

Simplified and Improved Separations Between Regular and General Resolution by Lifting

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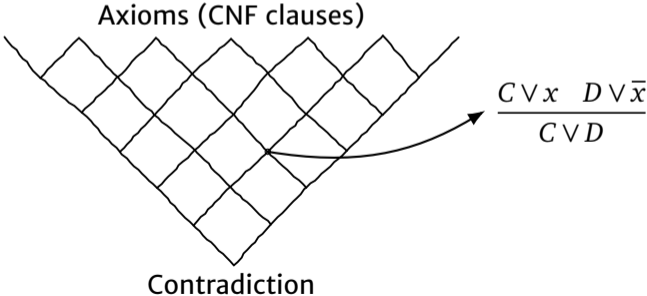
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joint work with Jan Elffers, Jan Johannsen, and Jakob Nordström

Regular Resolution

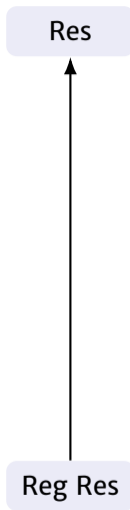
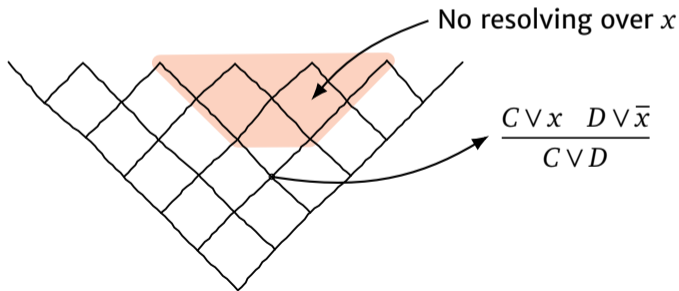
► Resolution.

Res



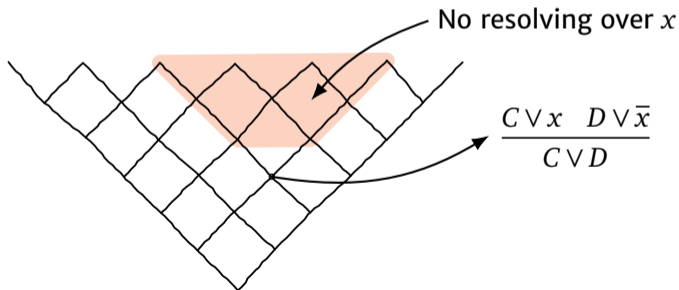
Regular Resolution

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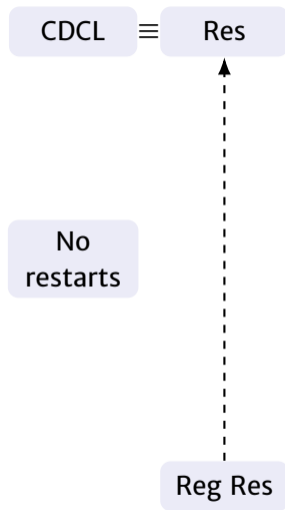
Regular Resolution

- ▶ Resolution.
- ▶ Regular resolution: do not resolve a variable twice on same path.
- ▶ Regular resolution exponentially weaker than general.
(Exist formulas with short proofs but exponentially long regular proofs)



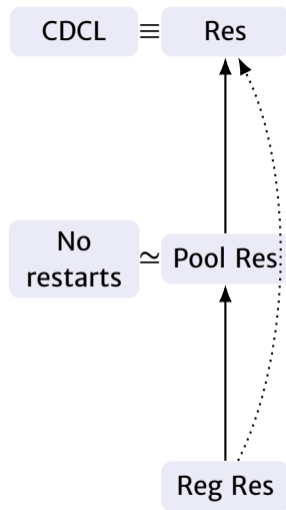
CDCL and Restarts

- ▶ CDCL as powerful as resolution.
- ▶ Crucially uses restarts.
- ▶ Restarts also seem very important in practice.
- Q Are restarts really needed?



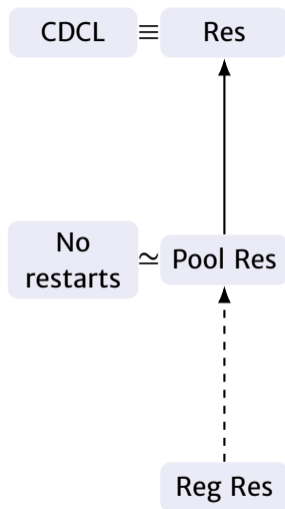
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- ▶ All such formulas easy for pool resolution.
 - ▶ Also: formulas not good to run experiments with.
 - ▶ **Need new formulas!**



Proving Resolution Lower Bounds

Largest clause in proof

Size–Width Relation

Resolution F requires width $W \Rightarrow F$ requires length $\exp(W^2/n)$

Regular resolution ??

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Lifting

Resolution F requires width $W \Rightarrow T(F)$ requires length $\exp(W)$

Regular resolution ??

Main Result (Informal)

Theorem

F requires large depth $\Rightarrow T(F)$ requires long regular proofs.

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- ▶ Simplifies separation between regular and general resolution.
 - ▶ If F has narrow proofs, then $T(F)$ still has short proofs.
 - ▶ Obtain separation from F with small width and large depth, e.g. pebbling formulas.

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 - ▶ Obtain separation from F with small width and large depth, e.g. pebbling formulas.
- ▶ New family of “sparse stone formulas”.
- ▶ Improved separation: $\exp(L/\log^3 L \log \log^5 L)$.
- ▶ Can use in experiments.