General Purpose Computing on the GPU

Characteristics of suitable problems

SIMON LJUNGSTRÖM and VIKTOR LJUNGSTRÖM



General Purpose Computing on the GPU

Characteristics of suitable problems

SIMON LJUNGSTRÖM and VIKTOR LJUNGSTRÖM

DD143X, Bachelor's Thesis in Computer Science (15 ECTS credits)
Degree Progr. in Computer Science and Engineering 300 credits
Royal Institute of Technology year 2012
Supervisor at CSC was Mårten Björkman
Examiner was Mårten Björkman

URL: www.csc.kth.se/utbildning/kandidatexjobb/datateknik/2012/ljungstrom_simon_OCH_ljungstrom_viktor_K12049.pdf

Kungliga tekniska högskolan Skolan för datavetenskap och kommunikation

KTH CSC 100 44 Stockholm

URL: www.kth.se/csc

Abstract

In a society that grows more and more dependent on fast digital data processing, many developers have turned their attention toward performing general-purpose computations on the graphics processing unit. This thesis explores what types of problems might be, or might not be, suitable for implementation on the GPU by taking a look at both classical and modern GPU concepts. Two computational problems – matrix multiplication and maximum value of a matrix – are implemented for both multi-core CPU and GPU and a comparison is presented. We reach the conclusion that the GPU can be an extremely potent computation unit as long as the problem is highly parallelizable, has no or very few branches and is computationally intensive.

Referat

Generella beräkningar på GPU:n

I ett samhälle som blir allt mer beroende av snabb digital databehandling har utvecklare och forskare börjat rikta sitt intresse åt att utföra generella beräkningar på datorns grafikprocessor, GPU:n. I detta examensarbete undersöks vilken typ av beräkningar som är lämpade, eller inte lämpade, att behandlas av GPU:n genom att ta en titt på både klassiska och moderna GPU koncept. Utöver detta tar vi också en djupare titt på hur två problem, matrismultiplikation och att hitta maxima i en matris, presterar på flerkärnig CPU och GPU och jämför resultaten. Vi har kommit till slutsatsen att GPU:n kan vara en mycket kraftfull beräkningsenhet, så länge problemet i fråga är högeligen paralleliserbart, saknar eller har väldigt få villkorliga förgreningar samt är beräkningsintensivt.

Statement of Collaboration

This text and associated code is a collaboration between the two authors Simon Ljungström and Viktor Ljungström. Work was divided as follows. Any sections of text not explicitly mentioned below are considered to be written with equal, or close to equal, effort from both authors.

Author	Sections / Code
Simon Ljungström	1, 2.3, 2.5, 5.2, Matrix Maximum for CPU, Matrix Multiplication for GPU
Viktor Ljungström	2.1, 2.2, 4.1, 4.2, 5.1, Matrix Maximum for GPU, Matrix Multiplication for CPU

Definitions

Abbreviation	Term	Definition				
AMD	Advanced Micro Devices	One of the worlds leading CPU and GPU manufacturers				
API	Application Programming Interface	An interface used for application programming. Often consists of header-files in C, abstracting complex assmbler routines				
CPU	Central Processing Unit	The processing unit that is normally used for computations				
CUDA	C for CUDA	An API for C, used to program CUDA devices				
	Compute Unified Device Architecture	An architecture implemented in recent NVIDIA devices				
DLP	Data Level Parallelism	When a problem can be parallelized by running a function on different data in parallel				
FLOPS	Floating Point Operations per Second	A common way to measure throughput				
GPGPU	General Purpose Computing on the GPU	Performing non-graphics computations on the GPU				
GPU	Graphics Processing Unit	A processing unit that is specialized on graphics-computations				
Latency	Latency	The time you have to wait for something to finish				
NVIDIA	NVIDIA	One of the worlds leading GPU manufacturers				

Abbreviation	Term	Definition			
OpenCL	Open Computing Language	An open programming language/API with a focus on portability between different systems and/or devices			
OpenGL	Open Graphics Library	An API for performing graphics computations on the GPU			
SIMD	Single Instruction Multiple Data	A processing model where the same instruction is applied to different data. Has historically been used mostly for image processing and graphics computations			
SPMD	Single Program Multiple Data	Same as SIMD but with support for conditional branching			
TLP	Task Level Parallelism	When a problem can be parallelized by dividing it into several sub-problems that can be performed independently			
Throughput	Throughput	The total number of computations performed during a time interval. Usually measured in FLOPS			

Contents

Statement of Collaboration

Definitions

1	Intr	oducti	io	or	n																										1
2	Bac	kgrour	nc	\mathbf{d}	ł																										3
	2.1	The C				sic	g (G:	ra	pł	nic	cs	Pi	ip	eli	ne															3
	2.2	Shader																													4
	2.3	Unified																													5
	2.4	Graph	nic	cs	\mathbf{s}	Ν	Ie	m	10!	rу																					6
	2.5	A Brie																													6
		2.5.1]	K	K	er	ne	əls	3																						7
		2.5.2	Ι	N	VI	eı	no	or	у	Μ	[00	de	el																		7
		2.5.3	1	V	W	O1	rk	i	te	m	s,	w	or	k	gr	ou	ps	a	nd	W	or	k	siz	zes							7
3	Met	hodolo	og	\mathbf{g}	gy	7																									9
4	Imp	lemen	ıta	a	ıt	ic	n	i.																							11
	4.1	Matrix	X	Ν	M	Ιu	lti	ip	lio	ca	tic	n																			11
	4.2	Matrix	X	N	V.	[a	xi	m	ıuı	m																					12
5	Res	ults an	nd	d	l	D	is	SC.	us	ssi	ioi	n																			15
	5.1	Matrix																													15
	5.2	Matrix																													17
6	Con	clusio	n	L																											21
Bi	bliog	graphy	7																												23
$\mathbf{A}_{]}$	ppen	dices																													23
\mathbf{A}	Cod	le																													2 5
	A.1	Matrix	x	Ν	V.	Ιu	lti	ip	lio	ca	tic	n																			25
		Matrix						-																							30

Introduction

As society grows ever more dependent on digital systems for everything from banking to elections, expectations on these systems are constantly rising. The systems are expected to be able to process an increasing amount of data without any increase in latency. This means that there is a constantly increasing demand for faster processing for these systems, and as new more data intensive systems emerge this demand grows at an even higher pace.

In the early days of digital computing this demand was met by CPU manufacturers optimizing for low latency by dividing the pipeline into a larger amount of smaller steps as well as increasing the CPU clock speed and number of cache levels. But this development could not continue indefinitely due to the difficulty of dividing the pipeline further, and the correlation between high clock speeds and high temperatures.

Today, we instead see an increase in parallelism with multi-core processors becoming the norm. More and more software is written to utilize these multiple cores, often leading to a great boost in performance. As it is, this development can not continue unhindered either. Due to the large amount of logic and hardware used to reduce latency, processor cores become rather large and it is not physically possible to squeeze more cores into the same area without removing some of the latency reducing functionality or reducing transistor size.

There is, fortunately, already a type of processor that is inherently parallel and massively multi-core to begin with – the graphics processing unit. The GPU is, however, a specialized piece of hardware focused on graphics calculations and making specialized hardware do general computations is not trivial. Thus, utilizing this innate parallelism has been a subject of research for some time now and has also more recently awakened an interest throughout the general development community, mainly due to the emergence of higher level APIs targeting this audience such as CUDA and OpenCL [1].

Many developers and consumers are interested in the topic of General-Purpose computing on the GPU (GPGPU or GPU computing), but do not fully understand what types of software may be suitable for GPU acceleration. One of the reasons for this is that in order to write efficient programs for the GPU, one needs to possess some basic knowledge of its architecture. This text aims to shed some light on this underlying architecture as well as identify some traits that make a problem or algorithm more or less suitable for GPU computing.

More explicitly, we are looking to answer the following two questions.

- What are the main characteristics of problems that perform well on the GPU?
- Is the GPU a viable alternative to the CPU?

To facilitate this, we shall also take a closer look at and implement two computational problems: matrix multiplication and finding the maximum element of a matrix. The first as an example of a problem that should see a significant performance boost on a GPU, and the second as a problem that should not. These implementations are tested and their performance evaluated in order to confirm whether or not they perform as anticipated.

We start off by explaining why the GPU is so parallel by taking a look at the graphics pipeline and its evolution including a brief overview of the Unified Shader Model, GPU latency hiding and the GPU memory model as well as a short introduction to OpenCL. This is followed by an explanation of the actual implementation details for our chosen problems. Finally, we present our results, discuss these and present our conclusions.

Background

Before the graphics processing unit was invented, developers of graphical user interfaces and games struggled to make their creations run smoothly and without delay on the CPU. There was clearly a need to offload the CPU by performing these demanding computations elsewhere. This was the birth of the GPU. In contrast to the very general CPU the GPU only had to do one thing, compute what colors the pixels on the screen should have. This meant that the GPU could be very specialized for this purpose. The key ingredient in this specialization is the fact that the color of each pixel can be computed almost entirely independently from the other pixels. This resulted in a GPU design that was much more parallel than the CPU, but one that ran at a slower clock speed. The focus of this design was to maximize throughput for multiple tasks, rather than minimizing latency for a single task [2, 3].

In the following sections we present some basic background information that helps with the understanding of why the graphics processing unit is so parallel and what problems it may be good for. We take a look at the graphics pipeline and its evolution including the Unified Shader concept and graphics memory along with a short overview of some OpenCL concepts that will be used both in the problem implementations and when discussing the results.

2.1 The Classic Graphics Pipeline

The classic graphics pipeline is built upon one simple fact: almost all operations used for computing graphics have both task level parallelism (TLP) and data level parallelism (DLP) [4]. This means that (1) there are several independent stages in the computations of the image output, and (2) the data in one stage can be processed in parallel. Graphics manufacturers exploit this parallelism in many ways, which we shall now take a closer look at.

The pipeline is split into several stages, all of which can be computed in parallel. The first step is *vertex operations*, where a programmer-provided input stream of geometric primitives (points, lines and triangles) represented by vertices is normalized

into screen space and shaded, typically by calculating their interaction with the light sources in the virtual environment. A typical scene can have hundreds of thousands of vertices, all of which can be computed independently in parallel [1, 2, 3, 5].

The next stage is *primitive assembly*, where the vertices are assembled into triangles, the fundamental hardware-supported building block in the GPU. The following step, rasterization, determines which screen-space triangles are covered by which triangles. Every triangle generates a fragment at every pixel location that it covers. Many triangles may cover the same pixel location and it may therefore be affected by multiple fragments in the next step, *fragment operations*.

By using color information from the vertices and possibly fetching textures from global memory, the fragments are shaded to determine their color. This can be done in parallel and is generally considered to be the most computationally demanding stage of the GPU pipeline. When all the fragment colors have been computed, we move on to the last step, *composition*.

In this final step of the pipeline, the fragments are merged together in order to calculate the final color of the pixels in the output image that will be sent to the screen.

Many of the steps in the pipeline above are performed on shader processors. These are special processors with a very limited instruction set, specific to the task they perform. Two examples of shader processors are fragment shaders and vertex shaders.

Historically, the vertex and fragment shaders have not been programmable, only configurable. The programmer was only in control of the positions and color of the lights, not their interaction with the objects in the scene [1]. In the following sections we will have a closer look at the actual hardware that has evolved from the challenges presented by the very parallel nature of graphics calculations.

2.2 Shader Processors

The graphics processor has evolved over time from a completely fixed-function pipeline to a relatively programmable, fairly flexible ditto. Programmers needed more flexibility in order to implement more advanced graphical effects. This lead to a dire need for more programmable shader units, which was also a first step toward making GPGPU possible at all.

The shader units, or shader processors, are implemented using the Single Instruction Multiple Data (SIMD) processing model. This means that the shader units perform the same instructions on multiple data at the same time by sharing control structures between multiple shader processors. The number of shader processors sharing these control structures is called the SIMD-width and is usually a power of two. A larger SIMD-width means that a larger part of the chip-area can be used for actual computations rather than instruction decoding [1, 3, 6].

There is, however, a large disadvantage with the SIMD-model when doing general computations. Since the data differs between the cores, different paths may be

2.3. UNIFIED SHADERS

followed when a conditional branch is reached. This kind of behaviour is not defined in the standard SIMD-model. To handle this behaviour, a new, similar model is required. This is called the Single Program Multiple Data (SPMD) model. It is the same as SIMD, but with the addition of branch-support. This is however a modified truth. When a branch is detected in a SPMD-unit, all threads that diverge are put on hold to be computed later. This means that if half of the threads go one way and the other half another way, the execution will take twice as long as if all threads had taken the same path. You can imagine how much impact this would have on a program with multiple conditional branches. Because of this, it is not recommended to use conditional branches unless it is absolutely necessary [1, 3, 6].

In the next section we shall have a look at how the different kinds of shaders have been merged together into a more general and programmable shader unit.

2.3 Unified Shaders

One problem with a pipeline using fixed vertex and fragment processors is the hugely varying level of demand on these resources. Certain calculations may require vertex processing almost exclusively, while only utilizing a small amount of the fragment processors and vice versa [1, 2, 3, 5]. A solution was needed to be able to make use of all the provided hardware, all the time. This lead to the conception of the Unified Shader Model, a term which is often used to describe two separate, but nonetheless intertwined, concepts [1].

The actual Unified Shader Model is the concept of using a unified instruction set to communicate with all different shader processor types. This greatly simplifies the task of writing shader programs. It is also a necessary step towards the related Unified Shader Architecture concept [1].

A "Unified Shader Architecture" is a GPU architecture where the shader processors are physically unified, that is, every shader unit is a more general computation device, able to do all types of shader work. If a computation only needs vertex shading, all the processors can do vertex computations, leading to much better load balancing. This is also a move away from the task-parallel hardware-fixed pipeline from earlier, allowing for a single step in the pipeline to operate almost exclusively instead of all steps executing in parallel all the time.

To facilitate an easier SIMD implementation, the unified shaders are usually grouped together into what NVIDIA calls "streaming multiprocessors". These contain several shader processors, sharing resources such as instruction fetching and caches. One or more of these multiprocessors can then be grouped together to form a larger SIMD array where every processor executes the same instruction at the same time. The size of these arrays is equal to the SIMD-width of the GPU [1].

2.4 Graphics Memory

As any experienced programmer knows, a lot of the run time of a program is spent fetching data from memory. While data is being fetched, the CPU is blocked. This is of course not very productive and slows execution by an unacceptable amount. CPU-manufacturers have solved this delay by implementing several layers of cachememory. The number and size of these caches is constantly increasing as new CPU models are introduced. The problem with this approach is that you will experience the full latency time the first time a memory block is accessed. The GPU has very little cache memory, often around 16k per stream multiprocessor, and thus handles the problem very differently.

When a thread starts fetching data from memory, the processing unit that handles that thread immediately switches to another thread. As long as the switch is done quickly, this behaviour allows for the processing unit to hide almost all latency. For this reason, modern GPUs support a huge amount of hardware threads. When the data fetch operation is finished for the waiting thread, it is queued for processing again. As long as there are more threads to switch to, most data fetching latency can be hidden [1, 2, 3, 4].

Lately, we have seen a large increase in the amount of on-board memory in high-end graphics cards. However, access time to this memory is often undesirably slow, even though GPUs generally do provide higher bandwidth to memory than CPUs do. As mentioned earlier, GPUs do have a bit of cache memory, even though they employ latency hiding. The cache memory is however different from the cache memory used by the CPU.

2.5 A Brief OpenCL Overview

OpenCL is an API for homogenous systems (systems with more than one type of computation device). In this thesis we will be using this API to implement the GPU versions of the chosen problems. More details on why this choice of API was made can be found in chapter 3.

At first glance, OpenCL programming may seem very daunting. As can be seen in Appendix A (specifically, the runOnGPU() methods), there is a fair amount of setup before you can actually use the GPU to perform calculations. But once you have overcome this hurdle, doing it again is not difficult since the setup is close to identical each time. At least when tackling the relatively simple problems we deal with in this text. We will not consider the setup further as it is outside the scope of this thesis.

In the following sections we will take a look at a few basic OpenCL concepts; kernels, the OpenCL memory model as well as work groups, work items and work sizes.

2.5.1 Kernels

In OpenCL (as well as in CUDA) the code which is run on the so called "OpenCL device" – in the case of this text, the GPU – is known as a kernel. Kernels are always declared using the keyword ___kernel and can be compiled either at runtime or not. To ensure correct compilation, the slightly performance-reducing runtime compilation should be used if the device the kernel will run on is not known beforehand [7, 8]. Note that code not run on the device, that is, the code that controls the device, is known as host code.

2.5.2 Memory Model

In the OpenCL memory model – which refers to the memory used by the device used to run kernels – there are four types of memory: global, constant, local and private [7, 8].

Global memory is the main memory used by the device; in the case of the GPU this refers to the on-board graphics memory [8].

Constant memory is the same as global memory, except it may be used more efficiently than global memory if the device has special hardware for handling constant memory caching. Most modern GPUs have such hardware.

Local memory is the shared memory on each compute unit [8]. On the GPU this corresponds to the shared memory within each stream multiprocessor, as discussed in sections 2.3 and 2.4.

Private memory is memory accesible only within the current unit of work [8]. For the GPU, this means the registers available to each stream processor.

2.5.3 Work items, work groups and work sizes

A work item is a unit of work that is to be executed on a single core.

The work items are then further grouped into work groups. All the work items in a work group can access memory shared within the work group, corresponding to the local memory discussed in the previous section.

How a work group is processed is not specified in OpenCL and thus depends on the device and its OpenCL drivers. On the GPU, however, the work group is usually represented as a group of threads (work items) executed on the same stream multiprocessor, using threads that cannot be processed straight away to hide latency as described in section 2.4.

The number of work items in a work group is called the local work size or simply work size. The maximum work size depends on the device to be used, and as we will se later on in the text, using a larger than maximum work size may lead to a system crash.

Methodology

As discussed earlier, we implement and evaluate two problems both on the GPU using a GPGPU API and on the CPU. Below we consider some options for the implementation and performance evaluation of the problems.

There are not many options to choose from when deciding which GPGPU API to use. The two most well known and widely supported alternatives are CUDA and OpenCL. The C for CUDA API from NVIDIA is most likely more mature since it has been around for quite some time, but the fact that it is a closed standard only supported on NVIDIA hardware makes it a less attractive choice than the fully open OpenCL [7, 9]. As for programming languages, there are several alternatives available. There are wrappers for the OpenCL API for more or less all widely used programming languages [10, 11]. In the end C was chosen, mainly for two reasons. First, the available implementations of OpenCL which are used for all the wrappers are written in C or assembler, and second, for the performance gained by writing code that is so close to the hardware.

Due to the above decision, the alternatives for the CPU implementations are quite limited. Choosing a different language than C would make the comparison of the CPU and GPU results much more difficult as most other languages are not as close to the hardware layer. Thus, C was used to write the CPU implementations of the problems as well. Since most modern CPUs are in fact multi-core the CPU implementations have been made as parallel as possible in order to be able to make fair comparisons between CPU and GPU performance. There are two appealing choices of threading libraries to use for this parallelization: pthreads and OpenMP. Using the low level pthreads library gives a higher level of control over the CPU, compared to OpenMPs higher abstraction level. As loss of control could possibly affect the end result in a negative fashion, pthreads was used.

The performance evaluation was performed as follows:

Hardware

A HP Pavilion dm4-2000eo laptop with:

- 6 GB RAM
- Intel Core i5 2410M @ 2.3 GHz (2 cores + hyperthreading)
- AMD Radeon HD 6470M Graphics (1 GB, 160 stream processors)

Note that the amount of stream processors is low compared to modern, high-end GPUs where the processor count can reach 2048 and above.

Method

- 1. Each problem was tested for both CPU and GPU using several different matrix sizes.
- 2. Each CPU test was run with 4 worker threads.
- 3. For the GPU, each matrix size was tested several times with different work sizes.
- 4. The maximum matrix and work sizes used in the tests were determined by trial and error, using "when the computer crashes due to the graphics card" as a cutoff. This cutoff was lower than expected due to a bug in the AMD OpenCL implementation leading to 75% of the graphics memory being inaccessible.
- 5. Each test was run ten times, taking the average runtime as the end result.
- 6. Any runtimes that were equal to or longer than twice the length of the median runtime were discarded and not used in the calculation of the average.

Implementation

In this chapter we take a closer look at the two problems we have chosen to implement and test, matrix multiplication and finding the maximum value in a matrix. We explain in-depth which parts of the problem solution that make the problems more or less suited for a GPGPU implementation as well as have a look at some sample code.

4.1 Matrix Multiplication

When we chose an algorithm that should perform well on the GPU we were looking for a compution intensive and highly parallelizable algorithm, without conditional branching. Naive matrix multiplication seemed to do the trick. At a time complexity of $O(n^3)$, it is definitely not a quick algorithm. There is no doubt that it is highly parallelizable; every element in the result matrix can be calculated independently of the others. To top it off, there is no branching. The sequential version of this algorithm is a simple triple-loop, which you can see below.

```
Pseudo code: sequential matrix multiplication

void matrix_mult(float * a, float * b, float * c, int n){

int i, j, k;

for(i = 0; i < n; i++){

for(j = 0; j < n; j++){

for(k = 0; k < n; k++){

    c[i][j] += a[i][k]*b[k][j];

}

}

}

}
```

The parallel version of this algorithm is quite simple, each thread asks for a row, and then computes all elements of that row. When this is finished, either compute a new row or terminate, depending on whether there are any rows left. See pseudo code below.

```
Pseudo code: parallel matrix multiplication

void matrix_mul(float * a, float * b, float * c, int n){
    int i, j, k;
    while (i=get_row()){
    for(j = 0; j < n; j++){
        for(k = 0; k < n; k++){
            c[i][j] = a[i][k]*b[k][j];
    }
}

}

}
```

The GPU version of the algorithm is actually the simplest of them all, since the OpenCL library hands out work to the cores, all we have to do is tell it how one element of the final matrix is calculated. See code below. The full code, including host code and comments, is available in appendix A.

```
OpenCL code: matrix multiplication

__kernel void matMul(__global float* a,
    __global float* b,
    __global float* c,

int width) {

int row = get_global_id(1);
int col = get_global_id(0);

float sum = 0;

for (int k = 0; k < width; k++) {

    sum += a[row*width+k] * b[k*width+col];
}

c[row*width+col] = sum;
}

c[row*width+col] = sum;
```

Now we need to clarify a few things. The code above is not optimized, in the sense that we have not made any variables local in the OpenCL version and have not spent time trying to polish the CPU version. The code is compiled with the -O2 flag using the g++ compiler, but that is all. Our intention is to make the different implementations as similar as possible. We believe that the results we have recorded speak a clear message even without optimization.

4.2 Matrix Maximum

The requirements for an algorithm that most likely would not perform well on the GPU are pretty much the opposite of the characteristics of matrix multiplication. We require an algorithm that performs few computations per data unit and has frequent conditional branching. Finding the maximum of a matrix fulfills these requirements. It is also easily parallelizable, making the comparison a bit more fair. The sequential algorithm is extremely simple, just look at all the elements in some order and save the current maximum. See below.

```
Pseudo code: sequential matrix maximum

float matrix_max(float * matrix, int n){
    int i, j;
    float max;
    for(i = 0; i < n; i++){
        for(j = 0; j < n; j++){
            if (matrix[i][j] > max){
                max = matrix[i][j];
            }
        }
     }
    return max;
}
```

This algorithm can be parallelized in many ways. We chose to calculate the maximum of each row, and then the maximum of that. In the CPU implementation, a thread keeps track of the largest value that it has encountered, and keeps calculating one row at a time until no rows remain. Each thread then compares their local maximum to a global maximum and changes it if necessary. See below.

```
Pseudo code: parallel matrix maximum
    global float max = FLOAT.MIN;
    global float * matrix;
     thread Worker(){
       int i,j;
       float local_max;
       while (i=get_row()) {
  for (j = 0; j < n; j++) {
    if (matrix[i][j] > local_max) {
              local_max = matrix[i][j]
10
11
12
13
       ĺock (max);
14
       if(local_max > max){
15
         max = local_max;
16
17
       unlock (max);
```

As with the matrix multiplication algorithm, we have tried to keep the GPU-and CPU-algorithms as similar as possible. However, when we are to merge together the maximums of the rows, we have to make a small adjustment in order to not cripple the GPU code. We perform the final maximum of maximums computation on the CPU, in order to avoid having to send data back and forth between the host and the device. See below.

CHAPTER 4. IMPLEMENTATION

Results and Discussion

In this section we present our findings and discuss them. The results from the GPU are those that were achieved with the optimal work size for the particular problem. The complete output of our tests can be seen in appendix B.

5.1 Matrix Multiplication

In this section we present and discuss the results from our matrix multiplication executions. We start off by comparing the execution times for different work sizes on our largest matrix to determine the optimal work size. As is depicted in fig. 5.1, the execution time decreases as we increase the work size. Since 16x16 - 256 work items – was the largest work size that did not induce a system crash, it is our optimum. All execution times mentioned from now on in this section will be from executions using work size 16x16.

A comparison of the runtime on the CPU and GPU is depicted in table 5.1 and fig. 5.2. Note that the scale of the horizontal axis is not uniform. As can be seen in the table, the GPU is only 2.44 times faster than the CPU on a 128x128 matrix, but as we increase the size of the matrix the achieved speed-up also increases. When we reach a matrix size of 1024x1024 the speed-up factor suddenly increases by 20. It is likely due to all the GPU's processors now being kept busy. At larger sizes the difference in performance slowly levels off to around 30-35. In section 4.1 we hypothesized that matrix multiplication would be very efficient on the GPU. It seems that we were correct. However, if a different matrix multiplication algorithm were to be chosen, there is room for an even larger performance gain from using the GPU. For example, there are algorithms with lower time complexity where the matrix is split up into squares that are calculated independently of each other and later multiplied together. An algorithm like that can utilize local memory in the GPU, something that our naive algorithm does not. This would likely lead to an even larger speed-up on the GPU compared to the same algorithm run on the CPU.

In the next section, we will have a look at how our implementation of finding the maximum of a matrix performed.

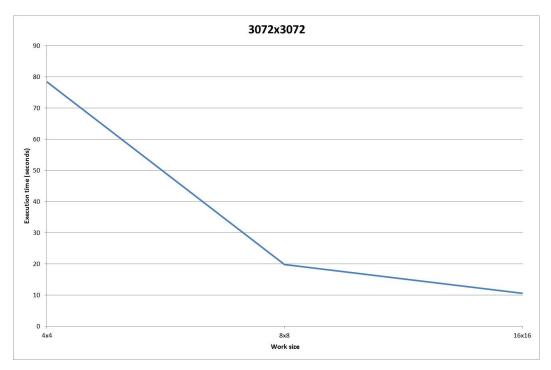


Figure 5.1. Matrix multiplication: Execution time in seconds for different work sizes on 3072x3072 matrices. Lower is better.

Matrix size	Execution time GPU	$\begin{array}{c} \text{Execution time} \\ \text{CPU} \end{array}$	GPU speed-up
128x128	0.0036s	0.0088s	2.44
256 x 256	0.0085s	$0.0637\mathrm{s}$	7.49
512x512	0.0535s	0.4738s	8.86
1024x1024	0.3998s	11.536s	28.9
1536×1536	1.3299s	39.417s	29.6
2048x2048	3.1459s	$102.57\mathrm{s}$	32.6
3072x3072	10.572s	361.99s	34.2

 ${\bf Table~5.1.} \ \, {\rm Matrix~multiplication:~ Comparison~of~ execution~time~in~ seconds~on~ CPU~ and~ GPU~ for~ different~ matrix~ sizes~ with~ work~ size~ 16x16~ on~ the~ GPU.$

5.2. MATRIX MAXIMUM

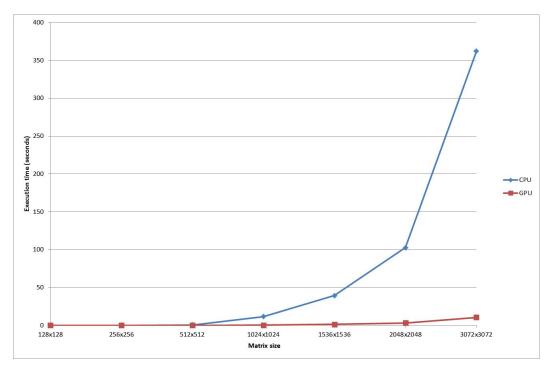


Figure 5.2. Matrix multiplication: Comparison of execution time in seconds on CPU and GPU for different matrix sizes with work size 16x16 on the GPU. Lower is better.

5.2 Matrix Maximum

In this section we take a look at the results of the performance evaluation of the maximum matrix value implementation. We begin, again, by taking a look at the GPU performance for different work sizes as shown in fig. 5.3. Recall from section 5.1 that the maximum work size for our hardware is 256 work items.

The figure depicts results quite different from those in the case of matrix multiplication. Instead of an execution time that always decreases, the faster execution only happens up to a certain point – work size 16 – followed by markedly diminishing performance. The initial performance increase is likely due to two reasons: at low work sizes not all shader processors in a work group can be put to use at the same time, and as a few more work items are added some latency hiding is possible. When the work size increases above 16, however, the negatives of branching under the SPMD model show themselves quite clearly. As noted in section 2.2, when a branch is detected in a SPMD unit, in this case the whole work group, some branches will have to be run later. This means that the work group will take at least twice as long to complete computation. Obviously, the impact gets larger as work size increases. Since the best work size for this problem on the specific GPU used for testing has been determined to be 16, the rest of the GPU results presented here use that work size.

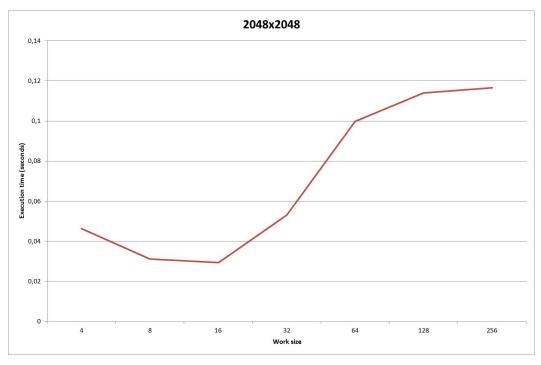


Figure 5.3. Matrix maximum: GPU execution time in seconds for different work sizes on a 2048x2048 matrix. Lower is better.

Let us now take a look at the differences in execution time between the CPU and GPU implementations, as seen in table 5.2 and fig. 5.4. Just as we hypothesized earlier in the text, the CPU wins over the GPU in terms of performance on a branch-intensive problem such as this. Since the resulting execution times are so low, they cannot give a definite answer due to differences in system load at the time of testing. We do, however, get a fair indication. For lower matrix sizes the CPU hovers around 4-5 times speed-up over the GPU after which the performance boost increases slightly more. This trend is expected to continue past the tested matrix sizes as larger matrices mean more potential branches as well as larger amounts of data to transfer to the GPU's global memory. Unfortunately, performance for larger matrices cannot be evaluated as the test-system crashes when trying to transfer the matrix to the graphics memory. A telling sign of the effects of branching on the GPU is the non-linear slowdown of execution as matrix size increases as depicted in fig. 5.4. Note that since the horizontal axis values increase in a quasi-exponential manner, linear slowdown will not be represented as a straight-line in the figure.

5.2. MATRIX MAXIMUM

Matrix size	Execution time GPU	Execution time CPU	CPU speed-up
128x128	0.00224s	0.00046s	4.87
$256 \mathrm{x} 256$	0.00269s	0.00051s	5.27
512x512	0.00359s	0.00075s	4.79
1024x1024	0.00854s	0.00187s	4.57
2048x2048	0.02940s	0.00474s	6.20
4096x4096	0.10520s	0.01679s	6.26

Table 5.2. Matrix maximum: Comparison of execution time in seconds on CPU and GPU for different matrix sizes with work size 16 on the GPU

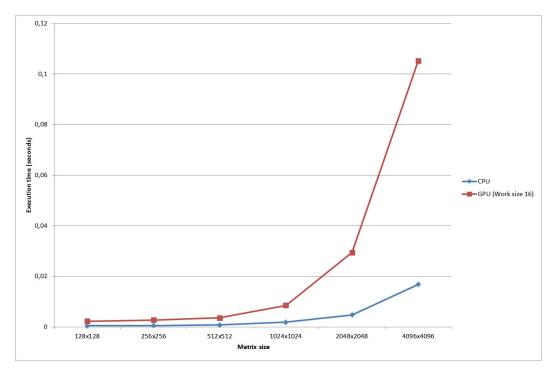


Figure 5.4. Matrix maximum: Comparison of execution time in seconds on CPU and GPU for different matrix sizes with work size 16 on the GPU. Lower is better.

Conclusion

Throughout this text we have identified three important characteristics of problems that are suited for the GPU. The first and most important characteristic is that the problem needs to be highly parallel to begin with. There needs to be a high level of data parallelism in order for a programmer to even implement a sensible solution for the GPU. The second characteristic is that the problem needs to be computationally intensive (high amount of work per data), in order for the GPU not to be idle, waiting for more data from memory. The third and final characteristic is that conditional branching should be non-existent or be triggered very seldom—if every work item takes the same branch, there will be no impact on performance. The architectural details presented in chapter 2 support these claims and the results presented in chapter 5 back this up; both of our hypotheses seem to be correct.

We have chosen not to take pricing into account in previous parts of our text, since it is not relevant from a scientific perspective. However, to determine whether the GPU is a viable alternative to the CPU, we need to; we are already at a point in time where scientist are far from alone in being interested in GPU computing. In practice, pricing is one of the larger factors affecting whether GPGPU will be a success or not. And the matter of the fact is, that GPU performance to currency ratio is very high. High-end consumer graphics cards have, during recent years, never had a price tag greater than 7000 SEK. A high-end consumer CPU, on the other hand, can cost almost twice as much. To top it off, as of OpenCL 1.1 it is quite simple to program for multiple GPUs. This has opened up the possibility for cheap GPU clusters and these are already deployed in some of the worlds fastest super computers.

To summarize, the GPU is not the successor of the CPU. It is however a very potent processing unit that should be considered for all problems that work on large sets of data. Sadly enough, all problems are not suited for GPU computing. However, what it lacks in generality it makes up for in performance.

Bibliography

- J. D. Owens et al. GPU Computing. Proceedings of the IEEE, 96(5):879–899, May 2008.
- [2] D. Luebke and G. Humpreys. How GPUs Work. IEEE Computer, 40(2):96–100, February 2007.
- [3] K. Fatahalian and M. Houston. A closer look at GPUs. Communications of the ACM, 51(10):50–57, October 2008.
- [4] V. W. Lee et al. Debunking the 100x GPU vs CPU myth: an evaluation of throughput computing on CPU and GPU. ACM SIGARCH Computer Architecture News ISCA '10, 38(3):451–460, June 2010.
- [5] J. D. Owens et al. A survey of general-purpose computation on graphics hardware. *Computer Graphics Forum*, 26(1):88–113, March 2007.
- [6] H. Wong et al. Demystifying GPU microarchitecture through microbenchmarking. In 2010 IEEE International Symposium on Performance Analysis of Systems and Software (ISPASS), pages 235–246, March 2010.
- [7] Khronos OpenCL API Registry. http://www.khronos.org/registry/cl/.
- [8] Fixstars Corporation. The OpenCL Programming Book. Fixstars Corporation, March 2010.
- [9] NVIDIA GPU Computing Documentation. http://developer.nvidia.com/nvidia-gpu-computing-documentation.
- [10] Java Bindings for OpenCL. http://jocl.org/.
- [11] PyOpenCL OpenCL for Python. http://mathema.tician.de/software/pyopencl.

Appendix A

Code

A.1 Matrix Multiplication

```
matmul.cpp

#ifndef __REENTRANT
#define __REENTRANT
#endif

#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include <ct/cl.h>
#include <CL/cl.h>
#include <pthread.h>

#include <pthread.h>

#define MAX_SOURCE_SIZE (0×10000)
#define LOCAL_TILE_SIZE (16)
#define DEFAULT_WIDTH (LOCAL_TILE_SIZE * 128)
#define NUM_REPETITIONS (10)
```

```
int row = 0; /* the bag of tasks */ unsigned int n = 1024; /* Width/Height of the matrix*/ unsigned int workGroupWidth = 16;
18
19
20
21
      /* matrices in host memory */
22
     float * h_mem_a;
23
      float * h_mem_b;
24
     float* h_mem_c;
25
     pthread_mutex_t block; /* mutex lock for the bag */
26
27
28
      /* timer */
     double read_timer() {
29
           static bool initialized = false;
30
           static struct timeval start;
struct timeval end;
31
32
33
34
           if (!initialized){
35
                gettimeofday(&start , NULL);
initialized = true;
36
37
           gettimeofday(&end, NULL);
return (end.tv_sec - start.tv_sec) + 1.0e-6 * (end.tv_usec - start.tv_usec);
38
39
40
41
42
      /st Each worker calculates the values in one strip of the matrix.st/
     void *Worker (void*) {
43
                unsigned int sum, i, j, k;
44
45
46
                 while (true) {
47
                            /* get a row number from the bag */
                            pthread_mutex_lock (&block);
48
49
                            i = row++;
50
                            pthread_mutex_unlock (&block);
51
52
                            if (i >= n) break;
53
                            /* multiply the row */ for (j = 0; j < n; j++) { sum = 0;
54
55
56
                                       for (k = 0; k < n; k++)\{

sum += h_mem_a[i*n+k]*h_mem_b[k*n+j];
57
58
59
60
                                      \dot{h}_mem_c[i*n+j] = sum;
61
62
63
                 pthread_exit (NULL);
                 return NULL;
64
65
66
67
     /* Multiplies two matrices and
     * returns the time the calculation took */ double runOnCPU(int numWorkers) \{
68
69
70
                int i:
71
                {\color{red}\textbf{double}} \ \ {\color{gray}\textbf{start\_time}} \ , \ \ {\color{gray}\textbf{end\_time}} \ ;
72
                 pthread_attr_t attr;
73
74
75
76
                 pthread_t workerid[numWorkers];
                /* set global thread attributes */
pthread_attr_init (& attr);
pthread_attr_setscope (& attr, PTHREAD_SCOPE_SYSTEM);
77
78
79
                 pthread_mutex_init (&block, NULL);
80
81
82
                 /st do the parallel work: create the workers st/
                 start_time = read_timer();
83
84
                 for (i = 0; i < numWorkers; i++)
                            pthread_create (&workerid[i], &attr, Worker, NULL);
85
86
87
                 \quad \textbf{for} \ (\ i \ = \ 0\,; \ \ i \ < \ numWorkers\,; \ \ i++)
```

A.1. MATRIX MULTIPLICATION

```
88
89
                         pthread_join (workerid[i], NULL);
 90
                /* get end time */
 91
               end_time = read_timer();
 92
 93
               return end_time - start_time;
 94
 95
 96
 97
      /* Matrix mutltiplication of n*n matrices on an opencl device,
 98
        assumes even numbered matrix width */
      double runOnGPU(int buffer_time_included) {
 99
100
                /* OpenCL kernel related variables
               char *source_str;
101
102
               size_t source_size;
103
               cl_program program;
cl_kernel kernel;
104
105
106
                /* Device and Platform related variables */
107
               cl_platform_id platform_id;
               cl_uint ret_num_platforms;
cl_device_id device_id;
108
109
110
               cl_uint ret_num_devices;
111
112
                /* context */
113
               cl_context context;
114
115
                /* command queue */
               cl_command_queue command_queue;
116
117
118
                 * memory buffers */
119
               cl_mem d_mem_a, d_mem_b, d_mem_c;
120
                /* error return value */
121
122
               cl_int ret;
123
124
               int size = n * n * sizeof(float);
125
               double startTime, endTime;
126
               /* OpenCL setup */
127
128
129
               /* Load OpenCL kernel */
130
               FILE *fp;
131
132
                {\bf char} \ \ {\tt fileName[]} \ = \ "./\, {\tt matmul.cl}\, "; \\
133
               /* Load kernel source */
fp = fopen(fileName, "r");
134
135
               if(!fp) {
    fprintf(stderr, "Failed to load kernel.\n");
136
137
138
139
140
               source_str = (char*) malloc (MAX_SOURCE_SIZE);
source_size = fread (source_str, 1, MAX_SOURCE_SIZE, fp);
141
142
143
               fclose (fp);
144
145
               /* Get Platform and Device info*/
146
               ret = clGetPlatformIDs(1, &platform_id, &ret_num_platforms);
ret = clGetDeviceIDs(platform_id, CL_DEVICE_TYPE_GPU, 1, &device_id,
147
148
149
                         &ret_num_devices);
150
151
                /* Context creation */
               context = clCreateContext(NULL, 1, &device_id, NULL, NULL, &ret);
152
153
154
                /* Command queue creation */
               command_queue = clCreateCommandQueue(context, device_id, 0, &ret);
155
156
157
                /* Create memory buffers */
158
                /* Write-buffers */
```

```
159
160
161
162
                                    /* Read—buffer */
                                  d_mem_c = clCreateBuffer(context, CL_MEM_WRITE_ONLY, size, NULL, &ret);
163
164
165
                                   /* create kernel program *
                                  program = clCreateProgramWithSource(context, 1,
166
167
                                                       (const char **)&source_str, (const size_t *)&source_size, &ret);
168
169
                                   /* build kernel program */
170
                                  ret = clBuildProgram(program, 1, &device_id, NULL, NULL, NULL);
171
172
                                   /* create kernel */
                                  kernel = clCreateKernel(program, "matMul", &ret);
173
174
175
                                  if (buffer_time_included)
176
                                                       startTime = read_timer();
177
                                  178
179
180
181
182
183
184
                                  if \ (\,!\, buffer\_time\_included\,)
185
                                                       startTime = read_timer();
186
                                  /* set kernel arguments */
clSetKernelArg(kernel, 0, sizeof(cl_mem), (void*)&d_mem_a);
clSetKernelArg(kernel, 1, sizeof(cl_mem), (void*)&d_mem_b);
clSetKernelArg(kernel, 2, sizeof(cl_mem), (void*)&d_mem_c);
187
188
189
190
191
                                  clSetKernelArg(kernel, 3, sizeof(int), (void*)&n);
192
                                  /* set work dimensions */
size_t globalWorkSize[2], localWorkSize[2];
193
194
                                  size_t globalWorkSize[0] = n;
globalWorkSize[1] = n;
localWorkSize[0] = workGroupWidth;
localWorkSize[1] = workGroupWidth;
195
196
197
198
199
200
                                     * Execute kernel */
201
                                  clEnqueueNDRangeKernel(command_queue, kernel, 2, NULL, globalWorkSize,
202
                                                       localWorkSize, 0, NULL, NULL);
203
                                  \label{eq:command_queue} /*\ read\ kernel\ result\ into\ C\ */\ clEnqueueReadBuffer(command_queue,\ d\_mem\_c,\ CL\_TRUE,\ 0,\ size\ ,\ (\verb"void*)h\_mem\_c,\ CL\_TRUE,\ 0,\ size\ ,\ (\verb"void*)h\_mem\_c,\ (\verb"void")h\_mem\_c,\ (\ "void")h\_mem\_c,\ (\ "
204
205
206
                                                        0, 0, NULL);
207
208
                                  endTime = read_timer();
209
210
211
                                  /* free resources */
cIFlush (command_queue);
cIFinish (command_queue);
212
213
214
                                  clReleaseMemObject(d_mem_a);
215
                                  clReleaseMemObject(d_mem_b);
                                  clReleaseMemObject(d_mem_c);
clReleaseCommandQueue(command_queue);
216
217
                                  clReleaseContext(context);
218
                                  clReleaseKernel(kernel);
219
220
                                  clReleaseProgram (program);
221
222
                                  free(source_str);
223
224
                                  return endTime-startTime;
225
226
227
              /st call with "matmul n work{\sf GroupWidth} num{\sf Workers} " st/
             int main(int argc, char *argv[]) {
228
                                  unsigned int i;
229
```

A.1. MATRIX MULTIPLICATION

```
230
                                              \begin{array}{lll} n=\left(\text{argc}>1\right)~?~\text{atoi}\left(\text{argv}\left[1\right]\right)~:~n;\\ \text{workGroupWidth}=\left(\text{argc}>2\right)~?~\text{atoi}\left(\text{argv}\left[2\right]\right)~:~\text{workGroupWidth};\\ \textbf{int}~\text{numWorkers}=\left(\text{argc}>3\right)~?~\text{atoi}\left(\text{argv}\left[3\right]\right)~:~2; \end{array}
231
232
233
234
235
                                               if (n%workGroupWidth != 0){
                                                                           printf("n needs to be a multiple of workGroupWidth\n"); exit(1);
236
237
238
                                              }
239
240
                                               /* Initialize matrices */
241
^{242}
                                              h_mem_a = (float*) malloc(n*n*sizeof(float));
                                              h_mem_b = (float*) malloc(n*n*sizeof(float));
h_mem_c = (float*) malloc(n*n*sizeof(float));
243
244
245
246
                                              srand((unsigned) time(NULL));
247
248
                                               \mbox{for} \, (\, i \, = \, 0\, ; \  \, i \, < \, n \! * \! n \, ; \  \, i + \! + \! ) \{ \,
                                                                          \begin{array}{lll} & & & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\
^{249}
250
251
252
253
                                               for (i = 0; i < n*n; i++){}
254
                                                                           h_mem_c[i] = 0;
255
256
                                              \label{eq:printf("Runnning matrix_multiplication on an %dx%d matrix \n", n, n); $$ printf("Local Work Size = %dx%d\n", workGroupWidth, workGroupWidth); $$ printf("Number of CPU workers = %d\n", numWorkers); $$ printf("ITER\tGPU(EX.BUF)\t\tGPU(INC.BUF)\t\tCPU\n"); $$
257
258
259
260
261
262
                                                /* Start performance testing */
263
                                               \label{eq:continuous_double_gpu_time_yes_buffer_sum} \ = \ 0;
                                              double gpu_time_no_buffer_sum = 0;
double cpu_time_sum = 0;
for(i = 0; i < NUM_REPETITIONS; i++){</pre>
264
265
266
                                                                           double tmp_no_buf = runOnGPU(0);
267
                                                                           double tmp_yes_buf = runOnGPU(1);
double tmp_cpu = runOnCPU(numWorkers);
268
269
270
271
                                                                            gpu_time_no_buffer_sum += tmp_no_buf;
272
                                                                           gpu_time_yes_buffer_sum += tmp_yes_buf;
273
                                                                            cpu_time_sum += tmp_cpu;
274
                                                                            printf("%d\t%gs\t\t%gs\t\t%gs\n",
275
276
                                                                                                       i, tmp_no_buf, tmp_yes_buf, tmp_cpu);
                                                                           row = 0;
277
278
279
                                              280
281
282
283
284
                                               285
286
                                                                           cpu_time_sum, cpu_time_sum/NUM_REPETITIONS);
287
288
                                              free(h_mem_a);
free(h_mem_b);
free(h_mem_c);
289
290
291
292
293
                                               return 0;
294
                 }
```

A.2 Matrix Max

```
matrixmax.cpp
     #ifndef _REENTRANT
#define _REENTRANT
     ...
#endif
     #include <pthread.h>
#include <stdio.h>
#include <stdlib.h>
     #include <math.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>
10
11
12
     #ifdef __APPLE__
#include <OpenCL/opencl.h>
14
15
     #else
     #include <CL/cl.h>
16
     #endif
17
18
19
     #define MAX_SOURCE_SIZE (0×100000)
20
21
22
     size_t matrixSize, rowSize;
     /* default size of local work on gpu */
size_t localSize = 16;
23
^{-24}
25
     /* default number of workers in cpu-implementation */unsigned int numWorkers = 4;
26
27
28
29
      /* default size of the n x n matrix */
     unsigned int n = 2048;
30
31
     /*\ default\ value\ for\ including\ matrix-to-gpu\ transfer\ time\ */bool\ gpu\_buffer\_time\_included\ =\ true\ ;
32
33
34
     \mbox{unsigned int} \ \mbox{NUM\_REPETITIONS} \ = \ 10;
35
36
     float *h_mem_matrix;
37
38
      int gmax;
39
      int row = 0; /* the bag of tasks */
40
     41
42
```

A.2. MATRIX MAX

```
44
       void *Worker (void *);
 45
      void getFileContent(const char* filename, char** source_str, size_t* source_size){
 46
 47
                  FILE* fp;
fp = fopen(filename, "r");
 48
                  if (! fp){
 49
                             fprintf(stdout, "Failed to load file\n");
 50
51
52
                             exit (1);
                  source_str = (char*) malloc(MAX_SOURCE_SIZE);
*source_size = fread(*source_str, 1, MAX_SOURCE_SIZE, fp);
 53
 54
 55
 56
 57
      /* timer */
double read_timer() {
    static bool initialized = false;
 58
 59
 60
            static struct timeval start;
struct timeval end;
 61
 62
 63
            if(!initialized)
 64
 65
                  {\tt gettimeofday(\&start, NULL);}\\
 66
                  initialized = true;
 67
 68
            gettimeofday ( &end , NULL );
 69
            return (end.tv_sec - start.tv_sec) + 1.0e-6 * (end.tv_usec - start.tv_usec);
\frac{70}{71}
 72
      /* Calculates the maximum value of a matrix and
 * returns the time the calculation took.
 * Uses multiple worker threads and a bag of tasks.*/
 73
 74
 75
      double runOnCPU(int numWorkers) {
76
77
78
                  int i;
                  double start_time, end_time;
pthread_attr_t attr;
pthread_t workerid[numWorkers];
 79
 80
 81
                  /* set global thread attributes */
                  pthread_attr_init (&attr);
pthread_attr_setscope (&attr, PTHREAD_SCOPE_SYSTEM);
82
83
 84
 85
                  /* initialize mutexes */
 86
                  pthread_mutex_init(&lmax, NULL);
 87
                  pthread_mutex_init (&block, NULL);
 88
89
90
                  gmax = h_mem_matrix[0];
                  /\ast do the parallel work: create the workers \ast/ start_time = read_timer();
 91
 92
 93
                  for (i = 0; i < numWorkers; i++)
 94
 95
                     pthread_create (&workerid[i], &attr, Worker, NULL);
 96
                  \begin{array}{lll} \textbf{for} & (i = 0; i < numWorkers; i++) \\ & \texttt{pthread\_join} & (workerid[i], NULL); \end{array}
 97
 98
 99
100
                  /* get end time */
                  end_time = read_timer();
101
102
103
                  return end_time - start_time;
104
105
106
      /* Each worker determines the max value in one strip of the matrix.
      ^{'}* After each updates the global max if the local is lager */ \mathbf{void} *Worker (\mathbf{void}*) {
107
108
                 int max, worked = 0;
unsigned int i, j;
109
110
111
                  max = h_mem_matrix[0];
112
113
                  while (true) {
114
                             /* get a row number from the bag */
```

```
115
                           pthread_mutex_lock (&block);
116
                           i = row++;
                           pthread_mutex_unlock (&block);
117
118
                           \begin{array}{ll} \mbox{if } (\mbox{i} >= \mbox{n}) \mbox{ break}; \\ \mbox{if } (\mbox{! worked}) \mbox{ worked} = 1; \end{array}
119
120
121
122
                           /st update local max with elements in the row st/
                           /* update local max ......

for (j = 0; j < n; j++) {
    if (max < h_mem_matrix[i*n+j]) {
        max = h_mem_matrix[i*n+j];
}
123
124
125
126
127
128
                if (worked) {
129
                           /* update global max */
if (gmax < max) {
130
131
132
                                     pthread_mutex_lock(&lmax);
133
                                     if (gmax < max) {
134
                                               gmax = max;
135
136
                                     pthread_mutex_unlock(&lmax);
137
                           }
138
139
                pthread_exit (NULL);
140
                return 0; /* avoid compiler warning */
141
142
      /* return the runtime on the GPU */
143
144
      double runOnGPU(){
145
                const char file_name[] = "./matrix_max.cl";
146
147
                char * source;
148
                size_t source_size;
149
150
                double start_time, end_time;
151
                start_time = end_time = 0.0;
152
153
                float *h_mem_result;
154
                cl_mem d_mem_matrix;
cl_mem d_mem_result;
155
156
157
158
                cl_platform_id platform;
159
                cl_device_id device;
160
                cl_context context;
161
                cl_command_queue queue;
162
                cl_program program;
163
                cl_kernel kernel;
164
                /* Load OpenCL kernel source code */
165
                getFileContent(file_name, &source, &source_size);
166
167
                h_mem_result = (float*) malloc(rowSize);
168
169
                size_t globalSize;
cl_int err;
170
171
172
                {\tt globalSize} \ = \ {\tt ceil} \, \big( \, {\tt n/(\,float\,)} \, {\tt localSize} \, \big) * {\tt localSize} \, ;
173
174
175
                /* Get platform and device info */
                err = clGetPlatformIDs(1, &platform, NULL);
176
                {\tt err = clGetDeviceIDs(platform, CL\_DEVICE\_TYPE\_GPU, 1, \&device, NULL);}
177
178
                context = clCreateContext(0, 1, &device, NULL, NULL, &err);
179
180
181
                queue = clCreateCommandQueue(context, device, 0, &err);
182
183
                program = clCreateProgramWithSource(context, 1, (const char **) & source,
184
                                     (const size_t*) &source_size, &err);
185
```

A.2. MATRIX MAX

```
186
187
188
189
                                   /* Create kernel */
                                  kernel = clCreateKernel(program, "matrixMax", &err);
190
191
                                 \label{lem:context}  \slashed{/*}  \slashed{/*} \slashed{/*}  \slashed{/*}  \slashed{/*}  \slashed{/*}  \slashed{/*}  \slashed{/*}  \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \slashed{/*} \sla
192
193
                                                                         NULL);
194
195
                                 {\tt d\_mem\_result} = {\tt clCreateBuffer(context, CL\_MEM\_WRITE\_ONLY, rowSize, NULL,}
196
                                                                         NULL);
197
198
                                 /* if we want to include time to copy data to GPU,
199
                                     * start timer now */
                                 200
201
202
203
                                  /* Copy matrix to GPU *
204
                                 err = clEnqueueWriteBuffer(queue, d_mem_matrix, CL_TRUE, 0, matrixSize, h_mem_matrix, 0, NULL, NULL);
205
206
                                 /* if we don't want to include time to copy data to GPU,
   * start timer now */
if(!gpu_buffer_time_included)
207
208
209
210
                                                      start_time = read_timer();
211
                                 /* Set kernel arguments */
err = clSetKernelArg(kernel, 0, sizeof(cl_mem), &d_mem_matrix);
err |= clSetKernelArg(kernel, 1, sizeof(cl_mem), &d_mem_result);
err |= clSetKernelArg(kernel, 2, sizeof(unsigned int), &n);
212
213
214
215
216
217
                                  /* Execute kernel */
218
                                  err = clEnqueueNDRangeKernel(queue, kernel, 1, NULL, &globalSize, &localSize,
219
                                                                         0, NŪLL, NULL);
220
221
                                 clFinish (queue);
222
223
                                   /* Read kernel result */
                                 clEnqueueReadBuffer(queue, d_mem_result, CL_TRUE, 0, rowSize, h_mem_result, 0, NULL, NULL);
224
225
226
227
                                  /* Calculate global max */
228
                                 float max = 0;
229
                                 unsigned int i;
                                 for (i = 0; i < n; i++){if (h_mem_result[i] > max)}
230
231
232
                                                                         max = h_mem_result[i];
233
                                                     }
234
235
236
                                 end_time = read_timer();
237
                                 /* Free resources */
cIFlush (queue);
cIFinish (queue);
238
239
240
241
                                 clReleaseMemObject(d_mem_matrix);
^{242}
                                 clReleaseMemObject (d_mem_result );
243
                                 clReleaseCommandQueue(queue);
                                 cIReleaseContext (context);
cIReleaseKernel (kernel);
244
245
                                 clReleaseProgram (program);
246
247
248
                                  free (source);
\frac{249}{250}
                                 free(h_mem_result);
251
                                 return end_time-start_time;
252
253
            /* Arg1: n (for an n x n matrix)
* Arg2: local work size when running on GPU
254
255
                * Arg3: numWorkers for CPU calculation */
256
```

```
257
               int main(int argc, char* argv[]){
258
259
                                        /* read command line */
                                       \begin{array}{lll} \text{The action main of } & \text{The action } & \text{The 
260
261
262
263
264
                                        matrixSize = n*n*sizeof(float);
265
                                       rowSize = n*sizeof(float);
266
267
                                       h_mem_matrix = (float*) malloc(matrixSize);
268
269
                                       srand((unsigned) time(NULL));
270
271
                                        /st initialize matrix with random floats st/
272
                                       unsigned int i; unsigned int j;
273
274
                                       for (i = 0; i < n; i++)
275
                                                               for (j = 0; j < n; j++){
                                                                                       h\_mem\_matrix[\hat{i*n}+j] = ((float)rand()/(float)RAND\_MAX)*2000;
276
277
278
                                       }
279
280
                                         /* Total times */
281
                                       double gpu_time_buffers_included_sum = 0.0;
282
                                        double gpu_time_buffers_excluded_sum = 0.0;
283
                                       double cpu\_time\_sum = 0.0;
284
                                       \label{eq:printf}  \begin{aligned} & printf("Running matrix_max on an %d x %d matrix \n", n, n); \\ & printf("Local Work Size = %d \n", localSize); \\ & printf("Number of CPU workers = %d \n", numWorkers); \\ & printf("ITER \tGPU(EX.BUF) \t\tGPU(INC.BUF) \t\tCPU \n"); \end{aligned}
285
286
287
288
289
290
                                       \begin{tabular}{lll} /* run & calculations */ \\ for ( i = 0; & i < NUM\_REPETITIONS; & i++) \{ \end{tabular}
291
292
293
                                                                gpu_buffer_time_included = true;
294
                                                                double tmp_gpu_included = runOnGPU();
295
296
                                                                gpu\_buffer\_time\_included = false;
297
                                                                \label{eq:double_double} \textbf{double} \  \, \texttt{tmp\_gpu\_excluded} \, = \, \texttt{runOnGPU} \, ( \, ) \, ;
298
299
                                                                double tmp_cpu = runOnCPU(numWorkers);
300
301
                                                                gpu_time_buffers_included_sum += tmp_gpu_included;
302
                                                                {\tt gpu\_time\_buffers\_excluded\_sum} \ +\!\!= \ {\tt tmp\_gpu\_excluded};
303
                                                                cpu_time_sum += tmp_cpu;
304
305
                                                                printf("%d\t%gs\t\t%gs\t\t%gs\n"
306
                                                                                                                 i, tmp_gpu_excluded, tmp_gpu_included, tmp_cpu);
307
                                                                row = 0:
308
                                       }
309
310
                                       printf("
                                        printf("WHAT\t\t\t\tTIME(TOT)\tTIME(AVG)\n");
311
                                        printf("GPU time (Transfer time excluded)\t %gs\t%gs\n",
312
                                       313
314
315
316
317
318
319
                                                                                        cpu_time_sum, cpu_time_sum/NUM_REPETITIONS);
320
```

Appendix B

Execution Output

```
Runnning matrix_multiplication on an 128x128 matrix
     Local Work Size = 4x4
Number of CPU workers = 4
ITER GPU(EX.BUF)
 2
3
4
5
6
7
8
9
                                                 GPU(INC.BUF)
                0.014544s
                                                 0.006908s
                                                                                   0.027684s
                0.006921\,s
                                                 0.008005 s
                                                                                   0.007776s
                0.006097 \, s
                                                 0.008721\,\mathsf{s}
                                                                                  0.007641s
                0.007118s
                                                 0.008943s
                                                                                  0.007415s
                0.006907s
                                                                                  0.008947s
                                                 0.008053s
10
                0.006178s
                                                 0.008495s
                                                                                  0.010691s
11
                0.006963s
                                                 0.006974s
                                                                                  0.007507s
12
                0.007048s
                                                 0.008876s
                                                                                  0.007663s
13
                0.006978s
                                                 0.008522\,s
                                                                                  0.00744 \, s
14
     9
                0.006996s
                                                 0.008615s
                                                                                  0.007399 \, s
15
     WHAT
                                                            TIME(TOT)
                                                                                  TIME(AVG)
16
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                              0.07575s
                                                                                  0.007575s
17
                                                              0.082112s
                                                                                   0.0082112s
19
     CPU time
                                                              0.100163\,\mathsf{s}
                                                                                  0.0100163\,\mathrm{s}
     Running matrix_multiplication on an 128x128 matrix
Local Work Size = 8x8
Number of CPU workers = 4
ITER GPU(EX.BUF) GPU(INC.BUF)
20
21
22
23
24
                0.002142s
                                                 0.004609s
                                                                                   0.012336s
25
                0.002447s
                                                 0.003991s
                                                                                   0.00741s
26
27
28
     2
                0.00295\,\mathsf{s}
                                                 0.004738\,\mathrm{s}
                                                                                  0.007363s
                0.00289s
                                                 0.004231s
                                                                                  0.007529s
                0.002992s
                                                 0.004124s
                                                                                  0.00739s
29
                0.002995s
                                                 0.004022s
                                                                                  0.007769s
30
                0.002773s
                                                                                  0.007511\,\mathrm{s}
                                                 0.00312\,\mathsf{s}
31
                0.002745s
                                                 0.003945\,s
                                                                                  0.007384 s
32
                0.00287\,\mathrm{s}
                                                 0.003956 \, s
                                                                                  0.007417 s
33
                0.00209 s
                                                 0.004641s
                                                                                  0.007871s
34
35
                                                            TIME(TOT)
                                                                                  TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
36
                                                                                  0.0026894s
                                                              0.041377s
                                                                                   0.0041377s
38
     CPU time
                                                              0.07998s
                                                                                  0.007998\,\mathsf{s}
     Runnning matrix_multiplication on an 128x128 matrix
39
     Local Work Size = 16 \times 16
Number of CPU workers = 4
40
41
42
                GPU(EX.BUF)
                                                 GPU(INC.BUF)
                                                                                  CPU
43
                0.002454s
                                                 0.003739s
                                                                                   0.010249s
44
                0.002217s
                                                 0.003702 s
                                                                                   0.007481s
\frac{45}{46}
     2
                0.00138s
                                                 0.003782 s
                                                                                  0.00745s
                0.002295 s
                                                 0.00393s
                                                                                  0.007502 s
47
                0.001965s
                                                 0.002118s
                                                                                  0.007514s
                0.002067s
                                                 0.003862s
                                                                                   0.010289s
```

```
49
                0.002362\,s
                                                0.003275s
                                                                               0.007396s
                0.0023185
                                                0.0039255
 50
                                                                               0.0075495
                0.002377s
                                               0.004092s
                                                                               0.010567s
 51
      8
 52
      9
                0.002124s
                                                0.003987s
                                                                               0.008268s
 53
 54
      WHAT
                                                          TIME(TOT)
                                                                              TIME(AVG)
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
 55
                                                           0.021559s
                                                                               0.0021559s
 56
                                                           0.0364125
                                                                               0.00364125
      CPU time
 57
                                                           0.084265s
                                                                               0.0084265 s
      Runnning matrix_multiplication on an 256x256 matrix
Local Work Size = 4x4
Number of CPU workers = 4
ITER GPU(EX.BUF) GPU(INC.BUF)
 58
 59
 60
                                                                              CPU
 61
                                                                              0.061802s
                0.047152s
 62
      0
                                               0.048985s
 63
                0.047153s
                                               0.048146s
      1
                                                                               0.05825s
 64
                0.047097s
                                               0.048916s
                                                                               0.06081s
                                               0.048837s
 65
                0.047138s
                                                                               0.059228s
 66
      4
                0.047322s
                                               0.048462s
                                                                               0.058271s
 67
      5
                0.046546s
                                               0.049\,s
                                                                    0.058146s
                                                                              0.05831s
68
69
      6
                0.047019s
                                               0.048893s
                0 0470995
                                               0 0499445
                                                                               0.058315s
 70
                0.046271s
                                               0.048926s
                                                                               0.058278s
      8
 71
      9
                0.046692s
                                               0.048705s
                                                                               0.060958s
 72
 73
      WHAT
                                                          TIME(TOT)
                                                                              TIME (AVG)
 74
      GPU time (Transfer time excluded)
                                                           0.469489s
                                                                              0.0469489s
      GPU time (Transfer time included)
75
76
                                                           0 4888145
                                                                              0.04888145
      CPU time
                                                           0.592368s
                                                                               0.0592368s
      Runnning matrix_multiplication on an 256x256 matrix Local Work Size = 8x8
 77
 78
      Number of CPU workers = 4
ITER GPU(EX.BUF)
 79
                                               GPU(INC.BUF)
                                                                              CPU
 80
 81
      0
                0.013063s
                                               0.013069\,\mathrm{s}
                                                                               0.062792s
 82
                0.012985s
                                               0.0144s
                                                                    0.058684s
 83
                0.012305s
                                               0.015225s
                                                                               0.06699s
 84
      3
                0.013208s
                                               0.014056s
                                                                               0.062544s
 85
      4
                0.012204s
                                               0.013156s
                                                                               0.060832s
 86
      5
                0.012187 s
                                               0.01369\,\mathsf{s}
                                                                               0.069126s
 87
      6
                0.012257s
                                               0.0148925
                                                                               0.05856s
 88
                0.012228s
                                               0.013297s
                                                                               0.065233 s
 89
      8
                0.012079s
                                               0.013533s
                                                                               0.060112s
 90
      9
                0.0123\,\mathrm{s}
                                     0.013678s
                                                                    0.059022s
 91
 92
      WHAT
                                                          TIME(TOT)
                                                                              TIME(AVG)
      GPU time (Transfer time excluded) GPU time (Transfer time included)
 93
                                                           0.124816s
                                                                              0.0124816s
 94
                                                           0 1389965
                                                                              0.01389965
 95
      CPU time
                                                                              0.0623895s
                                                           0.623895s
      Runnning matrix_multiplication on an 256x256 matrix
Local Work Size = 16x16
Number of CPU workers = 4
ITER GPU(EX.BUF) GPU(INC.BUF)
 96
 98
                                                                              CPU
 99
                0.006952s
                                                                               0.082145s
100
      0
                                               0.00807s
                0.006935s
                                               0.007612s
101
                                                                               0.078144s
                0.006991s
102
                                               0.007632s
                                                                               0.081516s
      3
                0.006937s
                                               0.007787s
                                                                               0.077798s
103
104
      4
                0.006924s
                                               0.007701s
                                                                               0.077695s
105
      5
                0.006826s
                                               0.008497s
                                                                               0.05988s
106
      6
                0.007877s
                                               0.009768s
                                                                               0.058996s
                0.007949s
107
                                               0.009742s
                                                                               0.058145s
                0.007892s
                                               0.009748s
108
      8
                                                                               0.058245s
                0.007751s
109
      9
                                                0.008269s
                                                                               0.061847s
110
111
     WHAT
                                                          TIME(TOT)
                                                                              TIME(AVG)
      GPU time (Transfer time excluded) GPU time (Transfer time included)
112
                                                           0.073034\,s
                                                                              0.0073034s
                                                           0.0848265
                                                                               0.00848265
113
      CPU time
                                                           0.694411s
                                                                               0.0694411s
114
      Runnning matrix_multiplication on an 512×512 matrix
115
      Local Work Size = 4x4
Number of CPU workers = 4
ITER GPU(EX.BUF)
116
117
                                                                              CPU
                                               GPU(INC.BUF)
118
119
      0
                0.36394s
                                               0.369404s
                                                                              0 4750855
```

```
120
               0.365542s
                                           0.368877s
                                                                       0.472146s
121
     2
               0 3644225
                                           0.3687295
                                                                       0.475139s
               0.364867s
                                           0.368139s
122
     3
                                                                       0.471157s
123
     4
               0.365211s
                                           0.367603s
                                                                       0.473709s
124
               0.364226s
                                           0.367755s
                                                                       0.473935s
125
               0.364707s
                                           0.367439s
                                                                       0.467896s
126
               0.365334s
                                           0.368103s
                                                                       0.469914s
127
     8
               0.364685
                                           0.3674935
                                                                       0.469593s
128
               0.365196s
                                           0.367271s
                                                                       0.46983s
     9
129
130
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
     GPU time (Transfer time excluded)
                                                     3.64812s
                                                                       0.364812s
131
                                                                       0.368081s
132
     GPU time (Transfer time included)
                                                      3.68081s
133
     CPU time
                                                      4.7184s
                                                                       0.47184s
     Runnning matrix_multiplication on an 512x512 matrix
134
     Local Work Size = 8x8
Number of CPU workers = 4
135
136
137
     ITER
               GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                       CPU
138
     0
               0.093214s
                                           0.101403s
                                                                       0.479275s
                                                                       0.479455s
139
               0.09406s
                                           0.09623s
               0.0942955
                                           0.0972465
                                                                       0.475197s
140
     2
               0.092446s
                                           0.096514s
                                                                       0.466775s
141
     3
142
     4
               0.093139s
                                           0.096261 s
                                                                       0.467282s
               0.093233s
                                           0.096633s
                                                                       0.472997s
143
144
     6
               0.093517s
                                           0.096412s
                                                                       0.467624s
145
               0.093264s
                                           0.095925\,\mathsf{s}
                                                                       0.479448s
146
     8
               0.0936875
                                           0 0959545
                                                                       0 468235
               0.093502s
                                           0.095945s
                                                                       0.478312s
147
     9
148
                                                    TIME(TOT)
149
     WHAT
                                                                       TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                      0.934357s
                                                                       0.0934357s
150
                                                                       0.0968523s
151
                                                     0.968523s
152
     CPU time
                                                      4.73459s
                                                                       0.473459s
     Running matrix_multiplication on an 512x512 matrix Local Work Size = 16x16 Number of CPU workers = 4
153
154
155
               GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                       CPU
156
     ITER
157
     0
               0.051152s
                                           0.054221s
                                                                       0.480828s
158
     1
               0.051684s
                                           0.053914s
                                                                       0.478244s
                                                                       0.474266s
159
     2
               0.050385s
                                           0.051905s
               0.050783s
                                                                       0.491792s
160
     3
                                           0.053953s
161
               0.050828s
                                           0.053437 \, s
                                                                       0.46962s
               0.050096s
                                           0.053974s
                                                                       0.470617s
162
163
               0.050905s
                                           0.053441s
                                                                       0.4863s
     6
164
               0.050841s
                                           0.053495\,\mathsf{s}
                                                                       0.470826s
165
     8
               0.0508215
                                           0.0534765
                                                                       0 4702125
               0.050831s
     9
                                           0.053445s
                                                                       0.469887s
166
167
168
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
169
     GPU time (Transfer time excluded)
                                                      0.508326s
                                                                       0.0508326s
170
     GPU time (Transfer time included)
                                                      0.535261s
                                                                       0.0535261s
     CPU time
171
                                                      4 762595
                                                                       0.476259s
     Runnning matrix_multiplication on an 1024 \times 1024 matrix
172
      Local Work Size = 4x4
173
     Number of CPU workers = 4
174
175
     ITER
               GPU(EX.BUF)
                                          GPU(INC.BUF)
                                                                       CPU
                                                              11.4591s
176
     0
               2.9067s
                                  2.91092\,\mathsf{s}
               2.90139s
                                           2.90811s
                                                                       11.3965s
177
     1
               2.90752s
                                           2.90925s
                                                                       11.4244s
178
     2
                                                                       11.4002s
179
     3
               2.89842s
                                           2.91144s
180
               2.90246s
                                           2.90893s
                                                                       11.5065s
181
               2.90252s
                                           2.90519s
                                                                       11.3982s
182
     6
               2.90845s
                                           2.91526s
                                                                       11.2742s
                                  2.9067s
183
               2.9083s
                                                    11.3131s
                                           2.90797s
               2 901045
                                                                       11 3955s
184
     8
185
     9
               2.90034s
                                           2.91257s
                                                                       11.2565s
186
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
187
188
     GPU time (Transfer time excluded)
                                                      29.0371s
                                                                       2.90371s
189
     GPU time (Transfer time included)
                                                      29 09635
                                                                       2 909635
190
     CPU time
                                                     113.824s
                                                                       11.3824s
```

```
191
     Runnning matrix_multiplication on an 1024×1024 matrix
     Local Work Size = 8x8
Number of CPU workers = 4
192
193
194
     ITER
              GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                       CPU
               0.732165s
                                                                       11.8158s
195
                                           0.745469s
196
               0.729733s
                                           0.736106s
                                                                       11.7965s
197
               0.731051s
                                           0.735587s
                                                                       11.7032s
198
     3
               0.7301535
                                           0.736223s
                                                                       11.7488s
199
     4
               0.730235 s
                                           0.736733s
                                                                       11.7934s
200
               0.73018s
                                           0.736837s
                                                                       11.6443s
     5
201
     6
               0.729615s
                                           0.735408s
                                                                       11.7064s
202
               0.729015 s
                                           0.738169 \, s
                                                                       11.8337s
203
     8
               0 7282925
                                           0 7357925
                                                                       11 7321s
204
     9
               0.728999s
                                           0.733398s
                                                                       11.8367s
205
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
206
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                     7.29944s
207
                                                                       0.729944s
208
                                                      7.36972s
                                                                       0.736972s
209
     CPU time
                                                      117.611s
                                                                       11.7611s
210
     Runnning matrix_multiplication on an 1024 \times 1024 matrix
     Local Work Size = 16x16
Number of CPU workers = 4
211
212
              GPU(EX.BUF)
213
     ITER
                                           GPU(INC.BUF)
                                                                       CPU
214
               0.394965s
                                           0.409668s
                                                                       11.5972s
215
               0.392015s
                                           0.398419s
                                                                       11.5878s
216
     2
               0.391848s
                                           0.39935\,\mathrm{s}
                                                                       11.4787\,\mathsf{s}
217
     3
               0 3917845
                                           0 3987545
                                                                       11.3879s
               0.391632s
218
     4
                                           0.398469s
                                                                       11.3492s
219
               0.391253s
                                           0.399194s
                                                                       11.4741s
     5
               0.391657s
220
     6
                                           0.39804s
                                                                       11.5484s
221
               0.391634s
                                           0.399436s
                                                                       11.324s
               0.392128s
222
     8
                                           0.398106s
                                                                       11 41235
223
     9
               0.39186s
                                           0.399134 s
                                                                       11.4966s
224
225
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
     GPU time (Transfer time excluded)
                                                     3.92078s
226
                                                                       0.392078s
227
     GPU time (Transfer time included)
                                                      3.99857s
                                                                       0.399857s
228
     CPU time
                                                      114.656s
                                                                       11.4656s
229
     Runnning matrix_multiplication on an 1536 \times 1536 matrix
     Local Work Size = 4x4
230
     Number of CPU workers = 4
231
232
     ITER
              GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                       CPU
233
               9.79704s
                                           9.80289s
                                                                       39.6244s
234
               9.79296s
                                           9.80364s
                                                                       39.5768s
235
     2
               9.79746s
                                           9.80418\,\mathsf{s}
                                                                       40.0286s
236
     3
               9 787645
                                           9 807145
                                                                       39 74195
237
                                                                       39.4886s
     4
               9.80338s
                                           9.81078s
238
               9.80744s
                                                                       39.4701s
     5
                                           9.82036s
               9.8015s
239
     6
                                  9.8157s
                                                    39.3647s
                                           9.81367s
240
               9.81195s
                                                                       39.4671s
241
     8
               9.80586s
                                           9.81067s
                                                                       39.349s
                                                             39.2807s
242
     9
               9.7926s
                                  9 80951s
243
                                                    TIME(TOT)
244
     WHAT
                                                                       TIME(AVG)
     GPU time (Transfer time excluded)
                                                      97.9978s
                                                                       9.79978s
245
246
     GPU time (Transfer time included)
                                                      98.0985s
                                                                       9.80985s
247
     CPU time
                                                      395.392s
                                                                       39.5392s
248
     Runnning matrix_multiplication on an 1536 \times 1536 matrix
     Local Work Size = 8x8
Number of CPU workers = 4
249
250
              GPU(EX.BUF)
                                           GPU(INC.BUF)
251
     ITER
                                                                       CPU
252
               2.46612s
                                           2.48582s
                                                                       40.0789s
253
               2.45816s
                                           2.4707s
                                                              40.2424s
     1
254
     2
               2.45953s
2.46116s
                                           2.47398s
                                                                       40 2495 s
255
     3
                                           2 474155
                                                                       40 04775
256
     4
               2.46024s
                                           2.47284s
                                                                       39.5299s
257
               2.45948s
                                           2.4725s
                                                              39.9242s
     5
258
               2.45944s
                                           2.47295s
                                                                       39.8136s
     6
259
               2.45958s
                                           2.47353s
                                                                       39.5433s
260
     8
               2 458245
                                           2 472395
                                                                       39 643 5
261
     9
               2.46103s
                                           2.47358s
                                                                       40.1815s
```

```
262
                                                                        TIME(AVG)
     WHAT
                                                     TIME (TOT)
263
     GPU time (Transfer time excluded)
264
                                                      24.603s
                                                                        2.4603s
     GPU time (Transfer time included)
                                                       24.7424s
                                                                        2.47424s
265
                                                       399.254s
266
     CPU time
267
      Runnning matrix_multiplication on an 1536×1536 matrix
     Local Work Size = 16 \times 16
Number of CPU workers = 4
268
269
               GPU(EX.BUF)
270
                                           GPU(INC.BUF)
                                                                        CPU
     ITER
271
               1.3248s
                                  1.33917s
                                                               39.0588s
     0
272
               1.31587s
                                            1.32876s
                                                                        39.1052s
               1.31722s
273
                                            1.3285\,\mathsf{s}
                                                               38.893s
274
     3
               1.31604s
                                            1 32837s
                                                                        38.6614s
275
     4
               1.31618s
                                            1.32476s
                                                                        39.1071s
276
     5
               1.31581s
                                            1.32906s
                                                                        38.9687s
                                            1.3292s
                                                               38.6317s
277
     6
               1.31627s
278
               1.31639s
                                            1.3309s
                                                               38.6012s
279
               1.31632s
                                            1.33138s
                                                                        38.2786s
280
     9
               1.31621s
                                            1.32875s
                                                                        38.5687s
281
     WHAT
                                                     TIME (TOT)
                                                                        TIME (AVG)
282
     GPU time (Transfer time excluded)
283
                                                      13.1711s
                                                                        1.31711s
284
     GPU time (Transfer time included)
                                                       13.2988s
                                                                        1.32988s
      CPU time
                                                       387.874s
285
286
      Runnning matrix_multiplication on an 2048×2048 matrix
     Local Work Size = 4x4
Number of CPU workers = 4
287
288
               GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                        CPU
289
     ITER
290
               23.209s
                                  23.2351s
                                                               104.623s
     0
               23.2174s
                                            23.2117s
                                                                        106.212s
291
292
               23.1939s
                                            23.2181s
                                                                        104.377s
293
     3
               23.1905s
                                            23.2172s
                                                                        104.21s
294
     4
               23.226s
                                  23.2317s
                                                               104.133s
295
               23.1873s
                                            23.2741s
                                                                        104.622s
     5
296
                                            23.2185s
                                                                        103.404s
     6
               23.1944s
               23.2106s
                                            23.2143s
                                                                        102.911s
297
298
     8
                                            23.2483s
                                                                        104.384s
               23.2202s
299
     9
               23.2119s
                                            23.2179s
                                                                        105.226s
300
                                                     TIME(TOT)
     WHAT
                                                                        TIME(AVG)
301
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
302
                                                      232.061s
                                                                        23.2061s
303
                                                       232.287s
                                                                        23.2287s
      CPU time
                                                       1044.1s
304
305
      Runnning matrix_multiplication on an 2048×2048 matrix
     Local Work Size = 8x8
Number of CPU workers = 4
306
307
               GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                        CPU
308
     ITER
309
               5.82678s
                                            5.85154s
                                                                        102.643s
     0
310
               5.81994s
                                            5.84253s
                                                                         100.954s
311
               5.81964s
                                            5.84165s
                                                                         102.538s
312
     3
               5.82112s
                                            5.83922s
                                                                        101.686s
313
     4
               5 8215s
                                  5.84715s
                                                               101 826s
                                                                        102.546s
                                            5.84122s
314
     5
               5.82161s
     6
                                                                        100.865s
315
               5.81516s
                                            5.84573s
316
               5.81573s
                                            5.83851s
                                                                        101.014s
     8
               5.8212s
                                  5.84242s
                                                               101.28s
317
318
     9
               5.82119s
                                            5.84431s
                                                                        101.131s
319
     WHAT
                                                     TIME(TOT)
                                                                        TIME(AVG)
320
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
321
                                                      58.2039s
                                                                        5.82039s
322
                                                       58.4343s
                                                                        5.84343s
323
     CPU time
                                                       1016.48s
                                                                         101.648s
324
     Runnning matrix_multiplication on an 2048x2048 matrix
     Local Work Size = 16 \times 16
Number of CPU workers = 4
325
326
               GPU(EX.BUF)
327
     ITER
                                           GPU(INC.BUF)
                                                                        CPU
328
               3.13105s
                                            3.15585s
                                                                        100.52s
     0
                                                                         101.578s
329
                                            3.14552s
               3.12284s
330
               3.12185s
                                            3.14306s
                                                                         100.062s
331
     3
               3 122145
                                            3 14661s
                                                                        100 9955
332
     4
               3.12221s
                                            3.14241s
                                                                        102.571s
```

```
333
                3.12263s
                                             3.14389s
                                                                           101.934s
334
      6
                3 121085
                                             3 142485
                                                                           101.878s
335
                3.11968s
                                             3.15114s
                                                                           101.923s
      7
336
                                                                           102.572s
      8
                3.12159s
                                             3.14272s
337
                                             3.14502s
338
339
     WHAT
                                                       TIME(TOT)
                                                                           TIME(AVG)
      GPU time (Transfer time excluded)
340
                                                        31.2263s
                                                                           3.12263s
                                                        31.4587s
      GPU time (Transfer time included)
                                                                           3.14587s
341
      CPU time
342
343
      Runnning matrix_multiplication on an 3072×3072 matrix
      Local Work Size = 4x4
Number of CPU workers = 4
344
345
               GPU(EX.BUF)
                                                                           CPU
346
      ITFR
                                             GPU(INC.BUF)
               78.3929s
                                                                           362.76s
                                             78.4727s
347
      0
                78.3602s
                                             78.4396s
348
                                                                           361.861s
                78.3588s
                                             78.4062s
349
                                                                           360.334s
350
      3
                78.4473s
                                             78.397s
                                                                 362.749s
351
      4
                78.4221 s
                                             78.4819s
                                                                           356.477s
352
      5
                78.4357s
                                             78.4805s
                                                                           363.152\,\mathsf{s}
353
      6
                78 3524s
                                             78 4014s
                                                                           362 4595
354
      7
                78.3476s
                                             78.4139s
                                                                           360.539s
355
      8
                78.3749s
                                             78.4088s
                                                                           360.499s
                78.4087s
                                             78.4945s
                                                                           359.463s
356
357
358
     WHAT
                                                       TIME(TOT)
                                                                           TIME (AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
359
                                                        783.901s
                                                                           78.3901s
                                                                           78.4397s
                                                         784.397s
360
      CPU time
                                                         3610.29s
361
                                                                           361.029s
362
      Runnning matrix_multiplication on an 3072×3072 matrix
      Local Work Size = 8x8
Number of CPU workers = 4
363
364
               GPU(EX.BUF)
365
      ITFR
                                             GPU(INC.BUF)
                                                                           CPU
               19.8566s
                                             19.8859s
                                                                           365.059s
366
      0
367
                19.8333s
                                             19.8493s
                                                                           363.537s
                19.8469s
                                             19.8502s
                                                                           363.345s
368
369
      3
                19.8299s
                                             19.8811s
                                                                           364.895s
                                             19.8395s
370
      4
                19.8087 \, s
                                                                           364.145\,\mathsf{s}
371
      5
                19.8425s
                                             19.8577s
                                                                           365.016s
372
      6
                19.8396s
                                             19.8495s
                                                                           362.937s
373
      7
                19.8175s
                                             19.846s
                                                                 361.469s
374
      8
                19.8262s
                                             19.8789s
                                                                           363.374s
      9
                19.8246s
                                             19.8713s
                                                                           362.91s
375
376
377
     \/\/HAT
                                                       TIME(TOT)
                                                                           TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
378
                                                        198 326s
                                                                           19 8326s
                                                         198.609s
379
                                                                           19.8609s
      CPU time
                                                         3636.69s
380
                                                                           363.669s
381
      Runnning matrix_multiplication on an 3072×3072 matrix
      Local Work Size = 16 \times 16
Number of CPU workers = 4
382
383
               GPU(EX.BUF)
                                                                           CPU
                                             GPU(INC.BUF)
384
      ITFR
               10.5373s
                                                                           361.763s
385
                                             10.5816s
      0
                10.5306s
386
                                             10.5658s
                                                                           361.001s
                10.5282s
                                             10.574s
                                                                 362.444s
387
388
      3
                10.5277s
                                             10.572s
                                                                 361.273s
389
      4
                10.5269s
                                             10.5765s
                                                                           362.639s
390
      5
                10.5272 s
                                             10.5745s
                                                                           360.781s
391
                                                                           362.028s
      6
                10.5328s
                                             10.5674s
392
      7
                10.5274s
                                             10.5645s
                                                                           360.654s
393
                10.528s
                                   10.5761s
                                                                 360.777s
394
      9
                10.5249s
                                             10.5667s
                                                                           359.035s
395
396
     Λ/ΗΔΤ
                                                       TIME(TOT)
                                                                           TIME (AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                        105.291s
397
                                                                           10 52915
                                                         105.719s
                                                                           10.5719s
398
399
      CPU time
                                                         3612.4s
                                                                           361.24s
      1016.44s
                        101.644s
      Running matrix_max on an 128 \times 128 matrix
Local Work Size = 4
Number of CPU workers = 4
400
401
402
```

```
403
     ITER
               GPU(EX.BUF)
                                           GPU(INC.BUF)
                                                                        CPII
                                                                        0.0029475
404
     0
               0.000897s
                                            0.0024595
               0.001386s
                                           0.002301s
                                                                        0.000126s
405
     1
               0.001552s
                                            0.002576s
                                                                        0.000204s
406
               0.001649s
407
                                            0.002389s
                                                                        0.000162 \, s
408
     4
               0.0014s
                                  0.00245s
                                                               0.000142s
409
               0.001004s
                                           0.002377s
                                                                        0.00013s
410
     6
               0.0014155
                                           0.000991s
                                                                        0.000243 s
411
               0.001462s
                                            0.002622s
                                                                        0.000157s
               0.001685s
                                           0.00261s
                                                                        0.000187s
412
     8
413
     9
               0.001537s
                                            0.002275s
                                                                        0.000162s
414
415
     WHAT
                                                     TIME(TOT)
                                                                        TIME(AVG)
416
     GPU time (Transfer time excluded)
                                                      0.013987s
                                                                        0.0013987s
     GPU time (Transfer time included)
417
                                                      0.02305s
                                                                        0.002305s
     CPU time
                                                                        0.000446s
418
                                                      0.00446s
419
      Runnning matrix_multiplication on an 3072×3072 matrix
     Local Work Size = 4x4
Number of CPU workers = 4
ITER GPU(EX.BUF)
420
421
422
                                           GPU(INC.BUF)
                                                                        CPU
               78.3929s
                                                                        362 76s
                                            78 47275
423
     n
               78.3602s
424
                                           78.4396s
                                                                        361.861s
     1
425
               78.3588s
                                            78.4062s
                                                                        360.334s
426
               78.4473 s
                                            78.397s
                                                               362.749s
427
     4
               78.4221s
                                            78.4819s
                                                                        356.477s
428
     5
               78.4357s
                                            78.4805s
                                                                        363.152\,\mathsf{s}
429
     6
               78 3524 s
                                            78 4014s
                                                                        362 459s
430
               78.3476s
                                            78.4139s
                                                                        360.539s
                                            78.4088s
                                                                        360.499s
431
               78.3749s
     8
432
               78.4087s
                                            78.4945s
                                                                        359.463s
433
434
     WHAT
                                                     TIME(TOT)
                                                                        TIME (AVG)
435
     GPU time (Transfer time excluded)
                                                      783.901s
                                                                        78.3901s
     GPU time (Transfer time included)
                                                      784.397s
                                                                        78.4397s
436
     CPU time
                                                                        361.029s
437
                                                      3610.29s
438
      Runnning matrix_multiplication on an 3072×3072 matrix
439
      Local Work Size = 8x8
     Number of CPU workers = 4
ITER GPU(EX.BUF)
440
                                           GPU(INC.BUF)
                                                                        CPII
441
               19.8566s
                                                                        365.059s
442
     0
                                           19.8859s
               19.8333s
443
                                           19.8493s
                                                                        363.537s
444
               19.8469s
                                            19.8502s
                                                                        363.345s
445
     3
               19.8299s
                                            19.8811s
                                                                        364.895s
446
     4
               19.8087s
                                           19.8395s
                                                                        364.145s
447
               19.8425s
                                           19.8577s
                                                                        365.016s
448
     6
               19 83965
                                           19.8495s
                                                                        362.937s
                                                               361.469s
449
               19.8175s
                                           19.846s
                                                                        363.374s
450
     8
               19.8262s
                                           19.8789s
451
               19.8246s
                                            19.8713s
                                                                        362.91s
452
                                                     TIME(TOT)
                                                                        TIME(AVG)
453
     WHAT
     GPU time (Transfer time excluded) GPU time (Transfer time included)
454
                                                      198.326s
                                                                        19.8326s
455
                                                      198.609s
                                                                        19.8609 s
     CPU time
                                                                        363.669s
                                                      3636.69s
456
      Runnning matrix_multiplication on an 3072×3072 matrix
457
     Local Work Size = 16×16
Number of CPU workers = 4
ITER GPU(EX.BUF)
458
459
                                           GPU(INC.BUF)
                                                                        CPU
460
               10.5373s
                                           10.5816s
                                                                        361.763s
461
     0
               10.5306s
                                            10.5658s
462
                                                                        361.001s
463
               10.5282s
                                            10.574s
                                                               362.444s
464
     3
               10.5277s
                                            10.572s
                                                               361.273s
465
     4
               10.5269s
                                           10.5765s
                                                                        362.639s
                                            10.5745s
466
               10.5272s
                                                                        360.781\,\mathsf{s}
467
     6
               10 53285
                                           10 56745
                                                                        362 0285
468
               10.5274s
                                           10.5645s
                                                                        360.654s
469
     8
                                  10.5761s
                                                               360.777s
               10.528s
470
               10.5249s
                                           10.5667s
                                                                        359.035s
471
     WHAT
                                                     TIME(TOT)
472
                                                                        TIME(AVG)
473
     GPU time (Transfer time excluded)
                                                      105.291s
                                                                        10.5291s
```

```
474
     GPU time (Transfer time included)
                                                       105.719s
                                                                         10.5719s
     CPU time
475
                                                       3612 45
                                                                         361 245
     Running matrix_max on an 128 x 128 matrix Local Work Size = 8
476
477
      Number of CPU workers = 4
478
479
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
480
     0
               0.000652s
                                            0.001052s
                                                                         0.00304s
481
     1
               0.0023275
                                            0.0019525
                                                                         0.0001385
               0.001269s
482
                                            0.002122s
                                                                         0.00017s
483
     3
               0.001411s
                                            0.001187s
                                                                         0.000215s
484
               0.000872s
                                            0.001357s
                                                                         0.000149s
485
               0.001387s
                                            0.002209 \, s
                                                                         0.000151s
486
     6
               0.001072s
                                            0.001346s
                                                                         0.000173 s
487
               0.001291s
                                            0.00224s
                                                                         0.000131s
               0.00136s
                                            0.001077s
488
     8
                                                                         0.000236s
               0.001343s
                                            0.001522s
                                                                         0.000167s
489
     9
490
491
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
492
     GPU time (Transfer time excluded)
                                                       0.012984s
                                                                         0.0012984s
493
     GPU time (Transfer time included)
                                                       0.016064\,s
                                                                         0.0016064s
     CPU time
                                                       0.00457s
                                                                         0.0004575
494
     Running matrix_max on an 128 \times 128 matrix Local Work Size = 16 Number of CPU workers = 4
495
496
497
498
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
499
     0
               0.001214s
                                            0.002315s
                                                                         0.003017 \, s
500
     1
               0.001161s
                                            0.0013055
                                                                         0.000147s
501
               0.001136s
                                                                         0.000152s
                                            0.002218s
     3
               0.001169s
                                            0.002483s
                                                                         0.000146s
502
               0.001069s
                                            0.002606s
                                                                         0.000147s
503
504
               0.001219s
                                            0.002912 s
                                                                         0.000159s
505
     6
               0.0014465
                                            0.002332s
                                                                         0.0001625
506
               0.001459s
                                            0.002196s
                                                                         0.000122 s
507
     8
               0.001116s
                                            0.002303s
                                                                         0.000129s
               0.000798s
                                            0.001805s
                                                                         0.00015s
508
     9
509
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
510
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                       0.011787s
                                                                         0.0011787s
511
512
                                                       0.022475s
                                                                         0.0022475s
     CPU time
                                                       0.004331s
                                                                         0.0004331s
513
     Running matrix_max on an 128 \times 128 matrix Local Work Size = 32 Number of CPU workers = 4
514
515
516
517
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
                                                                         0.003209s
518
     0
               0.002256s
                                            0.002659s
               0.0011045
                                                                         0.0001495
519
     1
                                            0.0024475
               0.001607s
                                            0.001493s
                                                                         0.000245s
520
521
     3
               0.0011s
                                  0.0022s
                                                     0.000137s
522
               0.001496s
                                            0.001978s
                                                                         0.000137s
523
               0.001525 s
                                            0.003187s
                                                                         0.000171 \, s
524
     6
               0.001503s
                                            0.002482s
                                                                         0.0001225
525
               0.001508s
                                            0.00247s
                                                                         0.000119s
526
               0.001451s
                                            0.002497s
                                                                         0.000155s
     8
527
     9
               0.001335s
                                            0.002292s
                                                                         0.000166s
528
529
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
530
                                                       0.014885s
                                                                         0.0014885s
531
                                                       0.023705 s
                                                                         0.0023705 s
     CPU time
                                                       0.00461s
                                                                         0.000461s
532
     Running matrix_max on an 128 x 128 matrix Local Work Size = 64 Number of CPU workers = 4
533
534
535
536
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
537
     0
               0.001582s
                                            0.002355s
                                                                         0.003233 s
               0.0014415
                                            0.0021085
                                                                         0.0001225
538
     1
539
               0.001541s
                                            0.002106s
                                                                         0.000159s
540
     3
               0.001085s
                                            0.002548s
                                                                         0.000146s
541
               0.001577s
                                            0.002477s
                                                                         0.000166s
542
     5
               0.001313s
                                            0.002331\,s
                                                                         0.000191s
543
     6
               0.0009065
                                            0.001385 s
                                                                         0.0001585
544
               0.001042s
                                            0.002219 s
                                                                         0.000145s
```

```
545
               0.001406s
                                            0.001531s
                                                                         0.000152 s
546
     9
               0.001475s
                                            0.0023225
                                                                         0.000273s
547
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
548
     GPU time (Transfer time excluded)
549
                                                       0.013368s
                                                                         0.0013368s
     GPU time (Transfer time included)
                                                       0.021382s
                                                                         0.0021382s
550
551
     CPU time
                                                       0.004745s
                                                                         0.0004745s
     Running matrix_max on an 128 x 128 matrix
Local Work Size = 128
Number of CPU workers = 4
552
553
554
555
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
556
               0.001166s
                                            0.002359s
                                                                         0.010251s
557
               0.001147s
                                            0.000991s
                                                                         0.000135 s
558
     2
               0.001552s
                                            0.002307s
                                                                         0.000128 s
     3
               0.001647s
                                            0.00249s
                                                                         0.000126 s
559
               0.000923s
                                            0.002384s
                                                                         0.000142s
560
     4
               0.001337s
561
                                            0.001148 \, s
                                                                         0.000165s
562
               0.000822s
                                            0.001856s
                                                                         0.000126s
563
               0.00089s
                                            0.002228s
                                                                         0.000127s
               0 0006925
564
     8
                                            0.001153s
                                                                         0.000123 s
               0.0013455
                                            0.0012945
565
     9
                                                                         0.0002885
566
567
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
568
                                                                         0.0011521s
569
                                                       0.01821s
                                                                         0.001821s
570
     CPU time
                                                       0.011611\,\mathsf{s}
                                                                         0.0011611s
     Running matrix_max on an 128 \times 128 matrix Local Work Size = 256 Number of CPU workers = 4
571
572
573
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
574
     ITER
575
               0.000592s
                                            0.002583s
                                                                         0.003145s
576
     1
               0.000597s
                                            0.0012165
                                                                         0.000133 s
577
     2
               0.001406s
                                            0.002086 s
                                                                         0.000151s
     3
               0.000706s
                                            0.00217s
                                                                         0.000152s
578
               0.000928s
                                            0.002013s
                                                                         0.00016s
579
               0.001447s
                                                                         0.000169s
580
                                            0.001383s
581
               0.000926s
                                            0.002435s
                                                                         0.000256s
582
               0.001596s
                                            0.002584s
                                                                         0.00015s
583
     8
               0.000921s
                                            0.002177s
                                                                         0.000386 s
                                  0.001418s
                                                               0.000186s
               0.0014s
584
     9
585
586
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
     GPU time (Transfer time excluded)
                                                       0.010519s
                                                                         0.0010519s
587
                                                                         0.0020065s
588
     GPU time (Transfer time included)
                                                       0.020065\,s
589
     CPU time
                                                       0.004888s
                                                                         0.0004888s
     Running matrix_max on an 256 \times 256 matrix Local Work Size = 4 Number of CPU workers = 4
590
591
592
593
      ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
594
               0.002058s
                                            0.002926s
                                                                         0.003533s
     0
595
     1
               0.001675s
                                            0.002633s
                                                                         0.000193 s
596
     2
               0.001843s
                                            0.002728s
                                                                         0.000257s
               0.001829s
597
     3
                                            0.002599s
                                                                         0.000196s
598
               0.001959s
                                            0.003095s
                                                                         0.000207s
               0.002033s
                                            0.002957s
                                                                         0.000193s
599
600
               0.002038s
                                            0.002937s
                                                                         0.000195s
     6
601
               0.001865s
                                            0.001716s
                                                                         0.000193 s
602
     8
               0.001564s
                                            0.00257s
                                                                         0.000196s
               0.002022s
603
     9
                                            0.003117 s
                                                                         0.000188s
604
605
                                                      TIME(TOT)
                                                                         TIME(AVG)
     GPU time (Transfer time excluded)
                                                       0.018886s
                                                                         0.0018886s
606
607
     GPU time (Transfer time included)
                                                       0.027278s
                                                                         0.0027278s
608
     CPU time
                                                       0.005351\,\mathsf{s}
                                                                         0.0005351s
     Running matrix_max on an 256 x 256 matrix Local Work Size = 8
609
610
611
     Number of CPU workers = 4
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
612
     ITER
613
     0
               0.001733s
                                            0.002496s
                                                                         0.003082 s
614
     1
               0.0015635
                                            0.001583s
                                                                         0.000292 s
615
     2
               0.000974s
                                            0.002618s
                                                                         0.000197s
```

```
616
               0.001788s
                                            0.002666s
                                                                         0.000205 s
617
     4
               0.0016915
                                            0.0029685
                                                                         0.0002065
               0.001525s
                                            0.002397s
                                                                         0.000253s
618
     5
619
     6
               0.001605s
                                            0.002426s
                                                                         0.000235s
               0.001568s
620
                                            0.002352s
                                                                         0.000216s
621
               0.001411s
                                            0.002353s
                                                                         0.000205s
     8
622
     9
               0.001358s
                                            0.002808s
                                                                         0.000195s
623
                                                      TIME(TOT)
624
     WHAT
                                                                         TIME (AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                       0.015216s
                                                                         0.0015216s
625
626
                                                       0.024667s
                                                                         0.0024667s
      CPU time
                                                       0.005086s
                                                                         0.0005086s
628
      Running matrix_max on an 256 \times 256 matrix
     Local Work Size = 16
Number of CPU workers = 4
629
630
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
631
     ITER
                                                                         0.00316s
               0.00172s
632
                                            0.002676s
633
               0.002563s
                                            0.002492s
                                                                         0.000188s
634
     2
               0.001315s
                                            0.002299s
                                                                         0.000207s
635
     3
               0.001518s
                                            0.002548s
                                                                         0.0002s
                                                                         0.000279s
     4
               0.0015325
                                            0.0023895
636
               0.001725s
                                                                         0.000199s
637
                                            0.002478s
     5
638
     6
               0.001715s
                                            0.002639 \, s
                                                                         0.000239s
639
               0.001725s
                                            0.002646s
                                                                         0.000185s
640
     8
               0.001766s
                                            0.002621s
                                                                         0.000204s
641
     9
               0.001716s
                                            0.004157s
                                                                         0.000208s
642
     WHAT
                                                      TIME(TOT)
                                                                         TIME (AVG)
643
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                                         0.0017295s
                                                       0.017295s
644
645
                                                       0.026945s
                                                                         0.0026945s
      CPU time
                                                       0.005069s
                                                                         0.0005069s
646
647
     Running matrix_max on an 256 \times 256 matrix
     Local Work Size = 32
Number of CPU workers = 4
648
649
     ITER
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
650
               0.001528s
                                            0.002729s
                                                                         0.010113s
651
652
               0.001557s
                                            0.002734s
                                                                         0.000177s
653
     2
               0.001804s
                                            0.002561\,s
                                                                         0.000209 \, s
                                                                         0 000227s
654
     3
               0.00175s
                                            0.002776s
               0.001835s
                                                                         0.000216s
655
     4
                                            0.002278s
               0.001528s
                                            0.002406s
656
                                                                         0.000194s
     5
657
     6
               0.001716 s
                                            0.00247s
                                                                         0.000208s
658
               0.001519s
                                            0.0023s
                                                               0.000205s
659
     8
               0.001529s
                                            0.002291\,\mathrm{s}
                                                                         0.000192 \, s
660
     9
               0.001776s
                                            0.002629s
                                                                         0.000286s
661
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
662
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                       0.016542s
                                                                         0.0016542s
663
664
                                                       0.025174 \, s
                                                                         0.0025174 s
665
      CPU time
                                                       0.012027s
                                                                         0.0012027s
666
     Running matrix_max on an 256 \times 256 matrix
     Local Work Size = 64
Number of CPU workers = 4
667
668
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
669
     ITER
               0.00159s
                                            0.002474s
                                                                         0.003036s
670
671
               0.001488s
                                            0.0022s
                                                               0.00029s
672
     2
               0.00157\,\mathsf{s}
                                            0.002674s
                                                                         0.0001915
673
     3
               0.001762 s
                                            0.002774s
                                                                         0.000193s
               0.001338s
                                            0.002666s
                                                                         0.00018s
674
     4
               0.000919s
                                            0.002831s
                                                                         0.000205s
675
     5
676
     6
               0.001885s
                                            0.00233s
                                                                         0.000356s
               0.001746s
                                            0.002618s
                                                                         0.000192s
677
678
     8
               0.00154s
                                            0.002643s
                                                                         0.000203s
679
     9
               0.001595s
                                            0.001548s
                                                                         0.000283 s
680
681
     WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
     GPU time (Transfer time excluded)
                                                       0.015433s
                                                                         0.0015433s
682
     GPU time (Transfer time included)
                                                       0.024758s
                                                                         0.0024758s
683
684
     CPU time
                                                       0.005129s
                                                                         0.0005129s
685
     Running matrix_max on an 256 \times 256 matrix
686
      Local Work Size = 128
```

```
687
      \begin{array}{ll} \text{Number of CPU workers} = 4 \\ \text{ITER} & \text{GPU(EX.BUF)} \end{array}
                                                                            CPU
688
                                             GPU(INC.BUF)
689
                0.001456s
                                                                            0.003016s
                                              0.002627s
      0
690
                0.001544s
                                              0.003878s
                                                                            0.000215s
                0.00148s
                                                                            0.000207s
691
                                              0.002359s
692
      3
                0.001736s
                                              0.002755s
                                                                            0.000207s
693
      4
                0.001806s
                                              0.002383s
                                                                            0.000213s
694
      5
                0.001759s
                                              0.002853s
                                                                            0.000185 s
695
                0.001758s
                                              0.002761s
                                                                            0.000207 s
      6
696
                0.001748s
                                              0.002698s
                                                                            0.000193s
697
                0.00144 \, s
                                              0.002319\,\mathrm{s}
                                                                            0.000211s
698
                0.001519s
                                              0.002384s
                                                                            0.000267s
699
     WHAT
700
                                                        TIME(TOT)
                                                                            TIME (AVG)
701
      GPU time (Transfer time excluded)
                                                                            0.0016246s
                                                         0.016246s
      GPU time (Transfer time included)
                                                         0.027017s
                                                                            0.0027017s
702
703
      CPU time
                                                         0.004921s
                                                                            0.0004921s
704
      Running matrix_max on an 256 x 256 matrix
705
      Local Work Size = 256
Number of CPU workers = 4
ITER GPU(EX.BUF)
706
                                             GPU(INC.BUF)
                                                                            CPU
707
                0.000763s
                                                                            0.002969s
708
                                              0.003019s
      0
709
                0.001494s
                                              0.001471 \, s
                                                                            0.000187s
710
                0.00149s
                                              0.002564s
                                                                            0.000203s
711
      3
                0.001676s
                                              0.002821s
                                                                            0.000196s
712
      4
                0.001794s
                                              0.001528\,\mathsf{s}
                                                                            0.000215s
713
      5
                0.0016925
                                              0.0022215
                                                                            0.000235
714
                0.001723s
                                              0.003359s
                                                                            0.000223 s
      6
715
                0.001805s
                                              0.002571s
                                                                            0.000202s
716
                0.001471s
                                              0.001451s
                                                                            0.000255s
717
                0.00136\,\mathrm{s}
                                              0.002619 s
                                                                            0.000285s
718
                                                                            TIME(AVG)
719
     WHAT
                                                        TIME(TOT)
720
     GPU time (Transfer time excluded) GPU time (Transfer time included)
                                                         0.015268s
                                                                            0.0015268s
                                                         0.023624s
                                                                            0.0023624s
721
722
      CPU time
                                                         0.004965 s
                                                                            0.0004965 s
723
      Running matrix_max on an 512 x 512 matrix
      Local Work Size = 4
Number of CPU workers = 4
ITER GPU(EX.BUF)
724
725
                                             GPU(INC.BUF)
                                                                            CPU
726
                0.003581s
                                                                            0.006013s
727
                                              0.005311s
      0
728
                0.003582 \, s
                                              0.004912 \, s
                                                                            0.000372s
729
                0.003549s
                                              0.004037s
                                                                            0.00039s
730
      3
                0.003546s
                                              0.005449s
                                                                            0.000385s
731
      4
                0.003427s
                                              0.005251s
                                                                            0.000399s
732
      5
                0.0033285
                                              0.0041185
                                                                            0.0004895
733
                0.003635s
      6
                                              0.005577s
                                                                            0.000389 s
734
                0.002633s
                                              0.005227s
                                                                            0.000379s
735
                0.003376s
                                              0.004945\,\mathsf{s}
                                                                            0.000402 s
736
      9
                0.003091s
                                              0.00494 s
                                                                            0.000373s
737
     WHAT
738
                                                        TIME(TOT)
                                                                            TIME (AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
739
                                                         0.033748s
                                                                            0.0033748s
                                                         0.049767s
                                                                            0.0049767s
740
741
                                                         0.009591s
                                                                            0.0009591s
742
      Running matrix_max on an 512 x 512 matrix
      Local Work Size = 8
Number of CPU workers = 4
743
744
               GPU(EX.BUF)
                                             GPU(INC.BUF)
                                                                            CPU
745
      ITER
                0.002372s
                                              0.002607s
                                                                            0.003433s
746
      0
                0.002362s
747
                                              0.003144 s
                                                                            0.00039s
748
      2
                0.002784s
                                              0.002736s
                                                                            0.000377s
749
      3
                0.002342s
                                              0.003731\,\mathrm{s}
                                                                            0.000516s
\frac{750}{751}
      4
                0.002386s
                                              0.003967s
                                                                            0.000393s
                0.0019585
      5
                                              0.003935
                                                                            0.0003965
752
      6
                0.002367s
                                              0.003991s
                                                                            0.000385s
753
                0.002134s
                                                                            0.00039s
                                              0.003115s
                0.002645s
754
                                              0.00306 s
                                                                            0.00036s
755
      9
                0.002637s
                                              0.004239s
                                                                            0.000376s
756
     WHAT
757
                                                        TIME(TOT)
                                                                            TIME (AVG)
```

```
0.023987s
758
     GPU time (Transfer time excluded)
                                                                     0.0023987s
759
     GPU time (Transfer time included)
                                                    0.034525
                                                                     0.0034525
760
     CPU time
                                                                     0.0007016s
                                                    0.007016s
     Running matrix_max on an 512 x 512 matrix Local Work Size = 16
761
762
763
     Number of CPU workers = 4
              GPU(EX.BUF)
764
     ITER
                                         GPU(INC.BUF)
                                                                     CPU
              0.001427s
                                                                     0.014051s
765
     0
                                         0.006169s
              0.002124s
                                          0.00367s
766
                                                                     0.000494s
767
              0.002405s
                                          0.00406s
                                                                     0.000411s
768
              0.001305s
                                          0.003754s
                                                                     0.000395s
769
     4
              0.002041s
                                          0.002682\,s
                                                                     0.000506s
770
     5
              0.002204s
                                          0.004033s
                                                                     0.0003985
771 \\ 772
     6
              0.001409s
                                          0.002312s
                                                                     0.000386s
              0.002095s
                                          0.003267s
                                                                     0.000383s
              0.00203s
                                                                     0.000504s
     8
                                          0.003635s
773
774
              0.002061s
                                          0.002369 \, s
                                                                     0.000417s
775
776
     WHAT
                                                   TIME(TOT)
                                                                     TIME(AVG)
     GPU time (Transfer time excluded) GPU time (Transfer time included)
777
                                                    0.019101s
                                                                     0.0019101s
778
                                                    0.035951s
                                                                     0.0035951s
     CPU time
779
                                                    0.017945s
                                                                     0.0017945s
780
     Running matrix_max on an 512 x 512 matrix
     Local Work Size = 32
781
782
     Number of CPU workers = 4
783
     ITER
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                     CPU
                                                                     0.0103125
              0.004625
784
     n
                                          0.005915s
              0.004092s
                                          0.006272s
785
                                                                     0.0004 s
     1
              0.004315s
                                          0.006022s
                                                                     0.000394s
786
              0.00464s
                                          0.006097s
                                                                     0.000398s
787
788
     4
              0.004555s
                                          0.006319 \, s
                                                                     0.000371s
789
     5
              0.0046185
                                          0.005079 s
                                                                     0.0004275
790
     6
              0.004564s
                                          0.006543s
                                                                     0.0003975
791
              0.004732s
                                          0.00629s
                                                                     0.000383s
              0.004601s
                                          0.005026s
                                                                     0.000438s
792
     8
793
              0.004628s
                                                                     0.000395s
794
795
     WHAT
                                                   TIME(TOT)
                                                                     TIME(AVG)
796
     GPU time (Transfer time excluded)
                                                    0.045365s
                                                                     0.0045365s
     GPU time (Transfer time included)
                                                    0.059598s
                                                                     0.0059598s
797
     CPU time
798
                                                    0.013915s
                                                                     0.0013915s
     Running matrix_max on an 512 x 512 matrix Local Work Size = 64
799
800
     Number of CPU workers = 4
801
802
     ITER
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                     CPII
              0.004532s
                                          0.006565
                                                                     0.0031895
803
     n
804
              0.004591s
                                          0.006119s
                                                                     0.000386s
805
              0.004628s
                                          0.004923s
                                                                     0.000406s
806
              0.004503s
                                          0.005256s
                                                                     0.000388s
807
     4
              0.004758s
                                          0.006546s
                                                                     0.000373 s
808
     5
              0.004707s
                                          0.006483s
                                                                     0.000368s
                                0.006491s
                                                           0.000397s
809
     6
              0.00485
                                         0.006485s
                                                                     0.000363s
              0.004751s
810
              0.004783s
                                          0.00559s
                                                                     0.0004s
811
              0.004844s
                                                                     0.000368s
812
813
814
     \Λ/ΗΔΤ
                                                   TIME(TOT)
                                                                     TIME(AVG)
     GPU time (Transfer time excluded)
GPU time (Transfer time included)
815
                                                    0.046897s
                                                                     0.0046897s
                                                    0.060612s
                                                                     0.0060612s
816
     CPU time
817
                                                    0.006638s
                                                                     0.0006638s
     Running matrix_max on an 512 x 512 matrix
818
819
     Local Work Size = 128
820
     Number of CPU workers = 4
821
     ITER
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                     CPII
              0.004497s
                                          0.0059945
                                                                     0.0032195
822
     n
823
              0.003917s
                                          0.005922s
                                                                     0.000391s
     1
824
              0.004545s
                                          0.006247s
                                                                     0.000376s
     2
                                          0.00636s
825
              0.004814s
                                                                     0.000387s
826
     4
              0.004244s
                                          0.005759 \, s
                                                                     0.000393s
827
     5
              0.0044575
                                          0.004633s
                                                                     0.0003915
828
     6
              0.005942s
                                          0.006509 s
                                                                     0.000373 s
```

```
0.000383s
829
              0.004811s
                                         0.006163s
              0.0048055
830
     8
                                         0.006115
                                                                    0.000385
              0.004453s
831
                                         0.005198s
                                                                    0.00041s
     9
832
     WHAT
                                                  TIME(TOT)
                                                                    TIME(AVG)
833
834
     GPU time (Transfer time excluded)
                                                   0.046485s
                                                                    0.0046485s
835
     GPU time (Transfer time included)
                                                   0.058895 \, s
                                                                    0.0058895s
836
     CPU time
                                                   0.006703s
                                                                    0.0006703s
     Running matrix_max on an 512 x 512 matrix Local Work Size = 256
837
838
839
     Number of CPU workers = 4
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                    CPU
840
     ITER
841
     0
              0.004612s
                                         0.005496s
                                                                    0.008763 s
842
     1
              0.004806s
                                         0.006357s
                                                                    0.000387s
              0.004594s
843
     2
                                         0.005249s
                                                                    0.00041s
     3
              0.004532s
                                         0.005147s
                                                                    0.000528s
844
              0.004701s
845
                                         0.006346 \, s
                                                                    0.000393s
846
              0.004534s
                                         0.006227s
                                                                    0.000379s
847
     6
              0.004788s
                                         0.00643s
                                                                    0.000467s
848
              0.004566s
                                         0.006088s
                                                                    0.000462s
              0 0047445
849
     8
                                         0.0054115
                                                                    0.0004555
              0.00381s
850
     9
                                         0.005289s
                                                                    0.000462s
851
     WHAT
                                                  TIME(TOT)
                                                                    TIME(AVG)
852
853
     GPU time (Transfer time excluded)
                                                   0.045687s
                                                                    0.0045687s
854
     GPU time (Transfer time included)
                                                   0.05804s
                                                                    0.005804s
     CPU time
855
                                                   0.012706s
                                                                    0.0012706s
     Running matrix_max on an 1024 \times 1024 matrix Local Work Size = 4
856
857
     Number of CPU workers = 4
858
859
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                    CPU
     ITER
860
     0
              0.009783s
                                         0.013521s
                                                                    0.0043295
861
     1
              0.009828s
                                         0.016197s
                                                                    0.001169s
              0.009617s
                                                          0.00118s
862
     2
                                         0.0138s
     3
              0.009765s
                                         0.011175s
                                                                    0.00116s
863
              0.009567s
864
                                         0.013369s
                                                                    0.001163s
865
              0.009699s
                                         0.013311s
                                                                    0.001209s
866
     6
              0.009629s
                                         0.01356\,\mathsf{s}
                                                                    0.00118\,\mathrm{s}
867
              0.009825
                                         0.012597s
                                                                    0.001182s
868
     8
              0.008899s
                                         0.013642s
                                                                    0.001167s
869
     9
              0.009707s
                                         0.012726s
                                                                    0.001168s
870
     WHAT
                                                  TIME(TOT)
                                                                    TIME(AVG)
871
     GPU time (Transfer time excluded)
                                                   0.096314s
                                                                    0.0096314s
872
873
     GPU time (Transfer time included)
                                                   0.133898s
                                                                    0.0133898s
     CPU time
874
                                                   0.014907s
                                                                    0.0014907s
     Running matrix_max on an 1024 x 1024 matrix
Local Work Size = 8
875
876
     Number of CPU workers = 4
877
878
     ITER
              GPU(EX.BUF)
                                         GPU(INC.BUF)
                                                                    CPU
879
     0
              0.00602s
                                         0.007891s
                                                                    0.011243 s
880
     1
              0.006157s
                                         0.009102s
                                                                    0.001177s
881
              0.00606s
                                         0.011089s
                                                                    0.001151s
882
              0.006023s
                                         0.009611s
                                                                    0.001272s
              0.006284s
                                         0.009754s
                                                                    0.001159s
883
884
              0.006116s
                                         0.009796s
                                                                    0.001167s
885
     6
              0.006089s
                                         0.00886s
                                                                    0.002135 s
886
              0.005985s
                                         0.009824 s
                                                                    0.00126s
887
     8
              0.006149s
                                         0.010218s
                                                                    0.001168s
              0.00585s
888
     9
                                         0.009659s
                                                                    0.001175s
889
890
     WHAT
                                                  TIME(TOT)
                                                                    TIME(AVG)
891
     GPU time (Transfer time excluded)
                                                   0.060733s
                                                                    0.0060733s
892
     GPU time (Transfer time included)
                                                   0.095804s
                                                                    0.0095804s
     CPU time
893
                                                   0.0229075
                                                                    0.0022907s
894
     Running matrix_max on an 1024 x 1024 matrix
895
     Local Work Size = 16
     Number of CPU workers = 4
ITER GPU(EX.BUF)
896
     ITER
897
                                         GPU(INC.BUF)
                                                                    CPII
                                                                    0.003961s
898
     0
              0.004852s
                                         0.007888s
                                         0.008571s
899
     1
              0.004668s
                                                                    0.00272s
```

```
900
               0.005048s
                                           0.008662s
                                                                       0.001186s
901
     3
               0 004985
                                           0.0089915
                                                                       0.0012115
               0.005059s
                                           0.008952s
                                                                       0.001186s
902
     4
903
               0.004067s
                                           0.008756s
                                                                       0.001177s
     5
               0.004773s
                                           0.008607s
                                                                       0.001454s
904
905
               0.004751s
                                           0.008759s
                                                                       0.001201s
906
     8
               0.004799s
                                           0.008623s
                                                                       0.001187s
907
     9
               0.0047645
                                           0.007621s
                                                                       0.001269 s
908
     WHAT
                                                    TIME(TOT)
                                                                       TIME (AVG)
909
910
     GPU time (Transfer time excluded)
                                                     0.047761s
                                                                       0.0047761s
     GPU time (Transfer time included)
                                                     0.08543s
                                                                       0.008543s
911
     CPU time
912
                                                     0.016552s
                                                                       0.0016552s
     Running matrix_max on an 1024 x 1024 matrix Local Work Size = 32 Number of CPU workers = 4 \,
913
914
915
               GPU(EX.BUF)
916
     ITER
                                           GPU(INC.BUF)
                                                                       CPU
917
     0
               0.015142s
                                           0.017043s
                                                                       0.003967s
918
               0.021241s
                                           0.025413s
                                                                       0.001209s
919
     2
               0.015065s
                                           0.025127s
                                                                       0.001166s
     3
               0.0152055
                                           0.0182235
                                                                       0.0011585
920
               0.020599s
                                                                       0.001195s
921
     4
                                           0.017911s
922
     5
               0.015305s
                                           0.018813s
                                                                       0.001155s
923
               0.020905s
                                                                       0.001162s
                                           0.019487\,\mathsf{s}
924
               0.0156s
                                 0.01823s
                                                             0.001307s
925
     8
               0.015499s
                                           0.024815s
                                                                       0.001143s
926
     9
               0.021106s
                                           0.018921s
                                                                       0.001163 s
927
     WHAT
                                                    TIME(TOT)
                                                                       TIME (AVG)
928
     GPU time (Transfer time excluded)
929
                                                     0.175667s
                                                                       0.0175667s
     GPU time (Transfer time included)
                                                     0.203983s
                                                                       0.0203983s
930
931
     CPU time
                                                     0.014625s
                                                                       0.0014625s
     Running matrix_max on an 1024 \times 1024 matrix Local Work Size = 64
932
933
934
     Number of CPU workers = 4
              GPU(EX.BUF)
935
     ITER
                                           GPU(INC.BUF)
                                                                       CPU
936
               0.027427s
                                           0.030744s
                                                                       0.007786s
     0
937
               0.028529 \, s
                                           0.02936s
                                                                       0.001164 s
938
     2
               0.029176s
                                           0.029449s
                                                                       0.001277s
                                           0.032546s
939
     3
               0.029704s
                                                                       0.001145 s
               0.028593s
                                           0.030487s
                                                                       0.00116s
940
     4
941
               0.028324s
                                           0.033259 \, s
                                                                       0.001152s
942
               0.027266s
                                           0.033412s
                                                                       0.001177s
943
               0.028815s
                                           0.029342\,s
                                                                       0.001182 \, s
944
     8
               0.027337s
                                           0.033338s
                                                                       0.001286s
945
     9
               0.0279465
                                           0.0303955
                                                                       0.001274 s
946
     WHAT
                                                    TIME(TOT)
                                                                       TIME(AVG)
947
948
     GPU time (Transfer time excluded)
                                                     0.283117s
                                                                       0.0283117s
949
     GPU time (Transfer time included)
                                                     0.312332s
                                                                       0.0312332s
950
     CPU time
                                                     0.018603s
                                                                       0.0018603s
     Running matrix_max on an 1024 \times 1024 matrix Local Work Size = 128 Number of CPU workers = 4
951
952
953
               GPU(EX.BUF)
                                          GPU(INC.BUF)
                                                                       CPU
954
     ITER
955
     0
               0.027443s
                                           0.031353s
                                                                       0.010925s
956
               0.028296s
                                           0.0322795
                                                                       0.001181s
957
     2
               0.029408s
                                           0.031936s
                                                                       0.001178s
     3
               0.029217s
                                           0.029927s
                                                                       0.001168s
958
               0.027704s
                                           0.032497s
                                                                       0.001176s
959
     4
960
               0.027188s
                                           0.031925\,s
                                                                       0.001178s
961
     6
               0.027231s
                                           0.030137s
                                                                       0.001185s
962
               0.028792s
                                           0.032743s
                                                                       0.001168s
                                           0.030945s
963
     8
               0.027958s
                                                                       0.00126 s
964
     9
               0.0274125
                                           0.033447s
                                                                       0.001167s
965
                                                    TIME(TOT)
                                                                       TIME(AVG)
966
     GPU time (Transfer time excluded)
                                                     0.280649s
967
                                                                       0.0280649s
968
     GPU time (Transfer time included)
                                                     0.317189 s
                                                                       0.0317189s
969
     CPU time
                                                     0.021586s
                                                                       0.0021586s
     Running matrix_max on an 1024 \times 1024 matrix
970
```

```
\begin{array}{lll} \mbox{Local Work Size} & = 256 \\ \mbox{Number of CPU workers} & = 4 \\ \mbox{ITER} & \mbox{GPU(EX.BUF)} \end{array}
 971
 972
                                              GPU(INC.BUF)
                                                                            CPU
 973
 974
                0.026769s
                                              0.029023s
                                                                            0.010639s
       0
                0.027628s
                                                                            0.001155s
                                              0.030637s
 975
 976
                0.02746s
                                              0.030373s
                                                                            0.001168s
       2
 977
       3
                0.027631s
                                              0.031535 \, s
                                                                            0.0015s
 978
       4
                0.027323s
                                              0.030836s
                                                                            0.001161s
 979
                0.027196s
                                              0.026719s
                                                                            0.001158s
       5
                0.027129s
                                              0.031038s
                                                                            0.001165 s
 980
       6
 981
                0.027836s
                                              0.030373s
                                                                            0.001154 \, s
                 0.026442s
                                              0.030461 s
                                                                            0.001177s
 982
 983
       9
                0.027806s
                                              0.032078s
                                                                            0.001159s
 984
      WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
 985
       GPU time (Transfer time excluded)
 986
                                                         0.27322s
                                                                            0.027322s
       GPU time (Transfer time included)
                                                         0.303073s
                                                                            0.0303073s
 987
 988
       CPU time
                                                         0.021436s
                                                                            0.0021436s
 989
       Running matrix_max on an 2048 \times 2048 matrix
       Local Work Size = 4
Number of CPU workers = 4
 990
 991
             GPU(EX.BUF)
                                                                            CPU
                                              GPU(INC.BUF)
 992
       ITER
 993
       0
                0.037975s
                                              0.044523s
                                                                            0.007314s
                0.034144s
                                              0.046313s
                                                                            0.004208s
 994
 995
       2
                0.034182s
                                              0.047215s
                                                                            0.004219s
 996
       3
                0.034537\,\mathsf{s}
                                              0.047141\,\mathsf{s}
                                                                            0.004189s
 997
       4
                0.034357s
                                              0.046892s
                                                                            0.0041765
 998
                0.033332s
                                              0.04485s
                                                                            0.004295 s
       5
                0.034521s
                                              0.047309s
                                                                            0.004217s
999
       6
1000
                0.034373s
                                              0.04499s
                                                                            0.004199s
                 0.034246s
                                              0.046476s
                                                                            0.004546s
1001
1002
       9
                0.034176s
                                              0.046729s
                                                                            0.004248s
1003
      WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
1004
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                                            0.0345843s
1005
                                                         0.345843s
                                                         0.462438s
1006
                                                                            0.0462438s
                                                                            0.0045611s
                                                         0.045611\,\mathrm{s}
1007
       CPU time
       Running matrix_max on an 2048 \times 2048 matrix Local Work Size = 8 Number of CPU workers = 4
1008
1009
1010
              GPU(EX.BUF)
                                              GPU(INC.BUF)
                                                                            CPU
1011
       ITER
       0
                0.019552s
                                              0.031673s
                                                                            0.01173s
1012
                0.019347s
                                              0.031547s
                                                                            0.004211s
1013
1014
       2
                0.019734s
                                              0.03188s
                                                                            0.004216s
1015
       3
                0.019609\,s
                                              0.029883s
                                                                            0.004774s
1016
       4
                0.0195465
                                              0.0318815
                                                                            0.004177s
                0.019449s
       5
                                              0.032877s
                                                                            0.004188s
1017
                0.019939s
                                                                            0.004882s
1018
       6
                                              0.03216s
1019
                 0.019309\,s
                                              0.031995\,s
                                                                            0.004182s
1020
                0.019084s
                                              0.030489 \, s
                                                                            0.004291s
1021
       9
                0.019495s
                                              0.026772s
                                                                            0.005307s
1022
      WHAT
                                                        TIME(TOT)
                                                                            TIME (AVG)
1023
       GPU time (Transfer time excluded)
                                                         0.195064s
                                                                            0.0195064s
1024
       GPU time (Transfer time included)
                                                         0.311157s
1025
                                                                            0.0311157s
1026
       CPU time
                                                         0.051958s
                                                                            0.0051958s
       Running matrix_max on an 2048 x 2048 matrix Local Work Size = 16 Number of CPU workers = 4
1027
1028
1029
              GPU(EX.BUF)
                                              GPU(INC.BUF)
                                                                            CPU
1030
       ITER
                0.016829s
                                                                            0.007165s
1031
       0
                                              0.028146s
1032
                0.017173s
                                              0.027926s
                                                                            0.004239s
1033
       2
                0.016533s
                                              0.030183s
                                                                            0.004226s
1034
       3
                0.017411\,s
                                              0.030866s
                                                                            0.004184 s
       4
                                              0.0297495
                                                                            0.0042075
1035
                0.01724s
1036
       5
                0.017385s
                                              0.030135s
                                                                            0.004214s
1037
                0.017242s
                                              0.027609s
                                                                            0.004327s
       6
1038
                 0.016919s
                                              0.030108\,s
                                                                            0.004211s
1039
       8
                0.017344s
                                              0.029764s
                                                                            0.004185s
1040
       9
                0.016754s
                                              0.02951s
                                                                            0.004325s
1041
```

```
1042
      WHAT
                                                     TIME(TOT)
                                                                        TIME(AVG)
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
1043
                                                       0 170835
                                                                        0.0170835
                                                       0.293996s
                                                                        0.0293996s
1044
      CPU time
                                                       0.045283s
                                                                        0.0045283s
1045
       Running matrix_max on an 2048 x 2048 matrix
1046
      Local Work Size = 32
Number of CPU workers = 4
ITER GPU(EX.BUF)
1047
1048
                                            GPU(INC.BUF)
                                                                        CPU
1049
                0.03854s
                                            0.059666s
1050
                                                                        0.011355s
      0
1051
                0.04447s
                                            0.053674s
                                                                        0.004299s
1052
                0.041393s
                                            0.051144s
                                                                        0.004589s
1053
      3
                0.037739s
                                            0.056971 \, s
                                                                        0.0042s
1054
      4
                0.063651s
                                            0.052132 s
                                                                        0.004199s
1055
      5
                0.038427s
                                            0.049212s
                                                                        0.00431s
                0.04236s
                                            0.051234s
                                                                        0.004207 s
1056
      6
                0.038562s
                                            0.058057s
                                                                        0.004187s
1057
1058
                0.053482s
                                            0.048596 s
                                                                        0.004185s
1059
      9
                0.044194s
                                            0.051001s
                                                                        0.00419s
1060
                                                     TIME(TOT)
                                                                        TIME(AVG)
1061
      V/HAT
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                       0 4428185
                                                                        0.04428185
1062
                                                       0.531687s
                                                                        0.0531687s
1063
1064
      CPU time
                                                       0.049721s
                                                                        0.0049721s
       Running matrix_max on an 2048 x 2048 matrix
1065
1066
       Local Work Size = 64
      Number of CPU workers = 4
ITER GPU(EX.BUF)
1067
                                                                        CPU
                                            GPU(INC.BUF)
1068
                0.09886s
                                            0.112406s
                                                                        0.007138s
1069
      0
1070
                0.07567s
                                            0.111075s
                                                                        0.004304s
                0.073248s
                                                                        0.004199s
1071
                                            0.117349\,\mathrm{s}
1072
      3
                0.095706s
                                            0.086189s
                                                                        0.004213s
1073
      4
                0.10048s
                                            0 1107655
                                                                        0.0042065
1074
      5
                0.10083s
                                            0.084656s
                                                                        0.0042145
                0.096937s
                                            0.0898285
                                                                        0.00423s
1075
      6
                0.09979s
                                            0.086881s
                                                                        0.004231s
1076
                0.096096s
1077
                                            0.085355s
                                                                        0.004208s
1078
      9
                0.098991s
                                            0.113271s
                                                                        0.004213s
1079
                                                      TIME(TOT)
      \/\/HAT
1080
                                                                        TIME (AVG)
      GPU time (Transfer time excluded)
                                                                        0.0936608s
1081
                                                       0.936608s
      GPU time (Transfer time included)
                                                       0.997775s
                                                                        0.0997775s
1082
1083
      CPU time
                                                       0.045156s
                                                                        0.0045156s
       Running matrix_max on an 2048 x 2048 matrix
1084
1085
       Local Work Size = 128
      Number of CPU workers = 4
ITER GPU(EX.BUF)
1086
                                                                        CPU
                                            GPU(INC.BUF)
1087
                0.104191s
                                                                        0.01054s
1088
                                            0.112773s
      0
1089
                0.102969s
                                            0.112369s
                                                                        0.004416s
      1
1090
                0.103673s
                                            0.112885\,\mathsf{s}
                                                                        0.005257s
1091
      3
                0.099153s
                                            0.115232 s
                                                                        0.004208s
1092
      4
                0.105941s
                                            0.114272s
                                                                        0.004288s
1093
      5
                0.105845s
                                            0.11226s
                                                                        0.00418s
                0.105462s
1094
                                                                        0.00418s
      6
                                            0.113682s
1095
                0.100953s
                                            0.114217s
                                                                        0.004221s
      8
                0.102782s
                                            0.114758s
1096
                                                                        0.004191s
1097
      9
                0.104498s
                                            0.11809s
                                                                        0.004178s
1098
      WHAT
                                                     TIME(TOT)
                                                                        TIME (AVG)
1099
      GPU time (Transfer time excluded)
                                                       1.03547s
1100
                                                                        0.103547 s
      GPU time (Transfer time included)
                                                       1.14054s
1101
                                                                        0.114054s
1102
      CPU time
                                                       0.049659s
                                                                        0.0049659s
       Running matrix_max on an 2048 x 2048 matrix
1103
1104
       Local Work Size = 256
      Number of CPU workers = 4
ITER GPU(EX.BUF)
1105
                                                                        CPU
                                            GPU(INC BUE)
1106
1107
      0
                0.107878s
                                            0.114359s
                                                                        0.006965s
1108
                0.107007s
                                            0.112568s
                                                                        0.004204s
1109
                0.109405s
                                            0.118287s
                                                                        0.004159s
1110
      3
                0.103465s
                                            0.11491s
                                                                        0.004184s
1111
      4
                0.10545s
                                            0 1164875
                                                                        0.0042165
1112
      5
                0.101241s
                                            0.123807s
                                                                        0.00422s
```

```
1113
                 0.102558s
                                              0.112428s
                                                                            0.004209 s
1114
                 0.1053215
                                              0 1201575
                                                                            0.0042175
1115
                 0.103111s
                                                                            0.004181s
       8
                                              0.114123s
                 0.102518s
                                                                            0.004182s
       9
                                              0.117968s
1116
1117
       WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
1118
       GPU time (Transfer time excluded)
GPU time (Transfer time included)
1119
                                                         1.04795 s
                                                                            0.104795s
1120
                                                         1 16509s
                                                                            0 1165095
       CPU time
                                                         0.044737s
                                                                            0.0044737s
1121
       Running matrix_max on an 4096 \times 4096 matrix Local Work Size = 4
1122
1123
       Number of CPU workers = 4
ITER GPU(EX.BUF)
1124
1125
                                              GPU(INC.BUF)
                                                                            CPU
1126
       0
                 0.131658s
                                              0.179302s
                                                                            0.019246s
1127
       1
                 0.131686s
                                              0.17934s
                                                                            0.016279s
                                              0.172332s
1128
                 0.133496s
                                                                            0.016338s
                 0.131699s
1129
                                              0.168124 s
                                                                            0.016302s
1130
                 0.131565s
                                              0.169192s
                                                                            0.017017s
1131
       5
                 0.131663s
                                              0.179068\,\mathsf{s}
                                                                            0.016488s
1132
       6
                 0.133576s
                                              0.176722s
                                                                            0.016252s
1133
                 0.133376s
                                              0 1764145
                                                                            0.0164985
1134
       8
                 0.133366s
                                              0.166915s
                                                                            0.01624s
1135
       9
                 0.13288s
                                              0.174505\,s
                                                                            0.016471s
1136
1137
      WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
1138
       GPU time (Transfer time excluded)
                                                        1.32497 s
                                                                            0.132497s
       GPU time (Transfer time included)
1139
                                                         1 741915
                                                                            0 1741915
       CPU time
1140
                                                         0.167131s
                                                                            0.0167131s
       Running matrix_max on an 4096 x 4096 matrix Local Work Size = 8
1141
1142
       Number of CPU workers = 4
ITER GPU(EX.BUF)
1143
                                              GPU(INC.BUF)
                                                                            CPU
1144
1145
       0
                 0.076053s
                                              0.115518s
                                                                            0.019122s
                 0.077227s
1146
       1
                                              0.118312s
                                                                            0.01623s
                 0.075462s
                                                                            0.016383s
                                              0.118964s
1147
                 0.077935s
                                              0.120758s
                                                                            0.016228s
1148
       4
                 0.086644s
                                              0.115309s
                                                                            0.016245s
1149
1150
                 0.080869s
                                              0.118394\,\mathsf{s}
                                                                            0.016238s
1151
       6
                 0.076934s
                                              0.117439s
                                                                            0.016219s
1152
                 0.075285s
                                              0.116531s
                                                                            0.016251s
1153
       8
                 0.074601s
                                              0.113045 s
                                                                            0.016265s
1154
       9
                 0.075773s
                                                                            0.016238s
                                              0.117732s
1155
1156
       WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
       GPU time (Transfer time excluded) GPU time (Transfer time included)
1157
                                                         0.776783s
                                                                            0.0776783s
                                                         1.172s 0.1172s
1158
       CPU time
                                                                            0.0165419s
1159
                                                         0.165419s
       Running matrix_max on an 4096 x 4096 matrix
Local Work Size = 16
Number of CPU workers = 4
ITER GPU(EX.BUF) GPU(INC.BUF
1160
1161
1162
                                              GPU(INC.BUF)
                                                                            CPU
1163
                 0.057033s
                                                                            0.019586s
1164
       O
                                              0.114978s
                 0.061941s
                                              0.09736s
                                                                            0.01624s
1165
       1
                 0.05934s
                                              0.10736s
                                                                            0.016233s
1166
                 0.066152s
                                              0.10568s
                                                                            0.016265s
1167
1168
       4
                 0.067301s
                                              0.103986s
                                                                            0.016206s
1169
                 0.081248s
                                              0.10584\,\mathsf{s}
                                                                            0.016221s
1170
       6
                 0.071491s
                                              0.112342s
                                                                            0.016224 s
1171
                 0.070696s
                                              0.104142s
                                                                            0.016285s
1172
       8
                 0.066575s
                                              0.098806s
                                                                            0.016206s
1173
       9
                 0.069086s
                                              0.101545\,s
                                                                            0.016407s
1174
1175
      WHAT
                                                        TIME(TOT)
                                                                            TIME(AVG)
1176
       GPU time (Transfer time excluded)
                                                         0.670863\,s
                                                                            0.0670863s
       GPU time (Transfer time included)
1177
                                                         1 05204s
                                                                            0 1052045
       CPU time
                                                         0.165873s
                                                                            0.0165873s
1178
       Running matrix_max on an 4096 x 4096 matrix
1179
       Local Work Size = 32
Number of CPU workers = 4
ITER GPU(EX.BUF)
1180
1181
                                              GPU(INC.BUF)
                                                                            CPU
1182
1183
       0
                 0.137356s
                                              0.161587s
                                                                            0.019011s
```

```
1184
                 0.130635s
                                               0.192045s
                                                                             0.016236s
1185
       2
                 0 1451295
                                               0 178795
                                                                             0.0163115
1186
       3
                 0.114893s
                                               0.169601s
                                                                             0.0163s
1187
       4
                 0.153264s
                                               0.169223s
                                                                             0.016209s
                                                                             0.016241s
1188
                 0.151884s
                                               0.184035s
1189
                 0.144728s
                                               0.211291s
                                                                             0.017821s
1190
                 0.116898s
                                               0.185365s
                                                                             0.016305s
1191
       8
                 0 1152485
                                               0 1896885
                                                                             0.0162745
                 0.15323s
                                               0.16913s
                                                                             0.016325 s
1192
       9
1193
1194
       WHAT
                                                         TIME(TOT)
                                                                            TIME(AVG)
       GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                         1.36327s
1195
                                                                             0.136327s
1196
                                                          1 81076s
                                                                             0 1810765
1197
       CPU time
                                                          0.167033s
                                                                             0.0167033s
       Running matrix_max on an 4096 x 4096 matrix
Local Work Size = 64
Number of CPU workers = 4
1198
1199
1200
1201
       ITER
                 GPU(EX.BUF)
                                               GPU(INC.BUF)
                                                                            CPU
1202
       0
                 0.237582s
                                               0.271394s
                                                                             0.019043s
1203
       1
                 0.248587s
                                               0.341792s
                                                                             0.016213 s
1204
                 0 3172445
                                               0.3570475
                                                                             0.0162275
       2
1205
       3
                 0.304983s
                                               0.361604s
                                                                             0.016221s
1206
       4
                 0.332338s
                                               0.278672s
                                                                             0.016305s
1207
                 0.305837s
                                               0.344257s
                                                                             0.016466s
1208
       6
                 0.33021s
                                               0.355925s
                                                                             0.017171s
1209
                 0.318338s
                                               0.348765\,s
                                                                             0.017051 s
1210
       8
                 0.331313s
                                               0.35637s
                                                                             0.017363s
                 0.33192s
                                                                             0.017246s
1211
       9
                                               0.335312s
1212
       WHAT
                                                         TIME(TOT)
1213
                                                                            TIME(AVG)
       GPU time (Transfer time excluded)
GPU time (Transfer time included)
1214
                                                          3.05835 s
                                                                             0.305835s
1215
                                                          3 351145
                                                                             0 335114s
1216
       CPU time
                                                          0.169306s
                                                                             0.0169306s
       Running matrix_max on an 4096 x 4096 matrix Local Work Size = 128 Number of CPU workers = 4
1217
1218
1219
1220
                 GPU(EX.BUF)
                                               GPU(INC.BUF)
                                                                            CPU
       ITER
1221
       0
                 0.331774s
                                               0.32765s
                                                                             0.019931s
1222
       1
                 0.285925s
                                               0.394738s
                                                                             0.0180285
                 0.279375s
1223
       2
                                               0.395744s
                                                                             0.017022s
1224
                 0.33099s
                                               0.397785s
                                                                             0.016458s
       3
1225
       4
                 0.281085s
                                               0.404075s
                                                                             0.016573s
1226
                 0.296238s
                                               0.33581s
                                                                             0.016262s
1227
       6
                 0.331469s
                                               0.392683s
                                                                             0.017332s
1228
                 0.279669 \, s
                                               0.402449\,s
                                                                             0.016792s
                                               0 3984935
1229
       8
                 0 2812855
                                                                             0.0166915
1230
                 0.328756s
       9
                                               0.39021s
                                                                             0.016373 s
1231
1232
       WHAT
                                                         TIME(TOT)
                                                                             TIME(AVG)
       GPU time (Transfer time excluded)
GPU time (Transfer time included)
1233
                                                          3.02657s
                                                                             0.302657s
1234
                                                          3.83964s
                                                                             0 3839645
       CPU time
1235
                                                          0.171462s
                                                                             0.0171462s
       Running matrix_max on an 4096 x 4096 matrix
Local Work Size = 256
Number of CPU workers = 4
1236
1237
1238
1239
       ITER
                 GPU(EX.BUF)
                                               GPU(INC.BUF)
                                                                            CPU
1240
       0
                 0.153446s
                                               0.187273s
                                                                             0.019068s
1241
       1
                 0.147271s
                                               0.188242s
                                                                             0.016242s
                 0.143291s
                                               0.19893s
                                                                             0.01631s
1242
       2
                                               0.188041s
                                                                             0.016246s
1243
       3
                 0.155284s
1244
                 0.148455s
                                               0.191923s
                                                                             0.016182s
1245
       5
                 0.146254s
                                               0.194932s
                                                                             0.016243s
                                                                             0.02001s
1246
       6
                 0.149059s
                                               0.190919\,\mathsf{s}
1247
                 0.147974s
                                               0.191543s
                                                                             0.016247s
1248
       8
                 0.154037s
                                               0 1828025
                                                                             0.0165895
1249
       9
                 0.153081s
                                               0.197119s
                                                                             0.016233s
1250
1251
       WHAT
                                                         TIME(TOT)
                                                                            TIME(AVG)
       GPU time (Transfer time excluded)
GPU time (Transfer time included)
1252
                                                          1.49815 s
                                                                             0.149815s
1253
                                                          1.91172s
                                                                             0.191172s
       CPU time
1254
                                                          0.16937s
                                                                             0.016937s
```

```
Running matrix_max on an 8192 x 8192 matrix Local Work Size = 4 Number of CPU workers = 4 \,
1255
1256
1257
1258
      ITER
                GPU(EX.BUF)
                                             GPU(INC.BUF)
                                                                          CPU
                0.486677s
                                                                          0.067399s
1259
                                             0.481426s
1260
                0.478481s
                                             0.486936s
                                                                          0.064348s
1261
                0.478843s
                                             0.475862s
                                                                          0.06434s
1262
      3
                0.47912s
                                             0.476938s
                                                                          0.06448s
1263
                0.477748s
                                             0.475909s
                                                                          0.06429s
1264
                0.474912s
                                             0.478002s
                                                                          0.064853s
      5
1265
      6
                0.474519s
                                             0.477661s
                                                                          0.064267s
1266
                0.477599s
                                             0.475624s
                                                                          0.064315s
                                             0.476265s
1267
      8
                0.475162s
                                                                          0.064265s
1268
      9
                0.475774s
                                             0.475413s
                                                                          0.064363s
1269
      WHAT
                                                      TIME(TOT)
                                                                          TIME(AVG)
1270
                                                        4.77883s
1271
      GPU time (Transfer time excluded)
                                                                          0.477883s
1272
      GPU time (Transfer time included)
                                                        4.78004s
                                                                          0.478004s
1273
      CPU time
                                                        0.64692 \, s
                                                                          0.064692s
      Running matrix_max on an 8192 x 8192 matrix
Local Work Size = 8
Number of CPU workers = 4
1274
1275
1276
                GPU(EX.BUF)
1277
      ITER
                                             GPU(INC.BUF)
                                                                          CPU
1278
                0.245722s
                                             0.244436s
                                                                          0.067372s
1279
                0.245343s
                                             0.2448s
                                                                0.065246s
1280
                0.245571s
                                             0.245364s
                                                                          0.064321s
1281
      3
                0.245069s
                                             0 2446485
                                                                          0.066329 s
                0.24552s
                                             0.245168s
                                                                          0.06469s
1282
1283
                0.243336s
                                             0.244409s
                                                                          0.064297s
      5
1284
      6
                0.245751s
                                             0.246419s
                                                                          0.064986s
1285
                0.254729s
                                             0.243795s
                                                                          0.064239s
1286
      8
                0 2450935
                                             0 2445385
                                                                          0.064255
1287
      9
                0.244242s
                                             0.243915s
                                                                          0.065069s
1288
1289
      WHAT
                                                      TIME(TOT)
                                                                          TIME(AVG)
      GPU time (Transfer time excluded)
1290
                                                       2.46038s
                                                                          0.246038s
      GPU time (Transfer time included)
                                                        2.44749s
                                                                          0.244749s
1291
      CPU time
1292
                                                        0.650799 \, s
                                                                          0.0650799s
      Running matrix_max on an 8192 x 8192 matrix Local Work Size = 16 Number of CPU workers = 4 \,
1293
1294
1295
1296
      ITER
                GPU(EX.BUF)
                                             GPU(INC.BUF)
                                                                          CPU
1297
                0.138015s
                                             0.138181s
                                                                          0.067772s
1298
                0.137435s
                                             0.138433s
                                                                          0.064527s
1299
                0.13731s
                                             0.138422\,\mathsf{s}
                                                                          0.064354s
1300
      3
                0.137315s
                                             0 1381385
                                                                          0.0643295
1301
      4
                0.137493s
                                             0.137672s
                                                                          0.06508s
                0.137433s
                                                                          0.066035s
1302
      5
                                             0.138425s
1303
      6
                0.136428\,\mathsf{s}
                                             0.137979s
                                                                          0.064333s
1304
                0.137609s
                                             0.137346s
                                                                          0.064326s
1305
      8
                0.138453s
                                             0 1379235
                                                                          0.064256s
1306
      9
                0.137136s
                                             0.136742s
                                                                          0.064212s
1307
                                                       TIME(TOT)
                                                                          TIME(AVG)
1308
                                                       1.37463s
                                                                          0.137463s
1309
      GPU time (Transfer time excluded)
      GPU time (Transfer time included)
                                                        1.37926s
                                                                          0.137926s
1310
1311
      CPU time
                                                        0.649224s
                                                                          0.0649224s
      Running matrix_max on an 8192 x 8192 matrix
Local Work Size = 32
Number of CPU workers = 4
1312
1313
1314
                GPU(EX.BUF)
      ITER
                                             GPU(INC.BUF)
1315
1316
                0.10669s
                                             0.115407s
                                                                          0.067092s
1317
                0.108893s
                                             0.115574\,\mathsf{s}
                                                                          0.064317s
1318
                0.115982\,\mathsf{s}
                                             0.120901\,\mathsf{s}
                                                                          0.06571s
      3
1319
                0 1158685
                                             0 1161445
                                                                          0.0642475
1320
      4
                0.115581s
                                             0.114323s
                                                                          0.066162s
1321
                0.115336s
                                             0.11988s
                                                                          0.064228s
      5
1322
      6
                0.115913s
                                             0.115885s
                                                                          0.064254 s
1323
                0.114442s
                                             0.108985s
                                                                          0.064281s
1324
      8
                0.106479s
                                             0.121078s
                                                                          0.064455
1325
      9
                0.105123s
                                             0.115441s
                                                                          0.064192s
```

```
1326
                                                                         TIME(AVG)
      WHAT
                                                      TIME(TOT)
1327
      GPU time (Transfer time excluded)
1328
                                                       1.12031s
                                                                         0.112031s
1329
      GPU time (Transfer time included)
                                                       1.16362s
                                                                         0.116362s
      CPU time
1330
1331
       Running matrix_max on an 8192 x 8192 matrix
      Local Work Size = 64
Number of CPU workers = 4
1332
1333
                GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
1334
      ITER
1335
                0.084819s
                                            0.084835s
                                                                         0.067364s
      0
1336
                0.084627s
                                            0.086459s
                                                                         0.064226s
1337
                0.086188s
                                            0.086259 s
                                                                         0.064337s
1338
      3
                0.084569s
                                            0.085386s
                                                                         0.06468s
1339
      4
                0.08664s
                                            0.087318s
                                                                         0.064252s
1340
      5
                0.083892s
                                            0.085589s
                                                                         0.064227 s
                0.084584s
1341
      6
                                            0.084735s
                                                                         0.064354 s
                0.085227s
1342
                                            0.083969s
                                                                         0.064289s
1343
                0.086087s
                                            0.085147s
                                                                         0.064296s
1344
      9
                0.086052s
                                            0.085495s
                                                                         0.064145s
1345
      WHAT
                                                      TIME (TOT)
                                                                         TIME (AVG)
1346
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
                                                      0.852685s
                                                                         0.0852685s
1347
1348
                                                       0.855192s
                                                                         0.0855192 \, s
      CPU time
                                                                         0.064617s
1349
                                                       0.64617s
1350
      Running matrix_max on an 8192 x 8192 matrix
      Local Work Size = 128
Number of CPU workers = 4
1351
1352
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
1353
      ITER
                0.106948s
                                            0.109526s
                                                                         0.067848s
1354
      0
                0.107187s
                                            0.113418s
                                                                         0.064364s
1355
1356
                0.105224s
                                            0.105687s
                                                                         0.064626s
1357
      3
                0.123325s
                                            0.112672s
                                                                         0.0647665
1358
      4
                0.117601s
                                            0.105831s
                                                                         0.064882s
                0.092825s
                                                                         0.064635 s
1359
      5
                                            0.109842s
1360
                0.117525s
                                            0.111299s
                                                                         0.064337s
      6
                0.105265s
                                            0.106353s
1361
                                                                         0.064383s
                0.113094s
1362
                                            0.115145s
                                                                         0.06435s
      8
1363
      9
                0.108792s
                                            0.107833s
                                                                         0.064297 s
1364
      WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
1365
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
1366
                                                       1.09779s
                                                                         0.109779s
1367
                                                       1.09761s
                                                                         0.109761s
       CPU time
                                                       0.648488s
                                                                         0.0648488s
1368
1369
      Running matrix_max on an 8192 x 8192 matrix
      Local Work Size = 256
Number of CPU workers = 4
1370
1371
               GPU(EX.BUF)
                                            GPU(INC.BUF)
                                                                         CPU
      ITER
1372
                0.114115s
                                            0.106475s
                                                                         0.068122s
1373
      0
1374
                0.076723s
                                            0.118151\,\mathsf{s}
                                                                         0.064366s
1375
                0.112423s
                                            0.108933s
                                                                         0.065309s
1376
      3
                0.111586s
                                            0.099482s
                                                                         0.064281s
1377
      4
                0.114877s
                                            0.111822s
                                                                         0.064221s
                0.108627s
                                                                         0.064338s
1378
      5
                                            0.112013s
      6
                0.112327s
                                            0.113743s
                                                                         0.064301s
1379
                0.105834s
                                                                         0.064213s
1380
                                            0.113864s
1381
      8
                0.110282s
                                            0.090779s
                                                                         0.064328s
1382
      9
                0.107973s
                                            0.110524s
                                                                         0.064308s
1383
      WHAT
                                                      TIME(TOT)
                                                                         TIME(AVG)
1384
      GPU time (Transfer time excluded)
GPU time (Transfer time included)
1385
                                                       1.07477s
                                                                         0.107477s
1386
                                                        1.08579s
                                                                         0.108579s
      CPU time
                                                       0.647787s
                                                                         0.0647787s
1387
```