

Variational formulation: find $u \in V$:

(4)

$$(V) \quad (\nabla u, \nabla v) = (f, v) \quad \forall v \in V$$

$$V = \left\{ v : \int_{\Omega} (|\nabla v|^2 + v^2) dx < \infty \text{ \& } v=0 \text{ on } \Gamma^? \right\}$$

$$V_h = \left\{ v \in V : v \text{ p.w. cont. on } \mathcal{T}_h \right\}$$

Degrees of freedom of $v \in V_h$: $\mathcal{N}_h = \{N\}$

set of internal nodes of \mathcal{T}_h

(homogeneous Dirichlet b.c.)

Let $\{N_1, \dots, N_n\}$ be an enumeration of \mathcal{N}_h

Let $\{\phi_1, \dots, \phi_n\}$ be corresponding nodal basis for V_h

Galerkin FEM: Find $U \in V_h$ such that

$$(G) \quad (\nabla U, \nabla v) = (f, v) \quad \forall v \in V_h$$

$V_h \subset V \Rightarrow$ Galerkin orthogonality: (V)-(G)

$$(\nabla u - \nabla U, \nabla v) = 0 \quad \forall v \in V_h$$