2D1260 Finite Element Methods: Written Examination
Monday 2003-05-19, kl 9-14

Aids: None. **Time:** 5 hours.

Answers may be given in English or Swedish.

Please note that answers should be explained and calculations shown unless the question states otherwise. A correct answer without explanation can thus be left without points.

Using a desk calculator is not allowed. It is thus allowed to leave simple expressions unsimplified which could easily be calculated on a **simple** desk calculator.
For example $\alpha = 0.3 \cdot 0.15^3 \cdot \cos(\pi/4)$ or $\beta = 0.3\left\{\frac{0.15^3}{3} \cdot 0.7 + 2\right\}$
Do not leave integrals or system of equations unsolved unless explicitly allowed.

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(5) **1.** Consider the boundary value problem

$$-(x^2\, u')' = x - 2u, \qquad 2 < x < 3$$

$$u(2) = 4,\ u'(3) = 5$$

Derive the final system of equations which must be solved when approximating the solution by a cubic polynomial using Galerkins method. What is the dimension of this linear system of equations? Show the expressions for all involved integrals, but you need not calculate them. (It is not necessary to solve the resulting final system of equations.)

(5) **2.** Derive the final system of equations which must be solved when approximating the solution to the previous differential equation problem using two linear finite elements. The discretization points are $x = [2,\ 2.6,\ 3]$. You should use a mid-point quadrature for the integrals. It is not necessary to solve the resulting final system of equations.

N.B. *The exam continues on the next page.*

(5) **3.** Let the differential equation

$$-\Delta u = 2 \qquad \text{on } \Omega$$

be given on the quadrilateral domain with vertices $(1,2)$, $(4,1)$, $(6,3)$ and $(4,9)$. The boundary values are

$$u = 3x \text{ on the boundary between } (1,2) \text{ and } (4,1)$$
$$\frac{\partial u}{\partial n} = 0 \text{ on the other boundaries}$$

Solve the problem using FEM and two linear finite elements obtained by subdividing Ω along the vertical diagonal connecting $(4,9)$ and $(4,1)$.

(5) **4.** Derive a weak formulation of the 2D-problem

$$\nabla(au) - \nabla \cdot (b\ \nabla u) + c\ u = f \qquad \text{on } \Omega$$

where Ω is the second quarter of the unit circle:

$$\Omega = \begin{cases} x = r\cos\varphi, \\ y = r\sin\varphi, \end{cases} \qquad \text{with} \qquad \begin{cases} 0 < r < 1 \\ \pi/2 < \varphi < \pi \end{cases}$$

and the boundary conditions are

$$\begin{aligned} (\nabla u) \cdot n = g, &\qquad \text{when } x = 0 \\ \frac{\partial u}{\partial y} = h, &\qquad \text{when } y = 0 \\ u = j, &\qquad \text{on the curved boundary} \end{aligned}$$

where a, c, f, g, h and j are smooth functions and $b > 0$ a positive constant.
(Integrals should be simplified when possible for full points).

(5) **5.** How many nodes are needed in a 1D-element with quadratic basis functions? Show these basis functions. How many nodes are needed in a 2D-element generalised from this 1D-element? Show the basis functions for this 2D-element.

Good luck!

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