The Monte Carlo method and option models

4a. In the risk neutral formulation a stock solves the SDE

$$dS/S(t) = rdt + \sigma dW(t),$$

with constant interest rate r and volatility σ. Show that

$$S(T) = S(0)\exp(rT - \sigma^2 T/2 + \sigma W(T)),$$

and use that to simulate the price

$$f(0, S(0)) = e^{-rT} E[\max(S(T) - K, 0)],$$

of an European call option by a Monte Carlo method, where $K = 35$, $S(0) = 40$, $r = 0.04$, $\sigma = 0.2$, $T = 1$. Compute also the corresponding delta

$$\Delta = \frac{\partial f(0, s)}{\partial s},$$

by approximating with a difference quotient and determine a good choise of your "∂s". Estimate the accuracy of your results. Suggest a better method to solve this problem.

4b. Assume that a system of stocks solve

$$dS_i/S_i(t) = rdt + \sum_{j=1}^{d} \sigma_{ij} dW_j(t), \ i = 1, \ldots, d,$$

where W_j are independent Brownian motions. Show that

$$S_i(T) = S_i(0)\exp(rT + \sum_{j=1}^{d}(\sigma_{ij} W_j(T) - \sigma_{ij}^2 T/2)).$$

Let

$$S_{av} := \sum_{i=1}^{d} S_i/d,$$

and simulate the price of the option above with $S(T)$ replaced by $S_{av}(T)$, where $d = 10$, $S_i(0) = 40$, $i = 1, \ldots, 10$, and some ad hoc non diagonal choice of the volatility matrix σ. Estimate the accuracy of your results. Can you find a better method to solve this problem?