



Average kinetic and heat energy



Average Kinetic Energy Left/Right



Irreversibility

- Kinetic energy increases under expansion.
- No tendency of gas to return to Chamber 1 (compression)
- Gas expands by itself but does not compress by itself.
- Compression produces heat: cooling: lost energy.

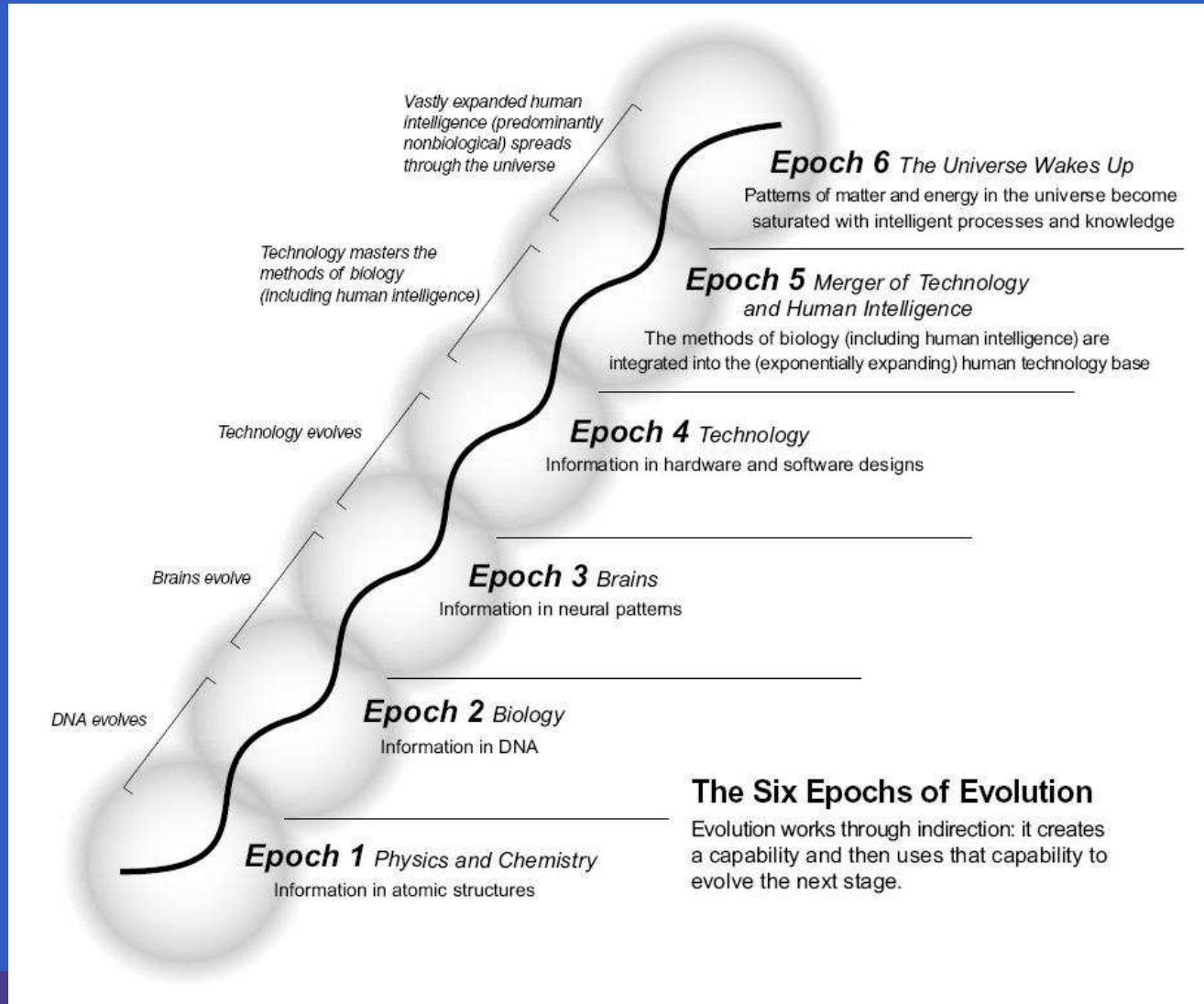
SUMMARY

- NEW 2ND LAW: $\dot{E} + W = D$
- EULER EQ INVISCID PERFECT GAS
- FINITE PREC + 1ST LAW \rightarrow 2ND LAW
- COMPLEXITY \rightarrow ARROW
- ARROW EMERGENT PHENOMENON
- FINITE PREC instead of STATISTICS
- APPL: QM, CHEM, BIO, GEO,...
- HYPERREALITY

Kurzweil: Singularity 2045

- Kurzweil: Synthesizer....
- Moore's law:
- Computational power doubles every 18 months
- DIGITAL SIMULATION
- Computational Technology Blow Up 2045
- Infinite speed of development

Kurzweil: Epochs of Evolution



Baudrillard: SIMULATION

- Copy of Reality
- Confusion Simulation–Reality
- Mask of Nonexisting Reality:
- HYPERREALITY
- HyperMarkets

HYPERREALITY (masking reality)



Deleuze: SIMULATION

- SIMULATION is REALITY
- REALITY is SIMULATION

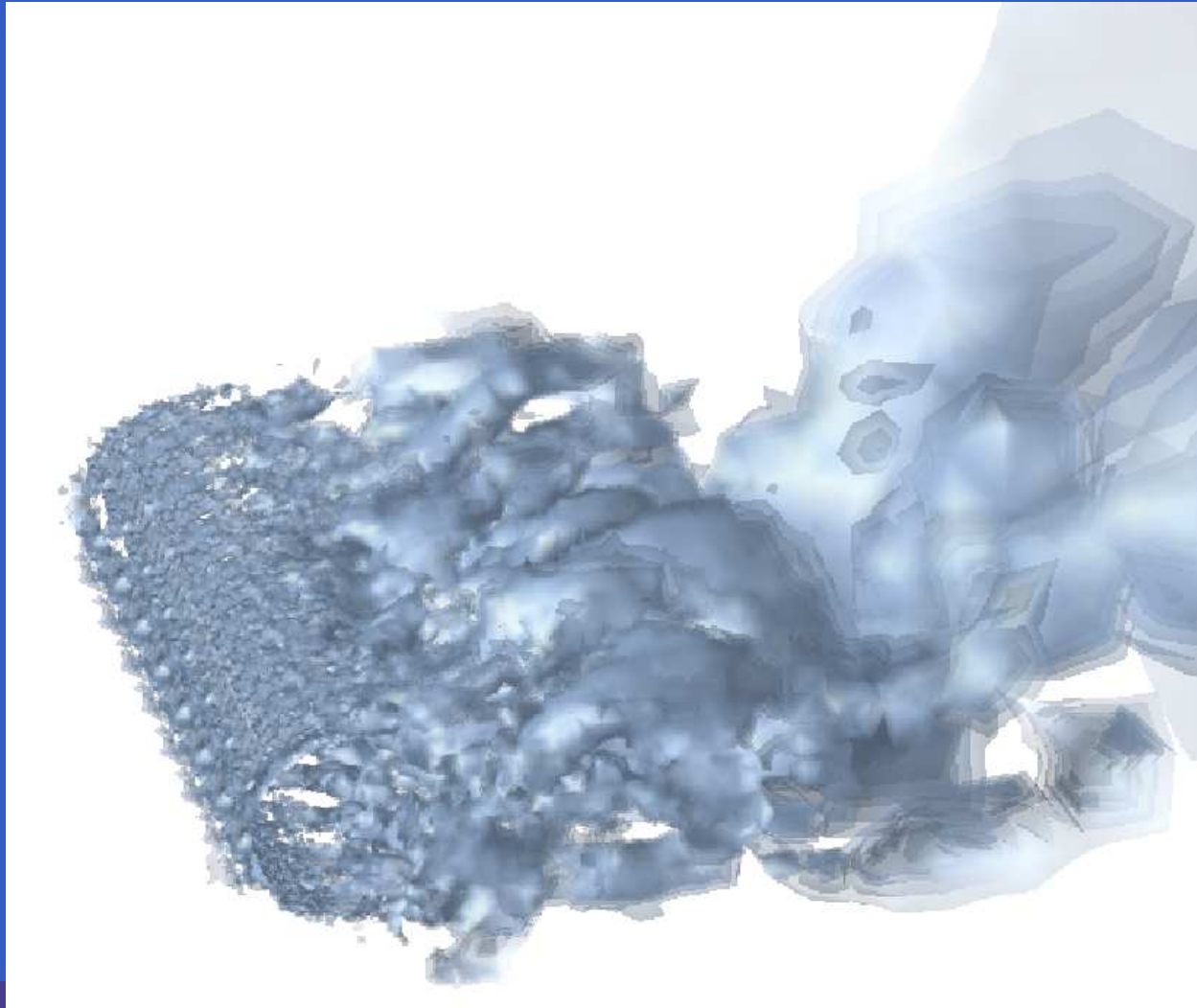
SIMULATION

- Math: COMPUTATIONAL CALCULUS
- Physics: Basic Conservation Laws
- Chemistry-Biology—: Constitutive Laws
- COMPUTATIONAL TECHNOLOGY

TEST: THERMODYNAMICS

- DIFFICULT!!
- Why?
- TURBULENCE/SHOCKS!!

CIRCULAR CYLINDER $RE=3900$



CLASSICAL THERMODYNAMICS

- DIFFICULT
- 2nd LAW?
- Who can understand and teach?
- Lars Onsager (1903-1976), Nobel Prize 1968
- Ilya Prigogine Nobel Prize 1989
- STATISTICAL MECHANICS
- Microscopic Games of Roulette
- PLANCK: ACT of DESPERATION!

1st LAW: EASY

- CONSERVATION of Mass, Momentum, Energy
- EULER EQ PERFECT IDEAL GAS
- UNDERSTANDABLE.

CLASSICAL 2ND LAW

First Law of Thermodynamics –

Conservation of Energy

Second Law of Thermodynamics – *It is*

not possible to create a cyclical heat engine that draws heat from a reservoir without wasting some heat energy.

Entropy – is a measure of the disorder in the Universe. It must always increase; local decreases make a bigger mess elsewhere.

Classical 2nd LAW

- The 2nd Law cannot be derived from purely mechanical laws. It carries the stamp of the essentially statistical nature of heat.
(Bergman in Basic Theories of Physics 1951)
- The total energy of the universe is constant; the total entropy is continually increasing.
(Rudolf Clausius 1865)
- PHYSICAL SIGNIFICANCE of ENTROPY??
- FORGET IT!!

HYPER-REALITY

- NON-EXIST EXACT EULER SOL!!
- Reason: TURBULENCE/SHOCKS
- WEAK SOL: NOT STRONG SOL
- APPROX TURBULENT SOL EXIST
- G2: GENERAL GALERKIN
- SIMULATION of NONEXIST EXACT SOL
- DETERMINISTIC NEW 2nd LAW
- ARROW of TIME

DETERMINISTIC 2ND LAW

- G2 satisfies 2nd LAW AUTOMATICALLY
- PENALTY for not being EXACT
- TURBULENT DISSIPATION
- Kinetic Energy \rightarrow Heat Energy
- LOSSES
- Cooling of engine
- ARROW of TIME: IRREVERSIBILITY
- DETERMINISTIC: No Statistics!!

2ND LAW

- FINITE PRECISION:
- ANALOG or DIGITAL COMPUTATION
- EDGE STABILITY: Not Stable, Not Unstable

EULER EQUATIONS

- air/water
- in fixed volume Ω in \mathbb{R}^3 with boundary Γ
- over a time interval I
- very small viscosity and heat conductivity
- density ρ
- momentum $m = \rho u$
- velocity $u = (u_1, u_2, u_3)$
- total energy ϵ

Conserv. Mass, Momentum, Energy

Find ρ , m and ϵ such that in $\Omega \times I$

$$\dot{\rho} + \nabla \cdot (\rho u) = 0$$

$$\dot{m} + \nabla \cdot (m u) + \nabla p = 0$$

$$\dot{\epsilon} + \nabla \cdot (\epsilon u + p u) = 0$$

$$u \cdot n = 0 \quad \text{on } \Gamma \times I$$

initial condition

- p pressure, $\dot{v} = \frac{\partial v}{\partial t}$
- SLIP BC

Constitutive Equations

- $\epsilon = k + e$ total energy
- $k = \frac{\rho|u|^2}{2}$ kinetic energy
- $e = \rho T$ internal energy
- T temperature.
- $p = (\gamma - 1)\rho T = (\gamma - 1)e$ perfect gas
- $\gamma > 1$ gas constant, $\gamma = 5/3$ monoatomic gas
- viscosity $\nu = 0$, heat conductivity $\kappa = 0$.

WHAT IS VISCOSITY?

- Nobody knows!!
- kinematic, dynamic, laminar, turbulent,
- molecular, eddy,....??
- solution dependent losses??
- experimental determination??
- ?????
- But we know it is small $\nu \leq 10^{-6}$
- Enough!! Euler: $\nu = 0!!$

SKIN FRICTION

- LAMINAR: $\nu^{0.5}$
- TURBULENT: $\nu^{0.2}$
- SLIP/SMALL FRICTION for $\nu < 10^{-5}$
- EULER with SLIP BC!!
- TURBULENT EULER SOLUTIONS

EINSTEIN'S DREAM

- $\gamma = 5/3, \nu = 0, \kappa = 0.$
- NO PARAMETER
- Predictive Power??
- YES!!
- The World as Analog Computation
- The World as Digital Computation

HYPERREALITY of EULER

- NON-EXISTENCE of EXACT SOL: Inf small scales
- COMPUTATIONAL TURBULENT SOL EXIST
- GIVE USEFUL INFO:
- Predict Drag and Lift of Car/Aircraft!!
- (CALCULUS USELESS)
- (COMPUTATIONAL CALCULUS USEFUL)

NS APPROX EULER

Find $\hat{u} = (\rho, m, \epsilon)$:

$$\dot{\rho} + \nabla \cdot (\rho u) = 0$$

$$R_m(\hat{u}) \equiv \dot{m} + \nabla \cdot (mu) + \nabla p = \nu \Delta u$$

$$\dot{\epsilon} + \nabla \cdot (\epsilon u + pu) = 0$$

$$\int R_m(\hat{u}) \varphi \, dx dt = \int \nu \nabla u \nabla \varphi \, dx dt = \sqrt{\nu} \|\varphi\|_{H^1}$$

- NS SOL: WEAK APPROX EULER SOL
- $\|R_m(\hat{u})\|_{H^{-1}} \approx \sqrt{\nu}$

NS APPROX EULER

$$\int (\dot{m} + \nabla \cdot (mu) + \nabla p) \cdot u \, dx = \int \nu |\nabla u|^2 \equiv D(u) \approx 1$$

- u Hölder 1/3
- $D(u) = 0$ if u smoother, but u is not (Onsager)
- NS not strong approx Euler: $R_m(\hat{u}) \sim \frac{1}{\sqrt{\nu}}$ pw,

$$\int (\dot{m} + \nabla \cdot (mu) + \nabla p) \cdot u = \int \nu |\nabla u|^2$$

LARGE = LARGE or LARGE - LARGE = 0

G2 APPROX EULER

- LEAST-SQUARES STABILIZED GALERKIN
- MESH SIZE h
- G2 SOL: WEAK APPROX EULER SOL
- $\|R_m(\hat{u})\|_{H^{-1}} \approx \sqrt{h}, R_m(\hat{u}) \sim \frac{1}{\sqrt{h}}$ pw
- $\nu \sim h$

STABILIZATION $\delta \sim h$

$$(\dot{\rho} + \nabla \cdot (\rho u), v) + (\delta u \cdot \nabla \rho, u \cdot \nabla v) = 0$$

$$(\dot{m} + \nabla \cdot (m u) + \nabla p, v) + (\delta u \cdot \nabla m, u \cdot \nabla v) = 0$$

$$(\dot{\epsilon} + \nabla \cdot (\epsilon u + p u), v) + (\delta u \cdot \nabla \epsilon, u \cdot \nabla v) = 0$$

STABILITY: $v = u$ in **MOMENTUM:**

$$D_h(u) = \int h \rho |u \cdot \nabla u|^2 dx dt \quad \text{PENALTY}$$

$$D(u) = \int \nu |\nabla u|^2 dx$$

PENALTY vs VIOLATION

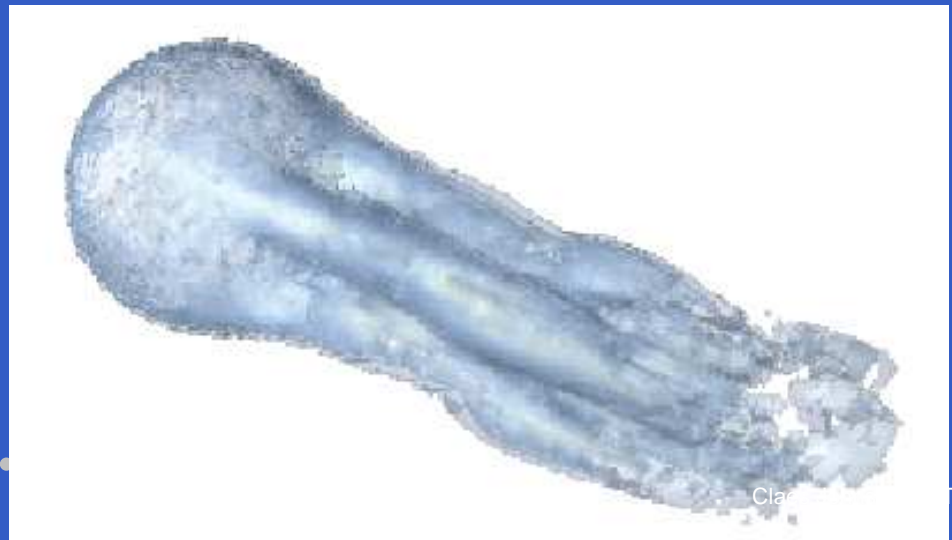
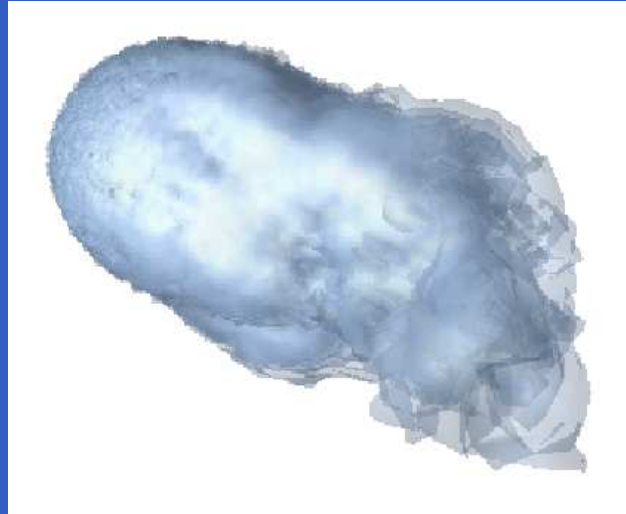
$$R_m(\hat{u}) = \dots + \rho u \cdot \nabla u + \dots = \text{LARGE}$$

- PENALTY on PART of $R_m(\hat{u})$
- STREAMLINE DIFFUSION
- SMART ARTIFICIAL VISCOSITY
- PENALTY = VIOLATION
- IDEAL according to FOUCAULT:
- Discipline and Punishment, The Birth of the Prison, 1991.

WEAK UNIQUENESS

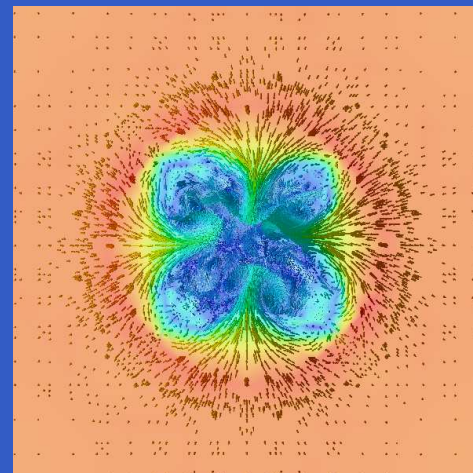
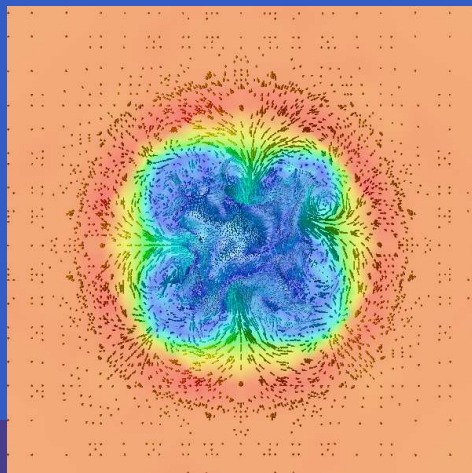
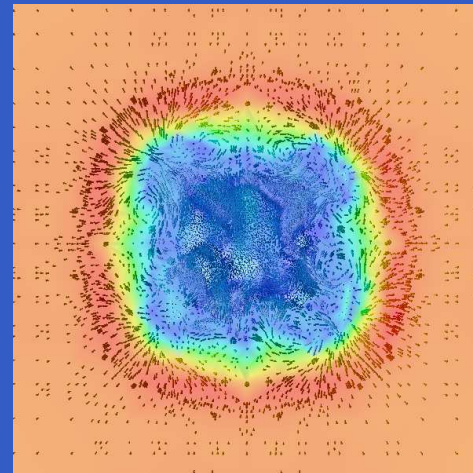
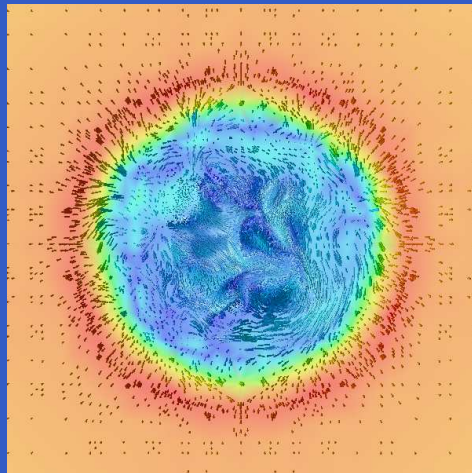
- MEAN-VALUE INDEPENDENT of h or ν
- INDEPENDENCE on STABILIZATION
- FOCUS on $R(\hat{u})$ NOT $-\nu\Delta u$
- $R(\hat{u})$ CANNOT BE STRONGLY SMALL!!

DRAG of SPHERE: Vorticity



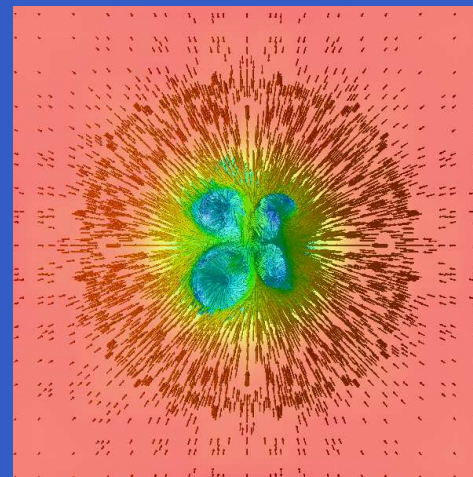
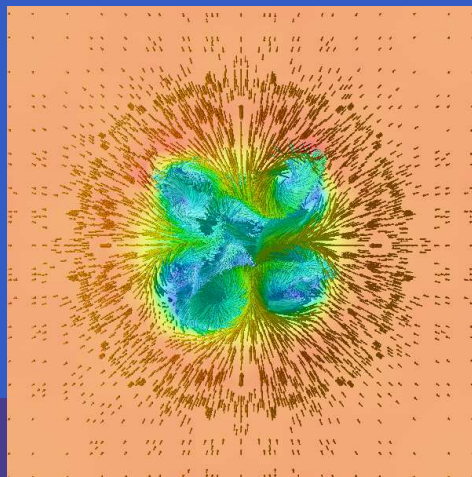
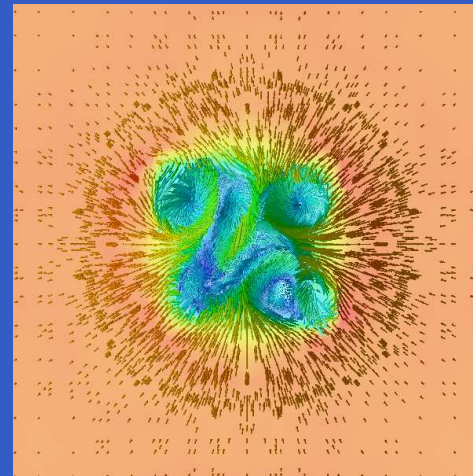
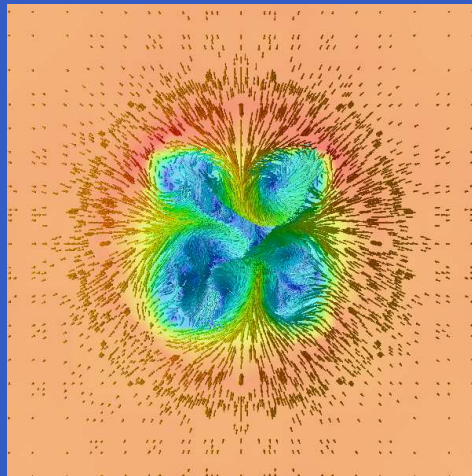
DRAG CRISIS $c_D = 0.5, 0.3, 0.2, 0.2$

$$\beta = 0.082, 0.032, 0.022, 0.018 \sim \nu^{0.2}$$



DRAG CRISIS $c_D = 0.2, 0.2, 0.2, 0.1$

$$\beta = 0.013, 0.012, 0.011, 0.0097$$



EG2: EULER G2

- STABILIZATION PENALTY: $D(u) = \int hR^2 dx$
- h mesh size, $R(u)$ Residual $\approx h^{-1/2} \gg 1$
- $D(u)$ NOT SMALL ≈ 1 : TURBULENCE
- 10^7 meshpoints for COMPLEX GEOM
- output error $\leq S \|hR\|_{L2} < 1$, S Stability factor
- NO VISCOUS BOUNDARY LAYER
- 10^{18} for DNS: IMPOSSIBLE USELESS

1D EULER: Find $\hat{u} \equiv (\rho, m, e)$:

$$\begin{aligned}R_\rho(\hat{u}) &\equiv \dot{\rho} + (\rho u)' = 0 && \text{in } Q, \\R_m(\hat{u}) &\equiv \dot{m} + (mu + p)' = 0 && \text{in } Q, \\R_e(\hat{u}) &\equiv \dot{e} + (eu)' + pu' = 0 && \text{in } Q, \\u(0, t) &= u(1, t) = 0 && t \in I, \\\hat{u}(\cdot, 0) &= \hat{u}^0 && \text{in } \Omega,\end{aligned}\tag{1}$$

where $p = (\gamma - 1)e$, $u = \frac{m}{\rho}$,

$$R(\hat{u}) = 0, \quad R = (R_\rho, R_m, R_e).$$

REG EULER: Find $\hat{u} = \hat{u}_{\nu, \mu}$:

$$\begin{aligned}\dot{\rho} + (\rho u)' &= 0 && \text{in } Q, \\ \dot{m} + (mu + p)' &= (\nu u')' + (\mu p u')' && \text{in } Q, \\ \dot{e} + (eu)' + pu' &= \nu(u')^2 && \text{in } Q, \\ u(0, t) = u(1, t) &= 0 && t \in I, \\ \hat{u}(\cdot, 0) &= \hat{u}^0 && \text{in } \Omega,\end{aligned}\tag{2}$$

where

- $\nu > 0$ shear viscosity
- $\mu \gg \nu$ small bulk viscosity ($\mu = 0$ if $u' < 0$).

REG SOL EXIST/SATISFIES

$$\|R_m(\hat{u})\|_{-1} \leq \frac{\sqrt{\nu}}{\sqrt{\mu}} + \sqrt{\mu} \ll 1 \quad (\text{small})$$

$$R_\rho(\hat{u}) = 0, \quad R_e(\hat{u}) \geq 0 \quad \text{pointwise.}$$

2nd Law:

$$\dot{K} \leq W - D, \quad \dot{E} = -W + D,$$

where $K = \int_J k dx, E = \int_J e dx,$

$$W = \int_J p u' dx, \quad D = \int_J \nu (u')^2 dx > 0.$$

NEW 2ND LAW

$$\dot{E} + w = D, \quad \dot{K} - W \leq -D$$

$$dE + pdV = D > 0 \quad dK - pdV = -D$$

CLASS 2ND LAW

Classical entropy S satisfies Classical 2nd Law:

$$S = \rho \log(e\rho^{-\gamma}) = \rho \log(T\rho^{-\gamma+1}) = \rho \log(TV^{\gamma-1}),$$

satisfies

$$\dot{S} + \nabla \cdot (Su) \geq 0 \quad \left(\frac{D}{T}\right)$$

or in symbolic form

$$TdS = dE + pdV, \quad dS \geq 0$$

$$TdS \geq 0 \quad \text{NEW}$$

MULT MOMENTUM by u :

Use the mass balance in the form

$$\frac{u^2}{2}(\dot{\rho} + (\rho u)') = 0$$

to get

$$\dot{k} + (ku)' + p'u - \mu(pu')'u - \nu u''u = 0.$$

By integration in space it follows that

$$\dot{K} - W \leq -D,$$

and similarly from the equation for e ,

$$\dot{E} + W = D.$$

ADD K and E :

$$\dot{K} + \dot{E} + \int_0^1 \mu p (u')^2 dx = 0,$$

$$K(1) + E(1) + \int_Q \mu p (u')^2 dx dt = K(0) + E(0).$$

Need to show that $E(1) \geq 0$. Energy eq:

$$\frac{De}{Dt} + \gamma e u' = \nu (u')^2,$$

where $\frac{De}{Dt} = \dot{e} + u e'$ is the material derivative of e following the fluid particles with velocity u . Assuming that $e(x, 0) > 0$ for $0 \leq x \leq 1$, it follows that $e(x, 1) > 0$ for $0 \leq x \leq 1$, and thus $E(1) > 0$.

CONTROL of WORK W

Assuming $K(0) + E(0) = 1$:

$$\int_Q \mu p (u')^2 dx dt \leq 1,$$

$$0 \leq E(t) \leq 1$$

INT KINETIC ENERGY:

$$\begin{aligned} K(1) + \int_Q \nu(u')^2 dx dt &= \int_Q pu' dx dt - \int_Q \mu p(u')^2 dx dt \\ &\leq \frac{1}{\mu} \int_Q p dx dt \leq \frac{1}{\mu}, \end{aligned}$$

where

$$\int_Q p dx dt = (\gamma - 1) \int_Q e dx dt \leq \int_I E(t) \leq 1.$$

Hence

$$\int_Q \nu(u')^2 dx dt \leq \frac{1}{\mu}.$$

EXISTENCE of REGULARIZED SOL

$$\|R_m(\hat{u})\|_{-1} \leq \sqrt{\mu} + \frac{\sqrt{\nu}}{\sqrt{\mu}} \quad (\text{SMALL}),$$

$$R_\rho(\hat{u}) = 0, \quad R_e(\hat{u}) \geq 0$$

$$\dot{E} + W = D \quad \dot{K} - W = -D$$

- MEANVALUE INDEP of REG
- WEAK UNIQUENESS of REG SOL
- CLAY PRIZE

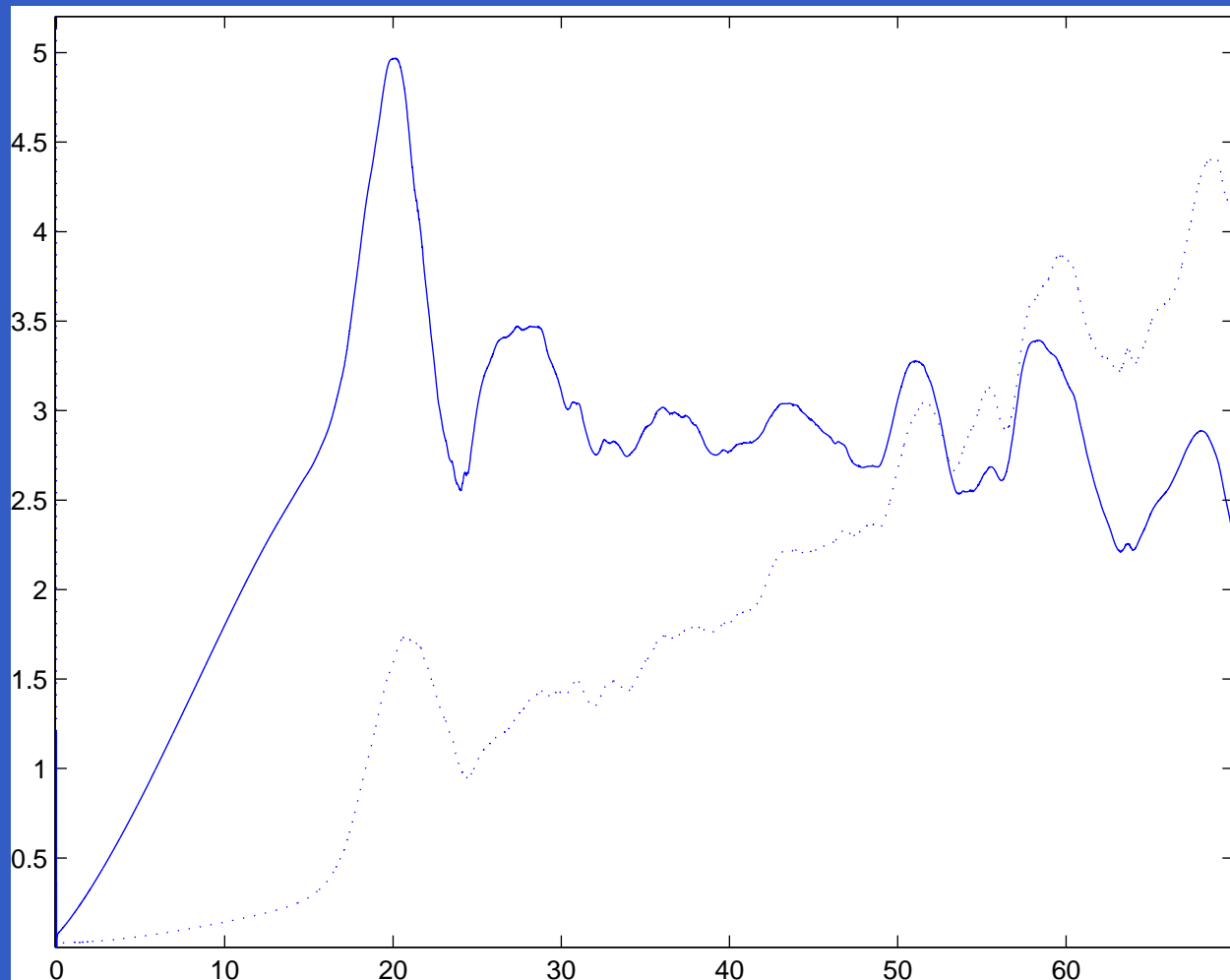
SUMMARY

- NEW 2ND LAW: $\dot{E} + W = D > 0$
- LARGE SCALE KE \rightarrow SMALL SCALE KE
- NO ENTROPY
- FINITE PRECISION COMPUTATION
- IRREVERSIBILITY by FINITE PRECISION
- MEAN VALUE OUTPUT STABLE
- COMPLEX FLOW EXISTS, SIMPLE NOT
- WORLD EXISTS BECAUSE IT IS COMPLEX
- FLYING POSSIBLE by TURBULENCE!!

SECRET of FLYING



LIFT DRAG vs ANGLE of ATTACK



EG2 BREAKTHROUGH

- NO VISCOUS BOUNDARY LAYER
- 10^7 meshpoints for COMPLEX GEOM
- OUTPUT ERROR $\leq S \|hR\|_{L2} < 1$,
- S Stability factor

2nd Law for EG2

MULT of MOMENTUM by u gives:

$$\dot{K} = W - D, \quad \dot{E} = -W + D$$

- $D > 0$ NOT SMALL = TURBULENCE
- $K(t)$ (total) KINETIC energy at time t
- $E(t)$ (total) HEAT energy
- $W = \int_{\Omega} p \nabla \cdot u \, dx$ WORK rate
- $W > / < 0$ under EXPANSION/COMPRESSION
- $W = 0$ incompressible flow

ESSENCE of THERMODYNAMICS

- $\dot{K} = W - D, \quad \dot{E} = -W + D$
- Transfer of kinetic energy K to heat energy E
- Irreversibility Arrow of Time
- K grows by expansion ONLY
- E grows by compression
- Entropy: NO ROLE
- NOBODY knows what Entropy is (Neumann)
- G2 THERMODYN: Understandable + Useful
- COMPUTATIONAL CALCULUS!!

PENDULUM

$$\dot{v} = -u, \quad \dot{u} = v$$

$$\frac{d}{dt}\left(\frac{v^2}{2}\right) = -uv, \quad \frac{d}{dt}\left(\frac{u^2}{2}\right) = uv,$$

$$\dot{K} = W, \quad \dot{E} = -W, \quad W = -uv$$

- K kinetic energy, E potential energy
- W work rate, $D = 0$: reversible
- Oscillation: kinetic–potential energy
- Thermodyn = Oscill: kinetic–heat energy

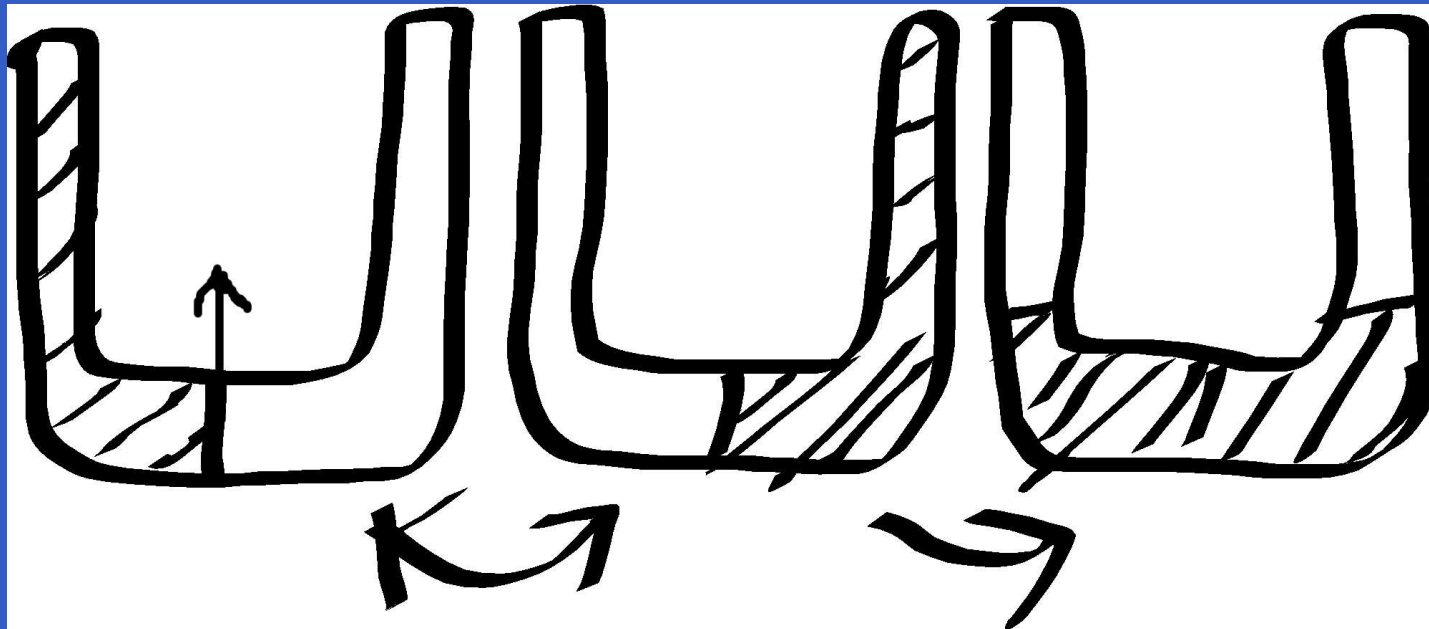
JOULE EXPERIMENT

- $T = 1$ in both chambers
- Gas expands from 1 into 2.
- Kinetic energy K increases
- Temperature T drops < 1
- Turbulence develops in 2
- Kinetic energy transforms into heat energy
- Temperature increases
- Final state $T = 1$ in both chambers.
- Simple Clear: Dynamics: No Mystery

SHEEP



U-GLASS



Boltzmann: Statistical Mechanics?

- Entropy/disorder increases
- More disorder in bigger volume
- Small probability that gas will return.
- Difficult Unclear: NO DYNAMICS: MYSTERY

CLASSICAL ENTROPY

CLASSICAL 2nd LAW:

$$TdS = T + pdV, \quad dS \geq 0$$

$$S = \log(p\rho^{-\gamma}) \sim \log(V)$$

- Physical significance of S ??

NEW 2nd LAW:

$$T + pdV \geq 0$$

- No significance of S

CLAY \$1 MILLION PRIZE

- NON-EXISTENCE: EXACT EULER SOL
- EXISTENCE: APPROX TURB EULER SOL:
- WEAK LERAY NS OR G2 SOL
- INCOMPRESS and COMPRESS (NEW)
- WEAK UNIQUENESS: OUTPUT ERROR CONTROL
- ANY REGULARIZATION!!
- NONTRIVIAL SOL of PRIZE PROBLEM??
- $\|R(\hat{u})\|_{H^{-1}} \leq \sqrt{h}, \quad \sqrt{\nu}$

SIGNIFICANCE of COMPUTATIONAL

- Referee: To me a computation means nothing
- COMPUTATIONAL SOL APPROX MATH
SOL!!

SOCIETY of FINITE PRECISION



- FLAG FLAT in PERFECT SOCIETY .

TURBULENCE

- Analytical Turbulence: IMPOSSIBLE
- Computational Turbulence: POSSIBLE

Computational Calculus vs Calculus?

- Mathematicians: MINOR Modification
- BUT MAJOR CHANGE!!
- Calculus: DIFFICULT
- Computational Calculus: EASY!!
- Calculus: IMPOSSIBLE to Teach
- Computational Calculus: POSSIBLE!!

CHINA CHALLENGE

- China: 400.000 Engineers/year
- Europe: Tradition
- Math Education stable for 100 years:
- Calculus–Classical–Analytical
- REFORM: COMPUTATIONAL CALCULUS
- COMPUTATIONAL TECHNOLOGY
- Start: First Day of First Year

INVESTMENTS

- CALCULUS 1700-2000: 300 years!!
- FLUID DYNAMICS: DNS Impossible!!
- RESISTANCE to REFORM!!

REFeree COMMENTS

- Well written, Interesting, Provocative
- I strongly recommend rejection
- I did not read their numerical papers
- To me their numerics proves nothing
- Not new, Too new
- Full of prentiousness, Sterile polemic
- Ignores modern work in fluid mech/numerics
- Play with words
- REJECT: SIAM SciComp, JFM, JMFM, M3AS

BODY&SOUL: www.bodysoulmath.org

Solve

$$\dot{u} = f(u)$$

- Derivative, integral, lin alg, Gauss, Stokes
- Compute general ODE
- Compute general PDE: Poisson, heat, wave, convection, Maxwell
- Compute Euler/Navier-Stokes, Schrödinger...
- Turbulence...Computer Games...

BOOKS

- Vol 1: Derivatives and Geometry in \mathbb{R}^3 2003
- Vol 2: Integrals and Geometry in \mathbb{R}^n 2003
- Vol 3: Calculus in Several Dimensions 2003
- Vol 4: Comp Turbulent Incompress Flow 2007
- Vol 5: Computational Thermodynamics 2007
- Vol 6. The Arrow of Time 2007
- Vol 7: Many-Minds Relativity 2000
- Vol 8: Many-Minds Quantum Mechanics 2008
- Vol 9: Comp Solid Mech 2008....Vol 10...

FENICS: AUTOMATION of CC

- G2 General PDE
- FEniCS Form Compiler
- Adaptivity–Duality
- A Posteriori Error Control
- Optimization- Control
- www.fenics.org

ICARUS: www.icarusmath.com

- Web version of BODY&SOUL
- Computer Game = CC
- DEMO: Crash Course Thermodynamics

COMPUTER GAMES

- $\dot{u} = f(u)$
- Interactive Model
- Input: Data
- Output: Solution
- CONTROL
- Stimulate Students: Active Learning

SUMMARY

- COMPUTATIONAL CALCULUS
- COMPLEX MODELING: TURBULENCE
- HYPERREALITY: SIMULATION
- KNOWLEDGE SOCIETY
- REFORM? WHEN?