

Lecture 3

Induction and the HD Method

What is Truth?

- In an obvious way science is about finding truths. But what is truth? There is at least two different types of truth:
 - Correspondence Truth.
 - Coherence Truth.
- The two types of truth are related to two ways of finding truths:
 - Check observations of reality.
 - Prove statements with logical methods.

The idea of Empiricism

- Logical Positivism aka. Logical Empiricism is a philosophy of science that was particularly influential in the first half of the 20th century.
- One of the principles of LP is demands we must put on a statement S in order for it to be *meaningful*.
- Let S be any statement put in a form that indicates that it should be true or false. It is meaningful if either:
 - it in principle can be proved or disproved using logical methods
 - there are some observations that would confirm or disconfirm the statement
- All other statements are *meaningless*.
- It is now generally thought that this demand is too strong, but it is still a good guiding principle.

The connection between theories and observations

- Can we use observations and form a theory from them?
- Can we first form theory and then check it against observations?
- First we shall study the famous induction method.

Induction

- The basic idea: We make observations and try to see a pattern in them.
- If the observations are many and all agree with the pattern we conjecture that the pattern always applies.
- There are at least two different standardized forms of the method.

Induction: Form 1

- We make observations of objects which all has property A.
- Let us assume that in all observations the objects also have property B.
- We conclude that all objects with property A also have property B.

Induction: Form 2

- This is a more general form.
- Assume that we make observations of situations of a certain type P .
- Then assume all these situations are of type Q .
- We conclude that all situations of type P also are of type Q .

Does induction work?

- Yes, basically. There are however counter-examples.
- The set of observations must be chosen in a sufficiently general way.
- What is the logical basis for induction?
- One motivation for induction is the weak principle Uniformity of Nature (UN), see Okasha ch. 2.

A critic

David Hume 1711-1776



There is no scientific ground for induction!

- Induction cannot be proved to be correct using logic.
- Induction cannot be proved using induction (circular reasoning).
- We believe in induction since it seems to work.
- But it cannot be used for scientific proofs.

A solution?

Karl Popper 1902-1994



- Popper claims that he has solved the riddle of induction.
- The solution is that we never really use induction!
- We can never verify hypothesis.
- We can only falsify them.

Can induction generate theories?

- The idea is that we can see patterns and we can generalize them into theories.
- By using the induction principle we can "prove" the theory.
- But can it be done? There are at least three objections.
- The fact (if it is a fact) that we must first have a theory before we can make observations.
- Underdetermination.
- Goodman's paradox.

Observations depend on theories and expectations

- "We see what we believe".
- Rosenthal's experiment: A group of medicine students was divided in two groups. They were supposed to make an intelligence test on mice. They are each given a set of mice.
- Group A is told that their mice are the most intelligent. Group B didn't get to know anything.
- Group A found that their mice performed better in the test than the mice in the other group.
- But A and B were given mice of the same type!
- It seems as if the expectations in group A influence the result.
- For reasons like this it is recommended that one should perform double blind tests.

Underdetermination

- To each set of observations there are always different theories that fits the data.
- Perhaps we should chose the simplest theory (Occam's razor). But will that always give the best result.
- Goodman's paradox: Let us say that a thing is *grue* if it either is green and has been observed before Christmas Eve 2013 or has not been observed before Christmas Eve 2013 and is blue.
- Induction seems to tell us that that all emeralds are *grue*. Is that true?

In spite of this ...

- It seems as if it is impossible not to use induction, at least in everyday situations
- But what should we do in science?
- We will describe a method that is a sort of development of the induction method.

The two methods of science

- In science we work both with deductions and observations.
- In mathematics it is almost always deductions.
- In physics we work with both methods.
- In social sciences and humanities the situation is more uncertain. But in a way observations must be used.

Is there a general scientific method?

- Science has at least four different components:
- To set up hypotheses.
- To verify the hypotheses with logic.
- To evaluate the hypotheses by doing observations.
- To do experiments that generate observations.

Is there a general scientific method?

- A suggestion: It could be the Hypothetico- Deductive Method.
- It is certainly used in physics and chemistry.
- In a specialized sense it is used in mathematics.
- It seems as if it used sometimes in Social Sciences.

Carl Hempel 1905-1997



The general method

- A general method for handling observations is the Hypothetico-Deductive Method (The HD Method).
- The HD Method and the way of thinking connected to it is a central theme in scientific thinking.
- But not all researchers agree.
- Physics, astronomy, chemistry and biology seem to be the most natural areas for the method.

How it works

- Let us assume that we have a hypothesis H . We want to know if it is true or not.
- H can be a single fact or a general law.
- We have different observations E_1, E_2, \dots, E_n .
- (The observations can be generated by an experiment. They can also exist before H .)
- Does the observation *confirm* or *disconfirm* the hypothesis H ?
- The HD Method is a way to find an answer to that question.

A special case: Induction

- Goodman's problem: What hypothesis is supported by the induction?
- We first decide which hypothesis we want to test. (Goodman's problem doesn't occur.).
- A common form: H says that "All objects of type has property B".
- The observations are of the type: E1 = "Object O1 that is of type A has property B", and so on.

The HD Method used for falsification

- We have a hypothesis and want to show that it is false.
- We have a set of observations E_1, E_2, \dots, E_n .
- Assume that there is an observation E_i such that $H \Rightarrow \text{not } E_i$.
- Then E_i falsifies H .

The Phlogiston Theory

Antoine Lavoisier



The Phlogiston Theory:
When an object is burning it
is phlogiston leaving the
object.

The Phlogiston Theory was
falsified by Lavoisier.

The falsification of The Phlogiston Theory

- Let H be The Phlogiston Theory.
- A consequence of The Phlogiston Theory must be that burning objects get lighter.
- But we can find certain metals that get heavier after burning. Let us call this observation E.
- Since $H \Rightarrow \text{not } E$, we have falsified H.

Supporting hypotheses

- It might not be possible to prove $H \Rightarrow \text{not } E$ *directly*. We might need a supporting hypothesis A such that $H \& A \Rightarrow \text{not } E$.
- A could be all our *background knowledge*. (Kuhn would call it the paradigm.)
- Eg: H = "The illness is caused by bacteria".
- A = "Penicillin kills bacteria".
- E = "The illness is not cured by penicillin".

Ad hoc hypotheses

- Supporting hypotheses should be well established and secure. Sometimes they are not:
- If $H \Rightarrow \text{not } E$ and E has been observed, someone might want to *save* H .
- This can maybe be done by assuming that the implication has the form $(H \& A \Rightarrow \text{not } E)$. Then one substitutes $A1$ for A and get $(H \& A1 \Rightarrow E)$.
- If $A1$ seems very unlikely, if considered by itself, we call $A1$ an ad hoc hypothesis.

Example: The Phlogiston Theory

- Let H = The Phlogiston Theory.
- E was the observation of a metal getting heavier after burning.
- We can argue that the implication is $H \& A \Rightarrow \text{not } E$, where A is "The phlogiston has positive weight".
- We can replace A with $A1$ = "The phlogiston in the metal has negative weight". Then $H \& A1 \Rightarrow E$!
- But how probable is $A1$?

A more critical example: Uranus and Neptune

- The planet Uranus was discovered with telescope in 1781.
- In the beginning of the 19th century it was observed that Uranus didn't move in the way Newton's laws predicted.
- Call this observation E and Newton's laws H. Then we have $H \Rightarrow \text{not } E$.
- So Newton's laws were falsified!?
- But wait! The implication is really $H \& A \Rightarrow \text{not } E$ where A, amongst other things contained the statement that there are seven planets.
- But if we replace A with A* where A* says that there are unknown planets we don't get a falsification.
- and in 1846 Neptune (the eighth planet) was observed!
- So A* wasn't really an ad hoc hypothesis (or?).

The HD Method for falsification. Summary.

- We have a hypothesis and want to test if it is false.
- We use a supporting hypothesis A and deduce $H \& A \Rightarrow \text{not } E$.
- We then observe E.
- We have then falsified H.

The HD Method used for verification

- Assume that we have a hypothesis H and observations E_1, E_2, \dots, E_n .
- When can we say that the observations confirm H ?
- One possibility is that $E_1 \& E_2 \& \dots \& E_n \Rightarrow H$. In that case H is verified.
- But let us assume that this is not the case.

Observations that confirm

- We have H and E_1, E_2, \dots, E_n .
- Assume that they are all rather improbable.
- Assume that we have a hypothesis A that we already believe is true and that $H \& A \Rightarrow E_1 \& E_2 \& \dots \& E_n$.
- Then the observations confirm H .

Arguments for and against a hypothesis

- Assume that we have observations E_1, E_2, \dots, E_n and a hypothesis H .
- Some of the observations confirm H if they together with a supporting hypothesis A_i gives $H \& A_i \Rightarrow E_i$.
- Other observations disconfirm H if they together with a supporting hypothesis B_k $H \& B_k \Rightarrow \text{not } E_k$. Observe that we don't know if B_k is true. We have not falsified H with absolute certainty.

Making a decision

- We form a type of weighted average. If the supporting hypotheses A_i are more natural than the B_k we say that H is strengthened, otherwise it is weakened.
- This works best if we can use probability theory.

A third form of the HD-Method. To choose between hypotheses.

- If we have a set of observations E_1, E_2, \dots, E_n and a hypothesis H we can try to find supporting hypotheses A_i such that $H \& A_i \Rightarrow E_i$ for all i .
- If another hypothesis H^* can do the same thing with more natural supporting hypotheses B_i (that is $H^* \& B_i \Rightarrow E_i$), then we say that H^* is a better hypothesis.

We use probability

- The previous methods were qualitative.
- We now try to do a probabilistic analysis of when observations confirm a hypothesis.
- So we have this problem: Given a hypothesis H and an observation E , when can we say that the observation confirms H ?

An important formula

Thomas Bayes 1702-1761



He found an important formula connecting different types of conditional probabilities.

This formula is the basis for so called Bayesian Statistics.

Bayes' formula

- We want to know what the conditional probability $P(H|E)$ is.
- Bayes' formula:
$$P(H|E) = P(E|H)P(H) / ((P(E|H)P(H) + P(E|\text{not } H)P(\text{not } H))$$
- Alternatively, we can write $P(H|E) = P(E|H)P(H) / P(E)$
- Which form we use depends on whether we know what $P(E)$ is or not.

Example: Test of medicine

- Let us assume that we have a certain medicine that is supposed to cure a disease. Call the hypothesis that the medicine works H .
- We make an observation. It is that a sick Patient gets well after been given the medicine. Call this observation E .
- Can we decide to what degree E confirms H ?

Test of medicine II

- We want to find $P(H|E)$.
- We need to estimate some probabilities in Bayes' formula.
- $P(E|H) = 1$ seems reasonable.
- $P(E|\text{not } H)$ is more complicated. Let us assume that we have the probability 0.25.
- $P(H)$ is even more complicated. Let us start with the guess $P(H)=0.5$.
- That gives us $P(H|E) = 0.8$.

Test of medicine III

- Let us now assume that we have the guess $P(H) = 0.1$.
- That gives us $P(H|E) = 0.36$.
- In both cases we find that $P(H|E) > P(H)$.
- We can use this relation to define strengthening.

Definition of strengthening

- We have a hypothesis H and an observation E .
- We say that E strengthens H if $P(H|E) > P(H)$.
- and we say that it weakens H if $P(H|E) < P(H)$.

Other ways of putting it

- We assume that $0 < P(E) < 1$.
- E strengthens H if $P(E|H)/P(E) > 1$, i.e.
 $P(E|H) > P(E)$.
- E weakens H if $P(E|H)/P(E) < 1$, i.e.
 $P(E|H) < P(E)$
- Or we can say it like this:
- E strengthens H if $P(E|H) > P(E| \text{not } H)$.
- E weakens H if $P(E|H) < P(E| \text{not } H)$.