Gunilla Kreiss,
NADA
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Numerical treatment of Differential Equations II, 2D1255, 2006

Problem set 5

Hand in a solution no later then April 21.

The relevant background for this problem is the material by Ken Mattsson on SBP operators and the SAT method.

Consider solving the linear advection equation,

$$q_t + (u(x)q)_x = 0, \quad 0 \leq x \leq 5,$$

$$u(x) = 1 + e^{-(x-2.5)^2},$$

with boundary condition

$$q(0,t) = \begin{cases} \sin^4(\pi t) & 0 \leq t \leq 1, \\ 0 & \text{otherwise} \end{cases},$$

and homogeneaus initial data, $u(x,0) = 0$, until $t = 6$. Use the SBP operator D_1 of section 1.2 in the appendix to approximate $\partial/\partial x$, and the classical Runge-Kutta method of order 4 in time. Impose the boundary condition using the SAT method. What should the coefficient of the penalty term be in order to guarantee stability? **Hint:** the last 4 rows of D_1 are obtained by

$$D_{n+1-m,n+1-j} = -D_{mj}, \; m = 1, \ldots 4, \; j = 1, \ldots 6.$$

1. Investigate, by analysis and numerical experiments, accuracy and stability. In particular, derive estimates for the continous problem and for the semi-discrete problem, and compare.

2. You should also look for reflections at $x = 5$. Can there be reflections in the mathematical problem?Does the resolution or the value of $\Delta t/\Delta x$ influence the appearence of reflections in your computation?

3. Design a test problem that shows that using a higher order method reduces dispersion errors.