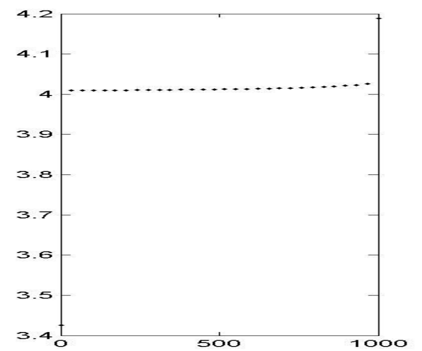
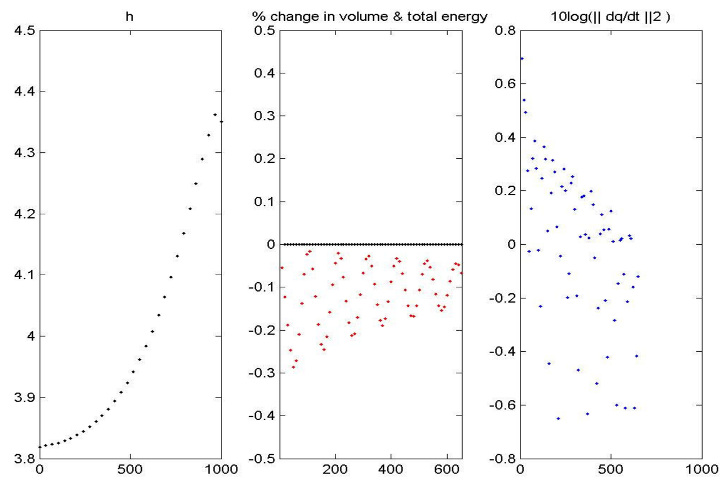
Comments on: wall boundary conditions, etc. Lab 4&5.



First, let us see what the Roe scheme does when started with perfectly flat, still sea at 4 m, a steady exact solution. The depth varies from 44m to 4m. The boundary conditions are “reflections”: copy *h* and reverse *hu*. After one step, the surface is no longer flat, and the first and last gridpoints have jumped (to 3.44 and 4.18). This tells us two things:

- the Roe-steady state, if there is one, is (slightly) different from the exact;
- the boundary conditions introduce jumps immediately.



So, we keep running and see if the sea will calm down to a Roe steady solution: Perhaps, but 600 steps or about six travel times of the waves across the ocean is nowhere near enough. The blue dots in the right plot show the magnitude of ||dq/dt|| (log scale) and 600 steps gave -0.4 so to get to -6 we need ten thousand steps. Clearly NOT a fast way to compute a steady solution!
Volume conservation is perfect, total energy not so, (middle plot, note this is % change).
The bottom line is that these BC introduce jumps (of size $O(\Delta x)$, see below) which live “forever” as waves, traveling and reflecting off the boundaries. When Hi-Res is turned on, they look like shocks, or maybe we should call them shock-lets because their magnitudes are small and they travel with the speed of “sound”.
None of this happens when the sea floor is horizontal, so Leveque does not discuss it.

We can make a modified equation analysis based on the fact that

$$u^2 << gh = c^2$$

because then
1. the equations simplify & become linear
2. the Roe matrix becomes $c\mathbf{I}$,

$$h_t + m_x = \Delta x\left(c(h+h_0)_x\right)_x = \Delta x\left(ch_x\right)_x + \Delta x S c_x, c^2 = gh_0(x)$$

$$m_t + c^2 h_x = \Delta x\left(cm_x\right)_x$$

S is the slope of the sea floor. The steady state solution of these equations has a length-scale of Δx but the magnitude depends on the BC. One may try to construct

BC (combinations of h and h_x, likewise m and m_x), still exactly conservative. We will not pursue this further.

However, we will explain the first plot above. It needs a look at the Roe-discretized h-equation:

$$h_j^1 - h_j^0 = -\frac{\Delta t}{2\Delta x}\left(\left(m_{j+1}^0 - m_{j-1}^0\right) + c_{j+1/2}^0\left(h_{j+1}^0 - h_j^0\right) - c_{j-1/2}^0\left(h_j^0 - h_{j-1}^0\right)\right) =$$
$$= -\frac{S\Delta t}{2}\left(c_{j+1/2}^0 - c_{j-1/2}^0\right) = -\frac{S\Delta h\Delta t}{4}\sqrt{g/h_j}\,, j > 1$$
$$= -\frac{\Delta t}{2\Delta x}\left(c_{3/2}^0\left(h_2^0 - h_1^0\right) - 0\right) = -\frac{S\Delta t}{2}\sqrt{gh_{3/2}}\,, j = 1$$

where S is the slope dH/dx of the sea floor. Δh is the change in h over a cell. We see that the Roe scheme built-in diffusion term is responsible. As the plot shows,

- the shallower the water, the larger the change;
- the change at the boundary (j = 1) is an order of magnitude larger than in the interior, $j > 1$
- the change at the deep boundary is larger (by a factor of the square root of the depths) than at the shallow boundary.