3a. Assume that $S(t)$ is the price of a single stock. Derive a Monte Carlo and a PDE method to determine the price of a contingent claim with the contract $I = \int_0^T h(S(t),t)dt$, for a given function h, instead of the ususal contract $\max(S(T)-K,0)$ for European call options. (This means that the holder of the contract at time T receives the amount of money I). *Hint:* Introduce an auxiliary variable, $Y(t)$, with $Y(0)=0$ and $dY = h(S(t),t)dt$. Let the price of the option be $f(t,S(t),Y(t))$.

3b. Derive the Black and Scholes equation for a general system of stocks $S(t) \in \mathbb{R}^d$ solving

$$dS_i = a_i(S(t),t)dt + \sum_{j=1}^{d} b_{ij}(S(t),t)dW_j(t),$$

with the independent Wiener processes W_i, $1 \le i \le d$, and a rainbow option with the contract $f(S(T),T) = g(S(T))$, for a given function $g : \mathbb{R}^d \to \mathbb{R}$, e.g. $g(s) = \max(d^{-1}\sum_{i=1}^{d} s_i - K, 0)$, where K is a constant. *Hint:* Generalize the derivation we did in class to a portfolio with the option and all stocks.