Integrate by parts over each subinterval I_j, and use that $(e-\pi_h e)(x_j)=0$ $\forall$ nodes x_j.

$$\Rightarrow \|e'\|_a^2 = \int_0^1 R(U)(e-\pi_h e)\,dx$$

with (discontinuous) residual

$$R(U) = f + (aU')' \quad \text{on each } I_j$$

$$\Rightarrow \|e'\|_a^2 \le \|hR(U)\|_{a^{-1}} \|h^{-1}(e-\pi_h e)\|_a$$

Inter polation error est. $\|h^{-1}(e-\pi_h e)\|_a \le C_i \|e'\|_a$

$$\Rightarrow \|u'-U'\|_a \le C_i \|hR(U)\|_{a^{-1}}$$

2D Poisson: FEM: find $U \in V_h$ s.t.

$$(\nabla U, \nabla v) = (f, v) \quad \forall v \in V_h$$

$V_h = \{v : v$ p.w. linear cont. on mesh $\tau_h\}$