

L is "continuous" since by linearity;

(3)

$$|L(v) - L(w)| = |L(v-w)| \leq \mathcal{H}_3 \|v-w\|_V$$

so that $L(v) \rightarrow L(w) \quad \forall \|v-w\|_V \rightarrow 0$
($v \rightarrow w$ in V)

Similar with $a(\cdot, \cdot)$.

Energy norm: $\|v\|_a = \sqrt{a(v,v)}$

• $\|v\|_a \geq 0$ since $\|v\|_a^2 = a(v,v) \geq \mathcal{H}_1 \|v\|_V^2 \geq 0$

• $\|\cdot\|_a$ & $\|\cdot\|_V$ are equivalent norms:

$$\mathcal{H}_1 \|v\|_V^2 \leq \|v\|_a^2 \leq \mathcal{H}_2 \|v\|_V^2$$

If we choose the norm on V to be $\|\cdot\|_a$

we get $\mathcal{H}_1 = \mathcal{H}_2 = 1$.

Abstract Galerkin method: Find $U \in V_h$ s.t.

$$a(U, v) = L(v) \quad \text{for all } v \in V_h$$

where $V_h \subset V$ finite dimensional subspace.

Galerkin orthogonality: $a(u-U, v) = 0 \quad \forall v \in V_h$