

# Lecture 2

Induction and the HD Method

# Logic and Experiments

- In the beginning science was all about logical reasoning. Scientists/philosophers tried to find theories about nature.
- What is a theory? In logic a theory is a set of axioms and all consequences following from them by deductions.
- The foremost demand on a theory is that it is *consistent*, i.e. that no contradictions follow from it.
- But then we have the demand that the theory should describe reality correctly. We must confront the theory with experiments. (Or must we?)

# The idea of Empiricism

- Logical Positivism aka. Logical Empiricism is a philosophy of science that was particularly influential in the first half of the 20th century.
- One of the principles of LP is demands we must put on a statement S in order for it to be *meaningful*.
- Let S be any statement put in a form that indicates that it should be true or false. It is meaningful if either:
  - it in principle can be proved or disproved using logical methods
  - there are some observations that would confirm or disconfirm the statement
- All other statements are *meaningless*.
- It is now generally thought that this demand is too strong, but it is still a good guiding principle.

# The connection between theories and observations

- We will spend some time on analyzing the positivistic theories and some questions related to them.
- Can we use observations and form a theory from them?
- Can we first form theory and then check it against observations?
- First we shall study the famous induction method.

# Induction

- The basic idea: We make observations and try to see a pattern in them.
- If the observations are many and all agree with the pattern we conjecture that the pattern always applies.
- There are at least two different standardized forms of the method.

# Induction: A basic form

- We make observations of objects which all has property A.
- Let us assume that in all observations the objects also have property B.
- We conclude that all objects with property A also have property B.

# Does induction work?

- Yes, basically. There are however counter-examples.
- The set of observations must be chosen in a sufficiently general way.
- What is the logical basis for induction?
- One motivation for induction is the weak principle Uniformity of Nature (UN), see Okasha ch. 2.

# A critic

**David Hume 1711-1776**



**There is no scientific ground for induction!**

- Induction cannot be proved to be correct using logic.
- Induction cannot be proved using induction (circular reasoning).
- We believe in induction since it seems to work.
- But it cannot be used for scientific proofs.



# A solution?

## Karl Popper 1902-1994



- Popper claims that he has solved the riddle of induction.
- The solution is that we never really use induction!
- We can never verify hypothesis.
- We can only falsify them.

# Can induction generate theories?

- The idea is that we can see patterns and we can generalize them into theories.
- By using the induction principle we can "prove" the theory.
- But can it be done? There are at least three objections.
- The fact (if it is a fact) that we must first have a theory before we can make observations.
- Underdetermination.
- Goodman's paradox.

# Observations depend on theories and expectations

- "We see what we believe".
- Rosenthal's experiment: A group of medicine students was divided in two groups. They were supposed to make an intelligence test on mice. They are each given a set of mice.
- Group A is told that their mice are the most intelligent. Group B didn't get to know anything.
- Group A found that their mice performed better in the test than the mice in the other group.
- But A and B were given mice of the same type!
- It seems as if the expectations in group A influence the result.
- For reasons like this it is recommended that one should perform double blind tests.

# Underdetermination

- To each set of observations there are always different theories that fits the data.
- Perhaps we should chose the simplest theory (Occam's razor). But will that always give the best result.
- Goodman's paradox: Let us say that a thing is *grue* if it either is green and has been observed before Christmas Eve 2013 or has not been observed before Christmas Eve 2013 and is blue.
- Induction seems to tell us that that all emeralds are *grue*. Is that true?

# In spite of this ...

- It seems as if it is impossible not to use induction, at least in everyday situations
- But what should we do in science?
- We will describe a method that is a sort of development of the induction method.

# Some history

We will now study the history of the *first scientific revolution* (as it is often called).

It is the history of how we changed from the *Geocentric* view of the world (Earth in center of the universe) to the *Heliocentric* view (Sun in center of the universe).



# The first revolution



Copernicus

# The Renaissance

During the Renaissance several scientific developments took place.

- The human body and the circulation of the blood
- Copernicus' heliocentric worldview



# The heliocentric worldview



Kepler

# The heliocentric worldview

- Tycho Brahe makes observations. He describes his own worldview: The Earth is at the center of the Universe. The Sun orbits the Earth. The planets orbit the Sun.
- Kepler describes a new heliocentric worldview where the planets move in ellipses.

# The scientific revolution



Galilei



Descartes

# The scientific revolution

- Galileo Galilei makes experiments.
- He discovers a law for the movements of pendulums.
- Bodies with different weights fall equally fast.
- He constructs telescopes. He discovers mountains on the surface of the moon.
- and moons circling around Jupiter.
- and rings around Saturn.
- He becomes convinced by Copernicus' model.
- He gets punished by the church.

# The scientific revolution II

- Descartes: "Cogito, ergo sum" (I think, therefore I am)
- He creates a program for how research should be done.
- He presents a totally mechanistic worldview: Everything can be explained by interactions between physical bodies.
- He invents analytical geometry.

# Newton's mechanics



Newton

# Newton's mechanics

- At the age of 23 Newton formulates three mechanical laws and the law of gravitation.
- He develops the Calculus (Differential-and Integral Calculus).
- The calculus and his mechanics form the cornerstone in the first modern science.
- At the end of the 17th century Newton's mechanics is internationally recognized.
- Newton is perhaps the first really socially esteemed scientist.

# Science established

- The Royal Society is established in England.
- Experiments are performed.
- Research on astronomy, gases and animals. Microscopes are used.
- Newton is at several times in conflict with the other scientists.
- Newton's optics.
- Conflict with Leibniz.



# The two methods of science

- In science we work both with deductions and observations.
- In mathematics it is almost always deductions.
- In physics we work with both methods.
- In social sciences and humanities the situation is more uncertain. But in a way observations must be used.

# Is there a general scientific method?

- Science has at least four different components:
- To set up hypotheses.
- To verify the hypotheses with logic.
- To evaluate the hypotheses by doing observations.
- To do experiments that generate observations.

# Is there a general scientific method?

- A suggestion: It could be the Hypothetico- Deductive Method.
- It is certainly used in physics and chemistry.
- In a specialized sense it is used in mathematics.
- It seems as if it used sometimes in Social Sciences.

# Carl Hempel 1905-1997



# The general method

- A general method for handling observations is the Hypothetico-Deductive Method (The HD Method).
- The HD Method and the way of thinking connected to it is a central theme in scientific thinking.
- But not all researchers agree.
- Physics, astronomy, chemistry and biology seem to be the most natural areas for the method.

# How it works

- Let us assume that we have a hypothesis  $H$ . We want to know if it is true or not.
- $H$  can be a single fact or a general law.
- We have different observations  $E_1, E_2, \dots, E_n$ .
- (The observations can be generated by an experiment. They can also exist before  $H$ .)
- Does the observation *confirm* or *disconfirm* the hypothesis  $H$ ?
- The HD Method is a way to find an answer to that question.

# A special case: Induction

- Goodman's problem: What hypothesis is supported by the induction?
- We first decide which hypothesis we want to test. (Goodman's problem doesn't occur.).
- A common form: H says that "All objects of type has property B".
- The observations are of the type: E1 = "Object O1 that is of type A has property B", and so on.

# The HD Method used for falsification

- We have a hypothesis and want to show that it is false.
- We have a set of observations  $E_1, E_2, \dots, E_n$ .
- Assume that there is an observation  $E_i$  such that  $H \Rightarrow \text{not } E_i$ .
- Then  $E_i$  falsifies  $H$ .



# Chemistry



Scheele



Lavoisier

# Chemistry

- Great steps are taken in the 18th century.
- At the beginning of the century almost nothing is known about atoms and chemical elements. There are only two known gases: Air and carbon dioxide.
- Oxygen is discovered. (Scheele/Priestley).
- Hydrogen is discovered (Cavendish). Man It is discovered that water is composed of hydrogen and oxygen.
- Lavoisier disproves the so called phlogiston theory of combustion.

# Chemistry II

- John Dalton discovers the atom.
- Berzelius describes the composition of elements.
- He creates the modern chemical notation for substances.
- Mendeleyev creates the periodic table.

# The Phlogiston Theory

## Antoine Lavoisier



The Phlogiston Theory:  
When an object is burning it  
is phlogiston leaving the  
object.

The Phlogiston Theory was  
falsified by Lavoisier.

# The falsification of The Phlogiston Theory

- Let H be The Phlogiston Theory.
- A consequence of The Phlogiston Theory must be that burning objects get lighter.
- But we can find certain metals that get heavier after burning. Let us call this observation E.
- Since  $H \Rightarrow \text{not } E$ , we have falsified H.

# Supporting hypotheses

- It might not be possible to prove  $H \Rightarrow \text{not } E$  *directly*. We might need a supporting hypothesis  $A$  such that  $H \& A \Rightarrow \text{not } E$ .
- $A$  could be all our *background knowledge*. (Kuhn would call it the paradigm.)
- Eg:  $H$  = "The illness is caused by bacteria".
- $A$  = "Penicillin kills bacteria".
- $E$  = "The illness is not cured by penicillin".

# Ad hoc hypotheses

- Supporting hypotheses should be well established and secure. Sometimes they are not:
- If  $H \Rightarrow \text{not } E$  and  $E$  has been observed, someone might want to *save*  $H$ .
- This can maybe be done by assuming that the implication has the form  $(H \& A \Rightarrow \text{not } E)$ . Then one substitutes  $A1$  for  $A$  and get  $(H \& A1 \Rightarrow E)$ .
- If  $A1$  seems very unlikely, if considered by itself, we call  $A1$  an ad hoc hypothesis.

# Example: The Phlogiston Theory

- Let  $H$  = The Phlogiston Theory.
- $E$  was the observation of a metal getting heavier after burning.
- We can argue that the implication is  $H \& A \Rightarrow \text{not } E$ , where  $A$  is "The phlogiston has positive weight".
- We can replace  $A$  with  $A1$  = "The phlogiston in the metal has negative weight". Then  $H \& A1 \Rightarrow E$ !
- But how probable is  $A1$ ?



# A more critical example: Uranus and Neptune

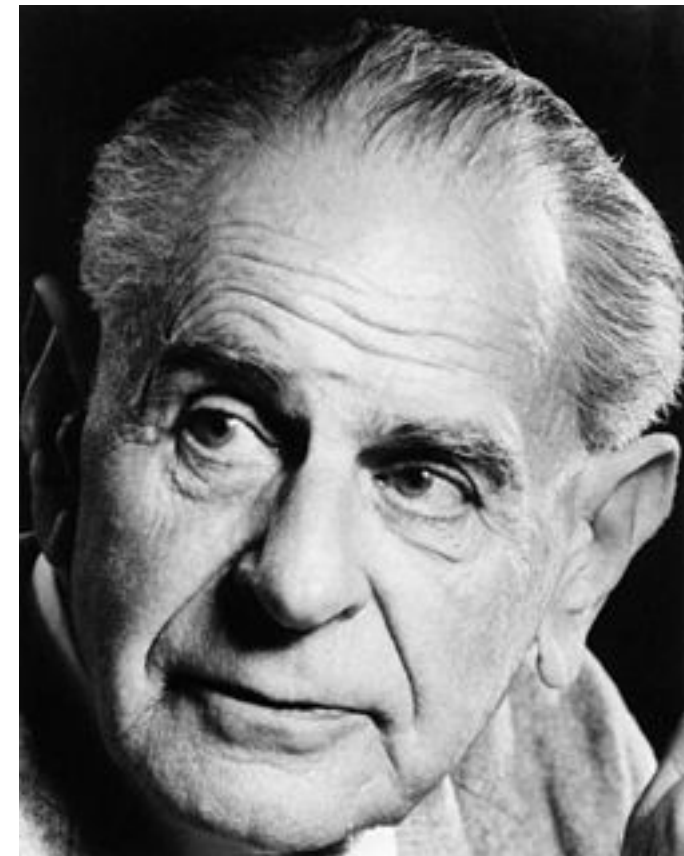
- The planet Uranus was discovered with telescope in 1781.
- In the beginning of the 19th century it was observed that Uranus didn't move in the way Newton's laws predicted.
- Call this observation E and Newton's laws H. Then we have  $H \Rightarrow \text{not } E$ .
- So Newton's laws were falsified!?
- But wait! The implication is really  $H \& A \Rightarrow \text{not } E$  where A, amongst other things contained the statement that there are seven planets.
- But if we replace A with  $A^*$  where  $A^*$  says that there are unknown planets we don't get a falsification.
- and in 1846 Neptune (the eighth planet) was observed!
- So  $A^*$  wasn't really an ad hoc hypothesis (or?).

# The HD Method for falsification. Summary.

- We have a hypothesis and want to test if it is false.
- We use a supporting hypothesis A and deduce  $H \& A \Rightarrow \text{not } E$ .
- We then observe E.
- We have then falsified H.

# This is what Popper believed in

- The HD-Method can be used for falsification
- But in some cases we feel that a theory can be *confirmed* by positive experiments
- Popper denied this but the logical positivists thought so
- A simple example is induction
- Now let's look at a more advanced form of induction



# The HD Method used for verification

- Assume that we have a hypothesis  $H$  and observations  $E_1, E_2, \dots, E_n$ .
- When can we say that the observations confirm  $H$ ?
- One possibility is that  $E_1 \& E_2 \& \dots \& E_n \Rightarrow H$ . In that case  $H$  is verified.
- But let us assume that this is not the case.

# Observations that confirm

- We have  $H$  and  $E_1, E_2, \dots, E_n$ .
- Assume that they are all rather improbable.
- Assume that we have a hypothesis  $A$  that we already believe is true and that  $H \& A \Rightarrow E_1 \& E_2 \& \dots \& E_n$ .
- Then the observations confirm  $H$ .

# Arguments for and against a hypothesis

- Assume that we have observations  $E_1, E_2, \dots, E_n$  and a hypothesis  $H$ .
- Some of the observations confirm  $H$  if they together with a supporting hypothesis  $A_i$  gives  $H \& A_i \Rightarrow E_i$ .
- Other observations disconfirm  $H$  if they together with a supporting hypothesis  $B_k$   $H \& B_k \Rightarrow \text{not } E_k$ . Observe that we don't know if  $B_k$  is true. We have not falsified  $H$  with absolute certainty.

# Making a decision

- We form a type of weighted average. If the supporting hypotheses  $A_i$  are more natural than the  $B_k$  we say that  $H$  is strengthened, otherwise it is weakened.
- This works best if we can use probability theory.

A third form of the HD-Method. To choose between hypotheses.

- If we have a set of observations  $E_1, E_2, \dots, E_n$  and a hypothesis  $H$  we can try to find supporting hypotheses  $A_i$  such that  $H \& A_i \Rightarrow E_i$  for all  $i$ .
- If another hypothesis  $H^*$  can do the same thing with more natural supporting hypotheses  $B_i$  (that is  $H^* \& B_i \Rightarrow E_i$ ), then we say that  $H^*$  is a better hypothesis.



# What is Truth?

- In an obvious way science is about finding truths. But what is truth? There is at least two different types of truth:
- Correspondence Truth.
- Coherence Truth.
- The two types of truth are related to two ways of finding truths:
- Check observations of reality.
- Prove statements with logical methods.

# Bayes' formula

- We want to know what the conditional probability  $P(H|E)$  is.
- Bayes' formula:  
$$P(H|E) = P(E|H)P(H) / ((P(E|H)P(H) + P(E|\text{not } H)P(\text{not } H)))$$
- Alternatively, we can write  $P(H|E) = P(E|H)P(H) / P(E)$
- Which form we use depends on whether we know what  $P(E)$  is or not.

# Example: Test of medicine

- Let us assume that we have a certain medicine that is supposed to cure a disease. Call the hypothesis that the medicine works  $H$ .
- We make an observation. It is that a sick Patient gets well after been given the medicine. Call this observation  $E$ .
- Can we decide to what degree  $E$  confirms  $H$ ?

# Test of medicine II

- We want to find  $P(H|E)$ .
- We need to estimate some probabilities in Bayes' formula.
- $P(E|H) = 1$  seems reasonable.
- $P(E|\text{not } H)$  is more complicated. Let us assume that we have the probability 0.25.
- $P(H)$  is even more complicated. Let us start with the guess  $P(H)=0.5$ .
- That gives us  $P(H|E) = 0.8$ .

# Test of medicine III

- Let us now assume that we have the guess  $P(H) = 0.1$ .
- That gives us  $P(H|E) = 0.36$ .
- In both cases we find that  $P(H|E) > P(H)$ .
- We can use this relation to define strengthening.

# Definition of strengthening

- We have a hypothesis  $H$  and an observation  $E$ .
- We say that  $E$  strengthens  $H$  if  $P(H|E) > P(H)$ .
- and we say that it weakens  $H$  if  $P(H|E) < P(H)$ .

# Other ways of putting it

- We assume that  $0 < P(E) < 1$ .
- E strengthens H if  $P(E|H)/P(E) > 1$ , i.e.  
 $P(E|H) > P(E)$ .
- E weakens H if  $P(E|H)/P(E) < 1$ , i.e.  
 $P(E|H) < P(E)$
- Or we can say it like this:
- E strengthens H if  $P(E|H) > P(E|\text{not } H)$ .
- E weakens H if  $P(E|H) < P(E|\text{not } H)$ .