L4

Polynomial approximation: $\mathcal{P}^q(a,b)$ is the set of polynomials $p(x) = \sum_{i=0}^{q} c_i x^i$. ($\dim(\mathcal{P}^q(a,b)) = q+1$)

$c_i \in \mathbb{R}$ coefficients in monomial basis $\{x^i\}_{i=0}^q = \{1, x, x^2, \dots, x^q\}$

Lagrange basis $\{\lambda_i\}_{i=0}^q$: associated with $q+1$ nodes:

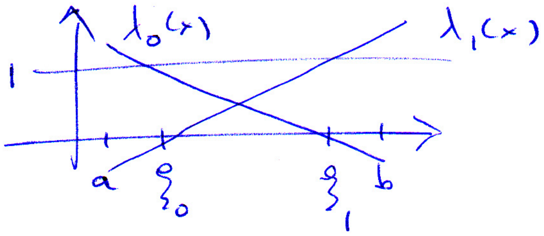
$\xi_0 < \xi_1 < \dots < \xi_q$ in (a,b).

Determined by: $\lambda_i(\xi_j) = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{else} \end{cases}$

$$\lambda_i(x) = \prod_{j \neq i} \frac{x-\xi_j}{\xi_i-\xi_j} = \frac{(x-\xi_0)(x-\xi_1)\cdots(x-\xi_{i-1})(x-\xi_{i+1})\cdots(x-\xi_q)}{(\xi_i-\xi_0)(\xi_i-\xi_1)\cdots(\xi_i-\xi_{i-1})(\xi_i-\xi_{i+1})\cdots(\xi_i-\xi_q)}$$

Lagrange basis is a nodal basis: $p \in \mathcal{P}^q(a,b)$ can be written as $p(x) = \sum_{i=0}^{q} p_i \lambda_i(x)$ with $p_i = p(\xi_i)$.

$q=1$: linear Lagrange basis $\{\lambda_0(x), \lambda_1(x)\}$



$$\lambda_0(x) = \frac{x-\xi_1}{\xi_0-\xi_1}$$

$$\lambda_1(x) = \frac{x-\xi_0}{\xi_1-\xi_0}$$