

# Lecture 3

The HD Method and Bayesianism

# What is Truth?

- In an obvious way science is about finding truths. But what is truth? There is at least two different types of truth:
  - Correspondence Truth.
  - Coherence Truth.
- The two types of truth are related to two ways of finding truths:
  - Check observations of reality.
  - Prove statements with logical methods.

# The general method

- A general method for handling observations is the Hypothetico-Deductive Method (The HD Method).
- The HD Method and the way of thinking connected to it is a central theme in scientific thinking.
- But not all researchers agree.
- Physics, astronomy, chemistry and biology seem to be the most natural areas for the method.

# How it works

- Let us assume that we have a hypothesis  $H$ . We want to know if it is true or not.
- $H$  can be a single fact or a general law.
- We have different observations  $E_1, E_2, \dots, E_n$ .
- (The observations can be generated by an experiment. They can also exist before  $H$ .)
- Does the observation *confirm* or *disconfirm* the hypothesis  $H$ ?
- The HD Method is a way to find an answer to that question.

# A special case: Induction

- A common form: H says that "All objects of type has property B".
- The observations are of the type: E1 = "Object O1 that is of type A has property B", and so on.

# The HD Method used for falsification

- We have a hypothesis and want to show that it is false.
- We have a set of observations  $E_1, E_2, \dots, E_n$ .
- Assume that there is an observation  $E_i$  such that  $H \Rightarrow \text{not } E_i$ .
- Then  $E_i$  falsifies  $H$ .

# The falsification of The Phlogiston Theory

- Let H be The Phlogiston Theory.
- A consequence of The Phlogiston Theory must be that burning objects get lighter.
- But we can find certain metals that get heavier after burning. Let us call this observation E.
- Since  $H \Rightarrow \text{not } E$ , we have falsified H.

# Supporting hypotheses

- It might not be possible to prove  $H \Rightarrow \text{not } E$  *directly*. We might need a supporting hypothesis  $A$  such that  $H \& A \Rightarrow \text{not } E$ .
- $A$  could be all our *background knowledge*.
- Eg:  $H$  = "The illness is caused by bacteria".
- $A$  = "Penicillin kills bacteria".
- $E$  = "The illness is not cured by penicillin".

This is very important! If  $A$  is wrong then the argument does not work.



# Ad hoc hypotheses

- Supporting hypotheses should be well established and secure. Sometimes they are not:
- If  $H \Rightarrow \text{not } E$  and  $E$  has been observed, someone might want to *save*  $H$ .
- This can maybe be done by assuming that the implication has the form  $(H \& A \Rightarrow \text{not } E)$ . Then one substitutes  $A1$  for  $A$  and get  $(H \& A1 \Rightarrow E)$ .
- If  $A1$  seems very unlikely, if considered by itself, we call  $A1$  an ad hoc hypothesis.

# Example: The Phlogiston Theory

- Let  $H$  = The Phlogiston Theory.
- $E$  was the observation of a metal getting heavier after burning.
- We can argue that the implication is  $H \& A \Rightarrow \text{not } E$ , where  $A$  is "The phlogiston has positive weight".
- We can replace  $A$  with  $A1$  = "The phlogiston in the metal has negative weight". Then  $H \& A1 \Rightarrow E$ !
- But how probable is  $A1$ ?

# A more critical example: Uranus and Neptune

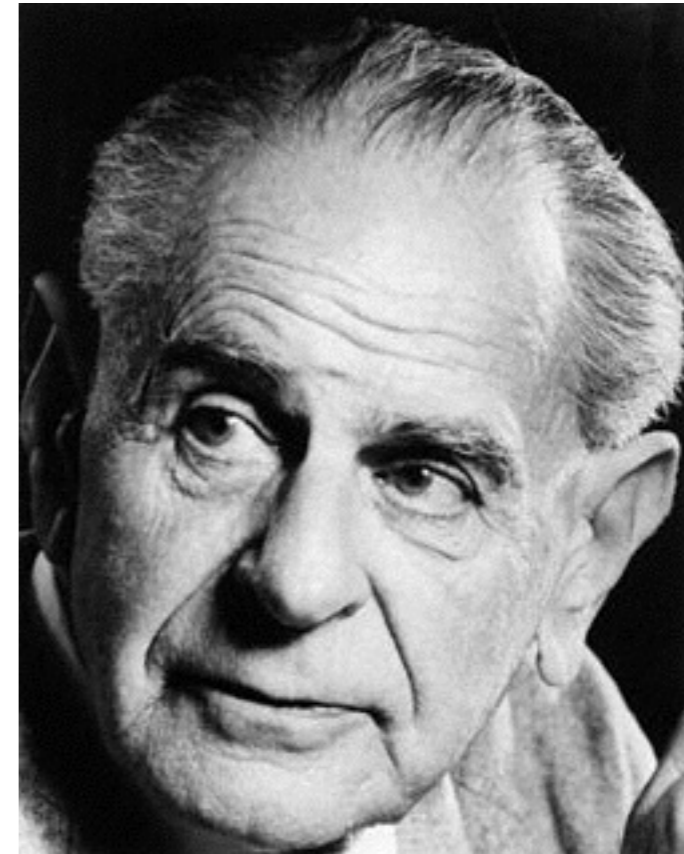
- The planet Uranus was discovered with telescope in 1781.
- In the beginning of the 19th century it was observed that Uranus didn't move in the way Newton's laws predicted.
- Call this observation E and Newton's laws H. Then we have  $H \Rightarrow \text{not } E$ .
- So Newton's laws were falsified!?
- But wait! The implication is really  $H \& A \Rightarrow \text{not } E$  where A, amongst other things contained the statement that there are seven planets.
- But if we replace A with A\* where A\* says that there are unknown planets we don't get a falsification.
- and in 1846 Neptune (the eighth planet) was observed!
- So A\* wasn't really an ad hoc hypothesis (or?).

# The HD Method for falsification. Summary.

- We have a hypothesis and want to test if it is false.
- We use a supporting hypothesis A and deduce  $H \& A \Rightarrow \text{not } E$ .
- We then observe E.
- We have then falsified H.

# This is what Popper believed in

- The HD-Method can be used for falsification
- But in some cases we feel that a theory can be *confirmed* by positive experiments
- Popper denied this but the logical positivists thought so
- A simple example is induction
- Now let's look at a more advanced form of induction



# The HD Method used for verification

- Assume that we have a hypothesis  $H$  and observations  $E_1, E_2, \dots, E_n$ .
- When can we say that the observations confirm  $H$ ?
- One possibility is that  $E_1 \& E_2 \& \dots \& E_n \Rightarrow H$ . In that case  $H$  is verified.
- But let us assume that this is not the case.

# Observations that confirm

- We have  $H$  and  $E_1, E_2, \dots, E_n$ .
- Assume that they are all rather improbable.
- Assume that we have a hypothesis  $A$  that we already believe is true and that  $H \& A \Rightarrow E_1 \& E_2 \& \dots \& E_n$ .
- Then the observations confirm  $H$ .

# Arguments for and against a hypothesis

- Assume that we have observations  $E_1, E_2, \dots, E_n$  and a hypothesis  $H$ .
- Some of the observations confirm  $H$  if they together with a supporting hypothesis  $A_i$  gives  $H \& A_i \Rightarrow E_i$ .
- Other observations disconfirm  $H$  if they together with a supporting hypothesis  $B_k$   $H \& B_k \Rightarrow \text{not } E_k$ . Observe that we don't know if  $B_k$  is true. We have not falsified  $H$  with absolute certainty.



# Making a decision

- We form a type of weighted average. If the supporting hypotheses  $A_i$  are more natural than the  $B_k$  we say that  $H$  is strengthened, otherwise it is weakened.
- This works best if we can use probability theory.

A third form of the HD-Method. To choose between hypotheses.

- If we have a set of observations  $E_1, E_2, \dots, E_n$  and a hypothesis  $H$  we can try to find supporting hypotheses  $A_i$  such that  $H \& A_i \Rightarrow E_i$  for all  $i$ .
- If another hypothesis  $H^*$  can do the same thing with more natural supporting hypotheses  $B_i$  (that is  $H^* \& B_i \Rightarrow E_i$ ), then we say that  $H^*$  is a better hypothesis.

# We use probability

- The previous methods were qualitative.
- We now try to do a probabilistic analysis of when observations confirm a hypothesis.
- So we have this problem: Given a hypothesis  $H$  and an observation  $E$ , when can we say that the observation confirms  $H$ ?

# An important formula

**Thomas Bayes 1702-1761**



He found an important formula connecting different types of conditional probabilities.

This formula is the basis for so called Bayesian Statistics.

# Bayes' formula

- We want to know what the conditional probability  $P(H|E)$  is.
- Bayes' formula:  
$$P(H|E) = P(E|H)P(H) / ((P(E|H)P(H) + P(E|\text{not } H)P(\text{not } H))$$
- Alternatively, we can write  $P(H|E) = P(E|H)P(H) / P(E)$
- Which form we use depends on whether we know what  $P(E)$  is or not.

# Example: Test of medicine

- Let us assume that we have a certain medicine that is supposed to cure a disease. Call the hypothesis that the medicine works  $H$ .
- We make an observation. It is that a sick Patient gets well after been given the medicine. Call this observation  $E$ .
- Can we decide to what degree  $E$  confirms  $H$ ?

# Test of medicine II

- We want to find  $P(H|E)$ .
- We need to estimate some probabilities in Bayes' formula.
- $P(E|H) = 1$  seems reasonable.
- $P(E|\text{not } H)$  is more complicated. Let us assume that we have the probability 0.25.
- $P(H)$  is even more complicated. Let us start with the guess  $P(H)=0.5$ .
- That gives us  $P(H|E) = 0.8$ .

# Test of medicine III

- Let us now assume that we have the guess  $P(H) = 0.1$ .
- That gives us  $P(H|E) = 0.36$ .
- In both cases we find that  $P(H|E) > P(H)$ .
- We can use this relation to define strengthening.



# Definition of strengthening

- We have a hypothesis  $H$  and an observation  $E$ .
- We say that  $E$  strengthens  $H$  if  $P(H|E) > P(H)$ .
- and we say that it weakens  $H$  if  $P(H|E) < P(H)$ .

# Other ways of putting it

- We assume that  $0 < P(E) < 1$ .
- E strengthens H if  $P(E|H)/P(E) > 1$ , i.e.  
 $P(E|H) > P(E)$ .
- E weakens H if  $P(E|H)/P(E) < 1$ , i.e.  
 $P(E|H) < P(E)$
- Or we can say it like this:
- E strengthens H if  $P(E|H) > P(E| \text{not } H)$ .
- E weakens H if  $P(E|H) < P(E| \text{not } H)$ .

# Different views of probability

There are three different ways in which probability can be interpreted.

- *Axiomatic*: We postulate a set of equally probable *elementary events*. Every other events is expressed as a combination of these events.
- *Frequency*: The probability for an event is roughly the frequency with which the event will occur in repeated experiments.
- *Subjective*: We give a measure for the "probability" of events without giving a formal basis for this measure.

It seems as if the Bayesian view of verification relies on an extensive use of subjective probability. This is a problem since subjective probability is not universally accepted as a stringent scientific concept.

# Geology and evolution

- Charles Lyell is considered the father of modern geology.
- He presents the theory of *uniformism* that says that the Earth has developed during a very long time by slow processes which are still at work today.
- Charles Darwin makes his famous journey with *Beagle* during the years 1831-1836.
- In 1859 he publishes *On the Origin of Species*.

# Details

- During his trip Darwin becomes convinced the the species have developed.
- Other thinkers, for instance Lamarck, had already come to the same conclusion.
- Darwin found an explanation how and why they had evolved.
- *Natural Selection!*
- But objections where not late to arrive: For instance, a process governed by natural selection would take to much time.
- The discussion continues ...