Non-homogeneous Dirichlet b.c.

Poisson 1D : $\begin{cases} -u'' = f & \text{on } (0,1) \\ u(0) = g,\ u(1) = 0 \end{cases}$

Variational formulation (mult. by test function & integrate over $(0,1)$) :

$$\int_0^1 -u'' v \, dx = \int_0^1 f v \, dx$$

Partial integration $\Rightarrow \int_0^1 u'v' dx - [u'v]_0^1 = \int_0^1 f v \, dx$

Choose $v \in V_0 = \{ v : \|v\|^2 + \|v'\|^2 < \infty,\ v(0) = v(1) = 0 \}$

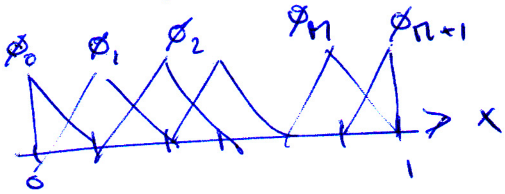
$\Rightarrow [u'v]_0^1 = 0$

Search for $u \in V_g = \{ v : \|v\|^2 + \|v'\|^2 < \infty,\ v(0) = g,\ v(1) = 0 \}$.

Find $u \in V_g$: $\int_0^1 u'v' dx = \int_0^1 f v \, dx \quad \forall v \in V_0$

FEM : Find $U \in V_h^g \subset V_g$: $\int_0^1 U'v' dx = \int_0^1 f v \, dx \quad \forall v \in V_h^0 \subset V_0$

$U = \sum_{j=0}^{M+1} \xi_j \phi_j(x)$



$U(1) = \xi_{M+1} = 0$; $U(0) = \xi_0 = g$

$\Rightarrow U = \sum_{j=1}^{M} \xi_j \phi_j(x) + g \phi_0(x)$

FEM : $\int_0^1 U'v' dx = \sum_{j=1}^{M} \xi_j \int_0^1 \phi_j' v' dx + g \int_0^1 \phi_0' v' dx = \int_0^1 f v \, dx \quad \forall v \in V_h^0$

$$\Rightarrow \underbrace{\sum_{j=1}^{M} \xi_j \int_0^1 \phi_j' \phi_i' dx}_{Ax} = \underbrace{\int_0^1 f \phi_i dx - g \int_0^1 \phi_0' \phi_i' dx}_{b} \quad i = 1, \dots, M$$