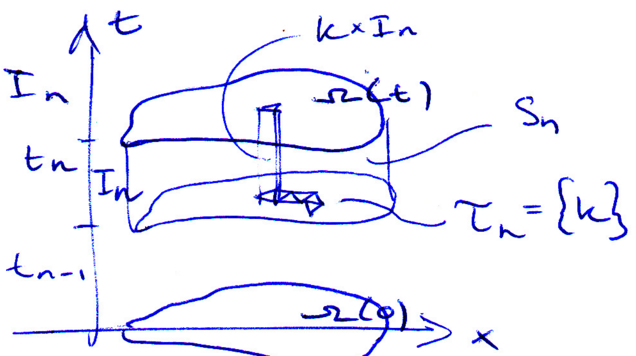
<u>Semidiscretization</u>: e.g. θ-method (time) + FEM (space)

<u>Space-time FEM</u>: Discretiz space-time simultaneously: basis functions $v = v(x,t)$

Space-time slab $S_n = \Omega \times I_n$

Space-time prism $K \times I_n$



$V_n = V_{h_n}$ space of hat functions on space mesh $\tau_n = \{K\}$

$$W_{kn}^{(r)} = \left\{ v(x,t) : v(x,t) = \sum_{j=0}^{r} t^j \varphi_j(x),\ \varphi_j \in V_h,\ (x,t) \in S_n \right\}$$

$$W_k^{(r)} = \left\{ v : v|_{S_n} \in W_{kn}^{(r)},\ n = 1, 2, \dots, N \right\}$$

$v \in W_k^{(r)}$ typically discont. across t_n: $[v_n] = v_n^+ - v_n^- > 0$

<u>cG(1) dG(0) for Heat equation</u>:

For $n = 1, \dots, N$ find $U \in W_k^{(0)}$:

$$\int_{I_n} \left((\dot{U}, v) + (\nabla U, \nabla v) \right) dt + \left([U_{n-1}], v_{n-1}^+ \right) = \int_{I_n} (f, v)\, dt \quad \forall v \in W_{kn}^{(0)}$$

$$\left(\dot{U} = 0,\ U|_{S_n} = U^n(x) \in V_h,\ [U_{n-1}] = U_n - U_{n-1} \right)$$

$$\Rightarrow \int_{I_n} (\nabla U, \nabla v)\, dt + (U_n, v_n) = (U_{n-1}, v_n) + \int_{I_n} (f, v)\, dt$$

<u>Further reading</u>: 9.1–9.2 (ODE), 16, 17 (Heat & Wave eqns)