

Puffin :

In Puffin the backward Euler method is typically used in the examples.

$$\text{ODE : } \begin{cases} \dot{u}(t) = f(u(t)) \\ u(0) = u_0 \end{cases}$$

FEM for ODE : $cG(\tau)$ & $dG(\tau)$

$cG(\tau)$: Find $U \in V_k^{(\tau)}$ with $U(0) = u_0$ s.t.

$$\int_0^T \dot{U} v \, dt = \int_0^T f(U) v \, dt \quad \forall v \in W_k^{(\tau-1)}$$

$dG(\tau)$: Find $U \in W_k^{(\tau)}$ with $U_0^- = u_0$ s.t.

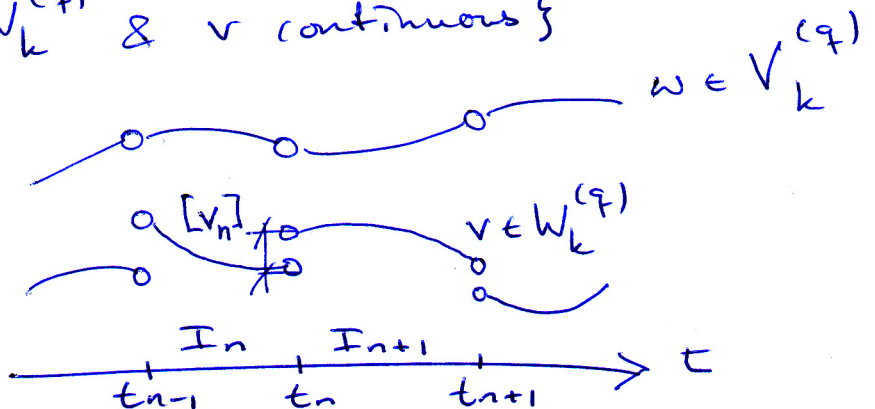
$$\int_0^T \dot{U} v \, dt + \sum_{n=1}^N [U_{n-1}] v_{n-1}^+ = \int_0^T f(U) v \, dt$$

$\forall v \in W_k^{(\tau)}$

$$W_k^{(\tau)} = \{ v : v |_{I_i} \in \mathcal{P}^{(\tau)}(I_i) \}$$

$$V_k^{(\tau)} = \{ v : v \in W_k^{(\tau)} \text{ \& } v \text{ continuous} \}$$

$$[v_n] = v_n^+ - v_n^-$$



$$k_n = t_n - t_{n-1}$$