To minimize the error $\|\nabla u_h - \nabla u\|$ we want to minimize $\|h R(u)\|$: so that we choose $h = h(x)$ small where the residual $R(u)$ is large.

Simple adaptive algorithm

Start from initial (coarse) mesh $\mathcal{T}_h^0$. Set $i=1$.

(1) Compute solution $u \in V_h$ by FEM

(2) Compute $R(u) = f + \Delta u \approx f + \Delta_h u$

(3) Mark 50% of the elements for refinement which have the largest residual $R(u)$.

(4) Refine the mesh $\mathcal{T}_h^{i-1}$, which then gives a new mesh $\mathcal{T}_h^i$.

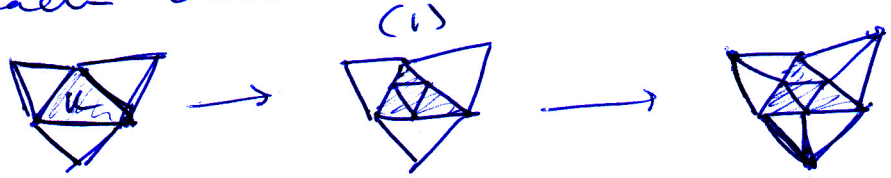
(5) Set $i = i+1$ then go to (1).

Red-green mesh refinement

(1) loop over all marked cells : insert new nodes at edge midpoints, and connect them by new edges $\Rightarrow$ 4 new cells.

(2) loop over all hanging nodes : connect ~~each~~ hanging node with the node opposite on each cell.

Ex :



Ex :