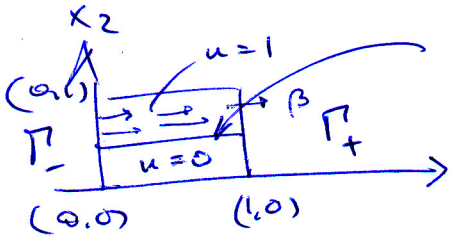
Ex: $\begin{cases} \dfrac{\partial u}{\partial x_1} = 0 & x \in \Omega = [0,1]\times[0,1] \\ u(0,x_2) = \begin{cases} 0 & 0 < x_2 < 1/2 \\ 1 & 1/2 \le x_2 < 1 \end{cases} \end{cases}$ $\left(\begin{array}{l} \beta = (1,0) \\ \alpha = 0 \\ f = 0 \end{array}\right)$

Solution: $u(x_1,x_2) = \begin{cases} 0 & 0 < x_2 < 1/2,\ 0 < x_1 < 1 \\ 1 & 1/2 \le x_2 < 1,\ 0 < x_1 < 1 \end{cases}$



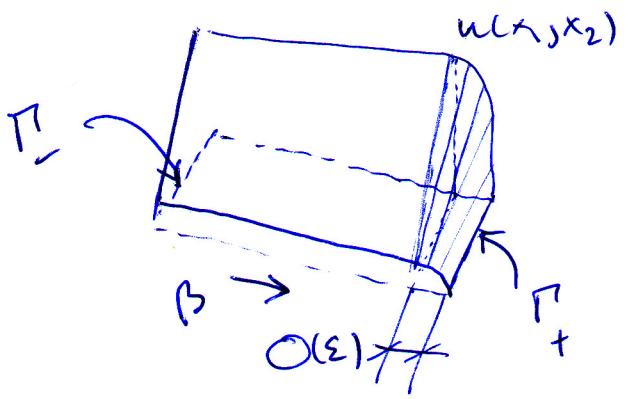
Discontinuity along streamline $x(s) = (s, \frac{1}{2})$

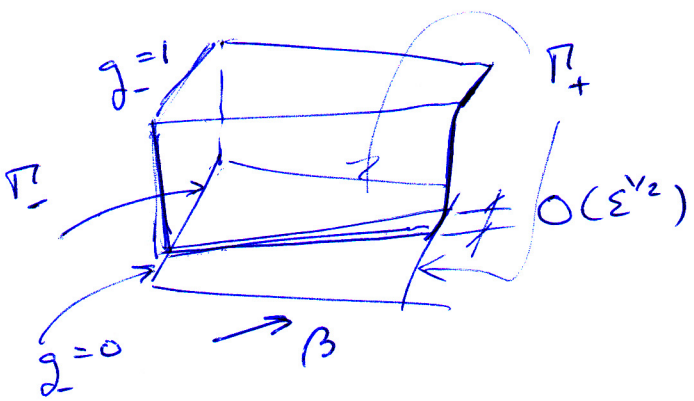
* The case $\dfrac{\varepsilon}{|\beta|}$ small similar to $\varepsilon \approx 0$.

* $\varepsilon > 0$ spreads out discontinuities $\Rightarrow$ continuous solution

(1) characteristic layer: $O(\sqrt{\varepsilon})$ (across streamline)

(2) boundary layer: $O(\varepsilon)$ (with Dirichlet b.c. Γ_+)



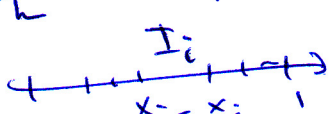


Ex (Problem 18.6) $\begin{cases} -\varepsilon u'' + u' = 0 & \text{on } \Omega = (0,1) \\ u(0) = 1,\ u(1) = 0 \end{cases}$ $(\Gamma_- = 0,\ \Gamma_+ = 1)$

cG(1): Find $U \in V_h$: $\int_0^1 \varepsilon U'v'\,dx + \int_0^1 U'v\,dx = 0 \quad \forall v \in \hat{V}_h$

$$\begin{cases} V_h = \{v \in H^1(0,1) : v(0) = 1,\ v(1) = 0\} \\ \hat{V}_h = \{v \in H^1(0,1) : v(0) = v(1) = 0\} = H_0^1(0,1) \end{cases}$$

$\mathcal{T}_h$: uniform mesh with M interior nodes:



$h = \dfrac{1}{M+1}$