Lecture 5

Scientific Methodology

Today: Overview

- Some general principles
- Deductive systems
- Some techniques for analyzing data

What is a scientific approach?

This question can be answered in a lot of different ways. We will try to do it by describing three somewhat different areas where we use science.

- Scientific attitude in every-day situations.
- Scientific methods in smaller research projects.
- Science in big scientific theories.

The scientific attitude

We can characterize the scientific method by the attitudes of scientists. According to Merton the following should be the attitudes. It is five principles gathered under the acronym CUDOS:

- Communalism – knowledge should be accessible for all people.
- Universalism – everyone should have the right to contribute.
- Disinterestedness – science should be objective and not ruled by special interests.
- Originality – the results should be new.
- Skepticism – scientists should be open to criticism.

Science in every-day situations

What does it mean to have a scientific attitude to things? Some suggestions:

- You are objective. Especially, you base your judgements on observations and verified facts.
- You realize to what extent you and everyone else can be biased by your/their perspective.
- You are curious and want to know facts.
- You have some knowledge of scientific methodology and try to apply it.

What scientific methodology?

Here are some scientific methods that also can be used in "simpler" situations:

- The HD-method for finding hypothesis. Use the formula H & A => E. (Lecture 2)
- Maximum Likelihood. Try to find H such that P(E l H) is maximal. (Lecture 5)
- If you are more advanced: Use Baye's formula for computing P(H l E). (Lecture 3)
- Realize that is A and B are correlated it doesn't have to mean that A is the cause of B. It can be the other way around, or neither. (Lecture 4)
- Use deduction. (Lecture 5)

Science in research projects

We identify three types of research projects:

- Exploratory research
- Testing-out research
- Problem-solving research

Exploratory research

- This is research on a new problem about which little is known.

- The problem may come from any part of the discipline; it may be a theoretical research puzzle or have an empirical basis.

- The research work will need to examine what theories and concepts are appropriate, developing new ones if necessary, and whether existing methodologies can be used.

- It obviously involves pushing out the frontiers of knowledge in the hope that something useful will be discovered.

Testing-out research

- In this type of research we are trying to find the limits of a previously proposed generalization.
- This is often termed the ‘null hypothesis', which we are bringing evidence to ‘overthrow’ - i.e. to show is inadequate.
- We can try to answer questions like: Does the theory apply at high temperatures? In new technology industries? With working-class parents? Before universal franchise was introduced?
- In this way we are able to make an original contribution and improve (by specifying, modifying, clarifying) the important generalizations in our discipline.

Problem-solving research

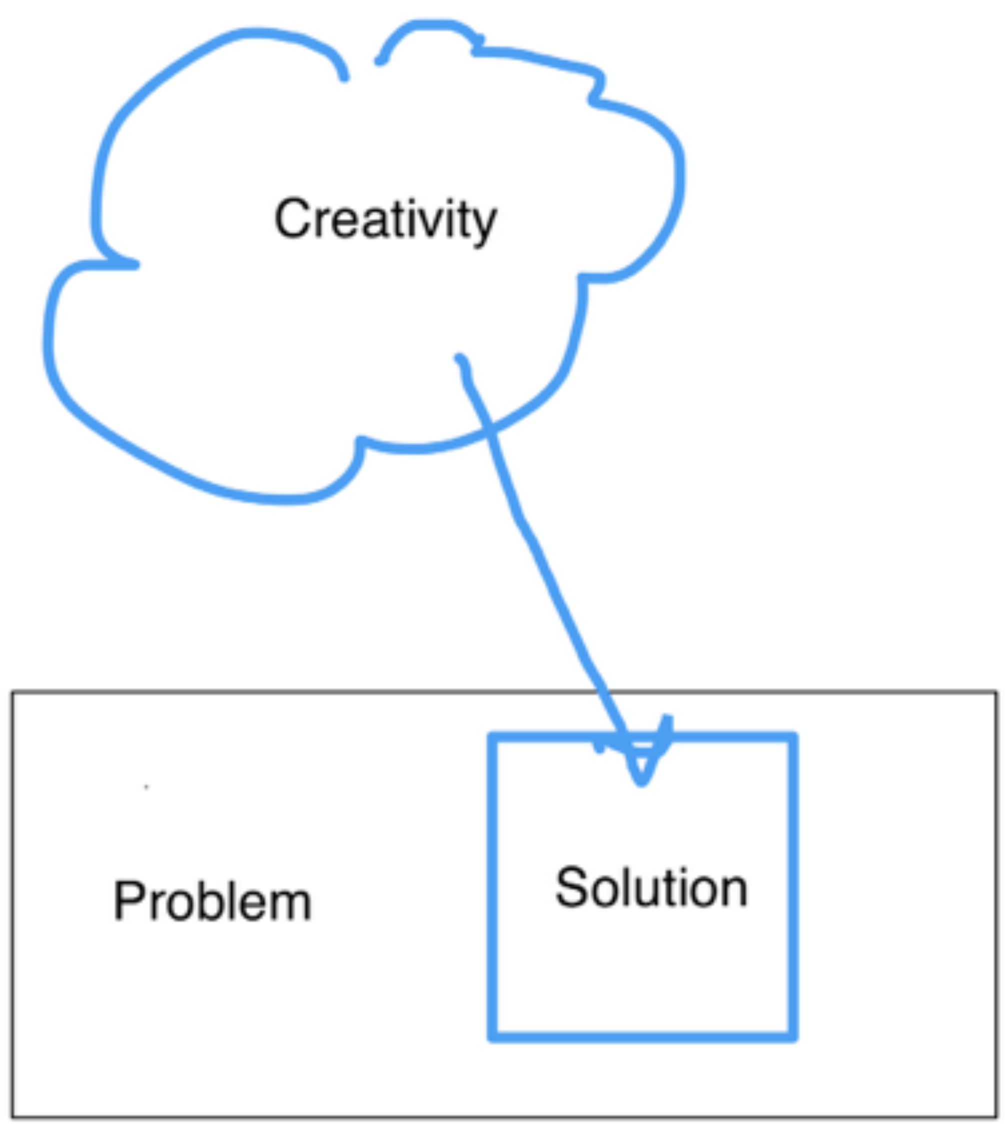
- In this type of research, we start from a particular problem in the real world, and bring together all the intellectual resources that can be brought to bear on its solution.

- The problem has to be defined and the method of solution has to be discovered.

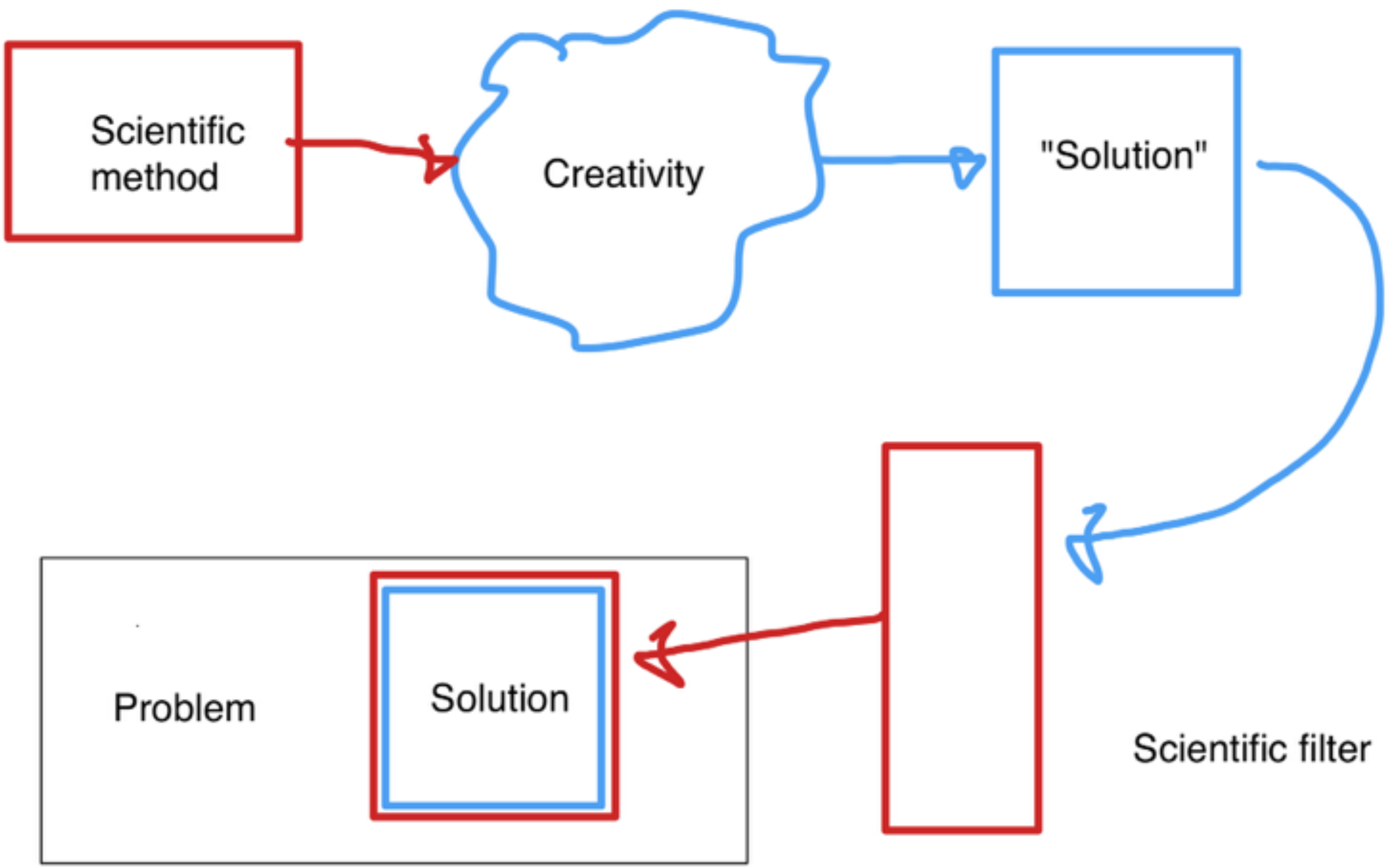
- The person working in this way may have to create and identify original problem solutions every step of the way. This will usually involve a variety of theories and methods, often ranging across more than one discipline since real-world problems are likely to be ‘messy’ and not soluble within the narrow confines of an academic discipline.

Science in an engineering project

The ordinary engineering process



The process with science "added"



What is the scientific filter?

1. We must put our solution in a broader scientific context. We must give references to other solutions and similar problems.
2. We must prove scientifically that our solution is *correct.*
3. We must put our solution in form of a report following scientific standards.

General questions

- What do you want to do? What is your project?
- Why do you want to do it? Is it important? Is it interesting?
- How do you plan to do it? Which methods will you use?
- When do you plan to do it? How long time will it take?

The subject

- It should be clearly stated.
- It should be significant. For instance, it should not be just a repetition of something already done.
- It should have clearly stated boundaries.
- It should be such that relevant data can be obtained.
- It should be such that significant conclusions can be drawn.

The form of the project

- It can be in form of a question, for instance, is functional programming better than imperative programming?
- It can be in the form of an hypothesis, for instance, functional programming is better than imperative programming.

The importance of being right

A famous mathematician once said that the most important thing is being right. You must have the talent for choosing hypotheses that are correct. You must have a sound intuition!

Scientific method in project work

We can characterize the project work by dividing it into four phases:

- Preparing Analysis
- Finding hypotheses
- Synthesis of partial results
- Validation of results

Analysis

The goal is to get an understanding of the problem/project. This understanding can involve the following steps:

- Describe the problem.
- Decide on a measure of success.
- Do studies on similar problems.
- Define goals.

Hypothesis

Here we have to be creative and try to find hypotheses and possible solutions to problems. This includes:

- State the hypothesis/solution clearly.
- Find consequences of the hyptothesis/solution.
- Find criteria for judging if the hypothesis/solution is true/works.

Synthesis

Here we test the hypothesis or implement and test the solution:

- If we have a solution to a problem we implement the solution.
- Do experiments for testing if the consequences of the hypothesis are true or if the solution works.
- Analyze the results.

Validation

Here we evaluate the hypothesis/solution and the results of the experiments:

- Try to measure how well the experiments confirm/falsify the hypothesis or how well the solution works.
- Try to decide if the hypothesis is true or if the solution works.
- Do documentation by writing a rapport or scientific paper.
- Submit your results for criticism from colleagues or independent referees.

Deductive systems

Deductive systems

The previous lectures have mainly been about the use of observations in science. This lecture will be concerned with the deductive side of science. The lecture is in two parts:

- A general discussion about formal systems.
- Paradoxes and impossibility theorems.

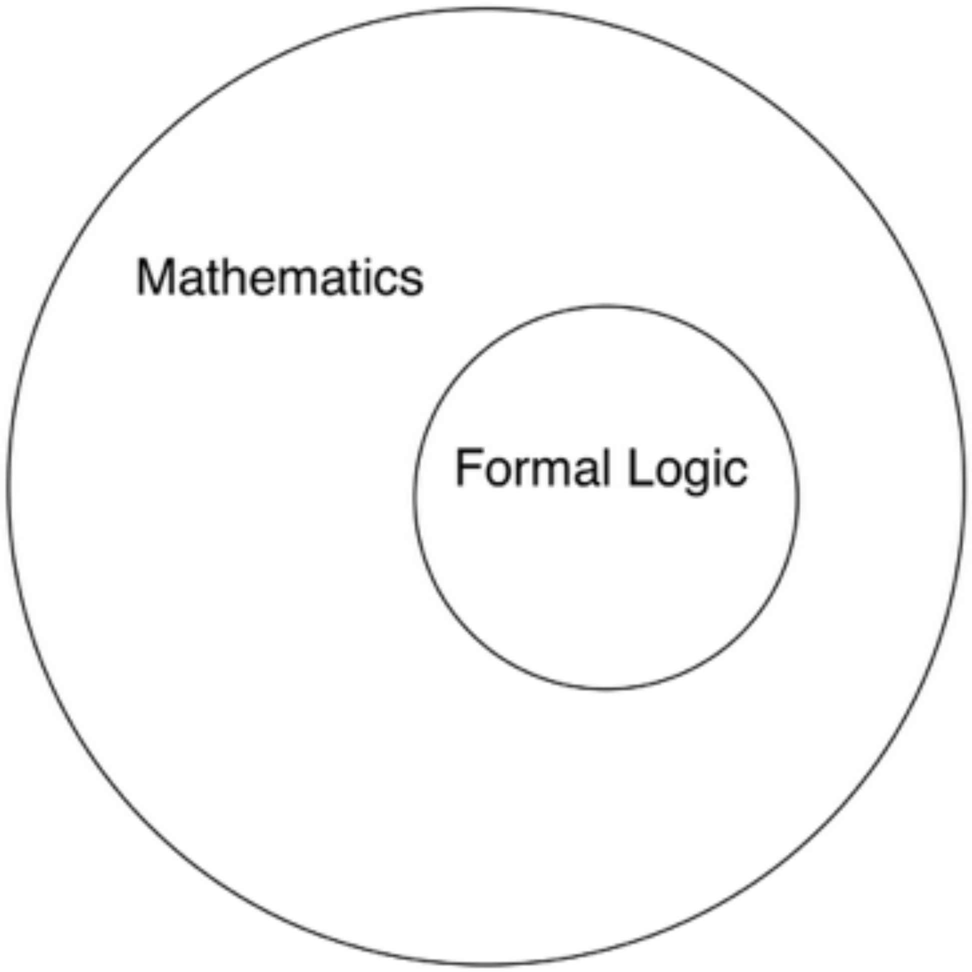
Users of formal systems

- Mathematicians - use them to prove mathematical theorems.
- Computer scientists - use them to design algorithms that solve problems.
- Philosophers - use them to define and analyze things.

Mathematics and Formal Logic

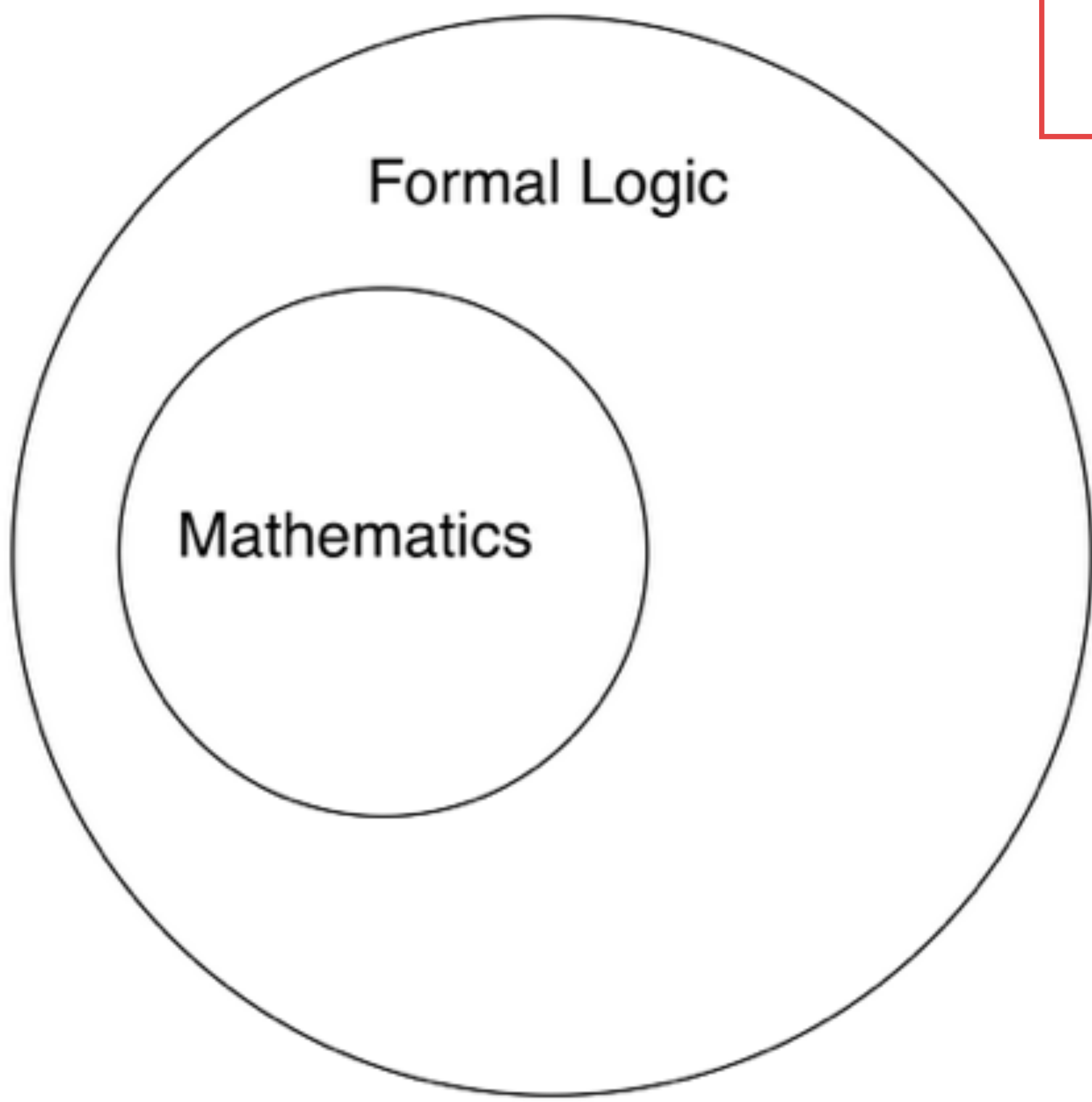
What is the connection between Mathematics and Formal Logic? Here are some suggestions:

Formal Logic is a part of Mathematics



This would probably be what mathematicians think

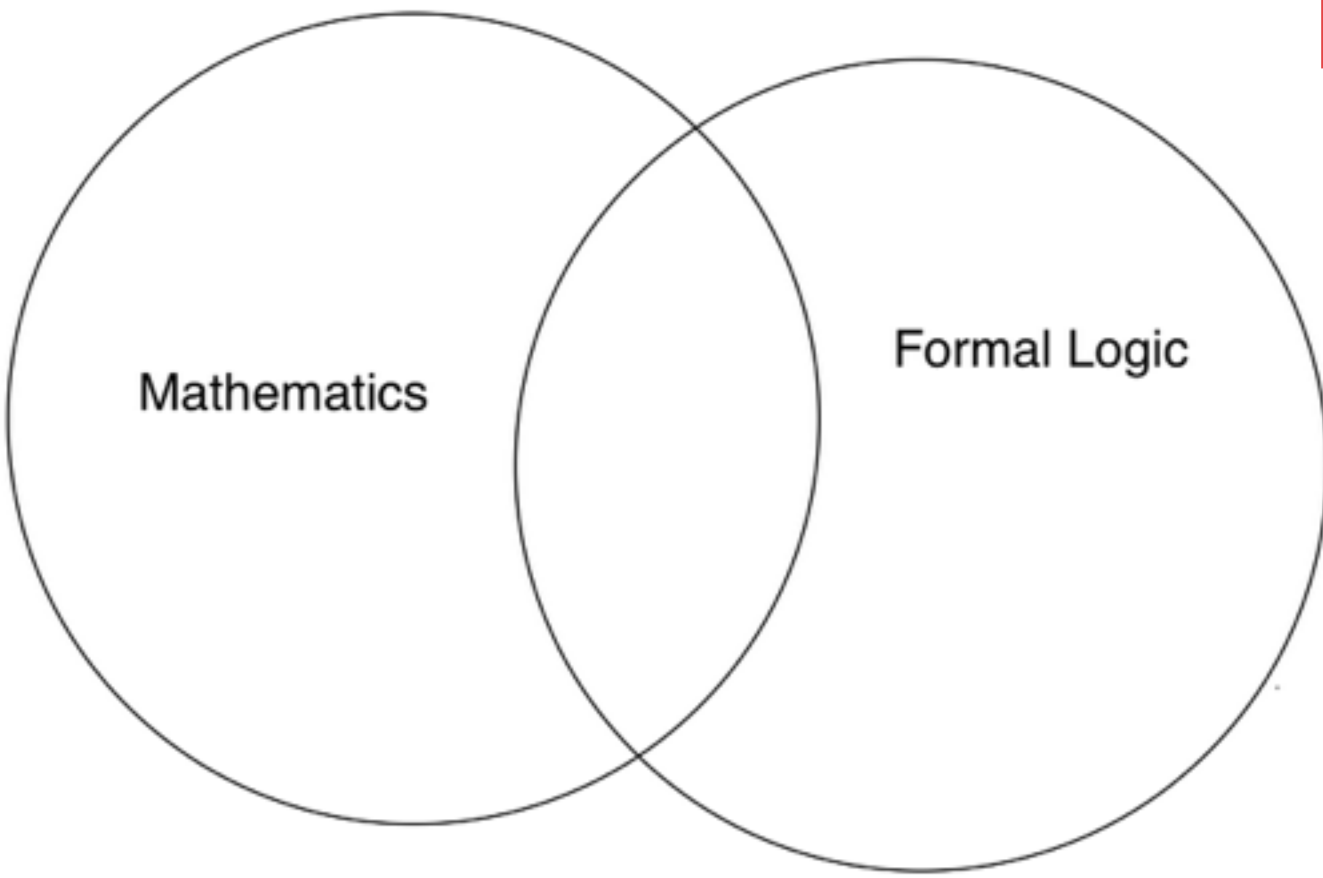
Mathematics is a part of Formal Logic



This is what the pioneers in Formal Logic thought

Neither is a part of the other

Nowadays, this seems to be natural view



Three components of a deductive system

- Vocabulary
- Deduction Rules
- Axioms

Vocabulary

We will look at some text from different disciplines all using formal syntax. It is normally rather easy to recognize the discipline.

Mathematics

Corollary 10.82 (**Künneth Formula for Homology, II**). *Let R be a right hereditary ring, let $(\mathbf{A}, \delta)$ be a complex of projective right R-modules, and let* $\mathbf{C}$ *be a complex of left R-modules.*

(i) *For all $n \geq 0$, there is an exact sequence*

$$\bigoplus_{p+q=n} H_p(\mathbf{A}) \otimes_R H_q(\mathbf{C}) \xrightarrow{\alpha} H_n(\mathbf{A} \otimes \mathbf{C}) \xrightarrow{\beta} \bigoplus_{p+q=n-1} \mathrm{Tor}_1^R(H_p(\mathbf{A}), H_q(\mathbf{C})),$$

where $\alpha_n \colon \sum_p \mathrm{cls}(b_p) \otimes c_{n-p} \mapsto \sum_p \mathrm{cls}(b_p \otimes c_{n-p})$, *and both λ_n and μ_n are natural.*

(ii) *For all $n \geq 0$, the exact sequence splits:*[8]

$$H_n(\mathbf{A} \otimes_R \mathbf{C}) \cong \Big[\bigoplus_{p+q=n} H_p(\mathbf{A}) \otimes_R H_q(\mathbf{C}) \Big] \oplus \bigoplus_{p+q=n-1} \mathrm{Tor}_1^R(H_p(\mathbf{A}), H_q(\mathbf{C})).$$

Theoretical physics

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t)$$

i is the imaginary number, $\sqrt{-1}$.

$\hbar$ is Planck's constant divided by 2π: 1.05459×10^{-34} joule·second.

$\psi(\mathbf{r},t)$ is the wave function, defined over space and time.

m is the mass of the particle.

∇^2 is the Laplacian operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Formal Logic

Ga	*1, 2, DM*
(~Fa → Ga)	*2-3, CD*
~Ga	***Hypothesis***
Fa	*1, 5, DM*
(~Ga → Fa)	*5-6, CD*
((~Fa → Ga) ^ (~Ga → Fa))	*4, 7, Conj*
((Fa v Ga) → ((~Fa → Ga) ^ (~Ga → Fa)))	*1-8, CD*
((~Fa → Ga) ^ (~Ga → Fa))	***Hypothesis***
~(Fa v Ga)	***Hypothesis***
~Fa	*11, DM*
(~Fa → Ga)	*10, Simp*
Ga	*12, 13, MP*
~Ga	*11, DM*

Computer Science

$$\overline{\ \ \rightsquigarrow a} \quad \textsc{EtaR}$$

$$\overline{x.(x \bullet k) \rightsquigarrow k} \quad \textsc{EtaL}$$

$$\overline{\ \ \rightsquigarrow [k/\alpha]s} \quad \textsc{BetaR}$$

$$\overline{a \bullet x.(s) \rightsquigarrow [a/x]s} \quad \textsc{BetaL}$$

$$\overline{\mathrm{not}[a] \rightsquigarrow a \bullet k} \quad \textsc{BetaNeg}$$

$$\overline{inl\ a \bullet [k, l] \rightsquigarrow a \bullet k} \quad \textsc{BetaCoP}$$

$$\overline{\ \ , l] \rightsquigarrow a \bullet l} \quad \textsc{BetaCoProd2}$$

$$\overline{<a, b> \bullet \mathrm{fst}\ k \rightsquigarrow a \bullet k} \quad \textsc{BetaPr}$$

$$\overline{\bullet \mathrm{snd}\ k \rightsquigarrow b \bullet k} \quad \textsc{BetaProd2}$$

Chemistry

$$2\mathrm{NaX} + \mathrm{Ca}^{2+} \leftrightarrow \mathrm{CaX}_2 + 2\mathrm{Na}^+ \quad \log K_{\mathrm{NaCa}} = -0.20$$

$$K_{\mathrm{NaCa}} = \frac{\beta_{\mathrm{Ca}}[\mathrm{Na}^+]^2}{\beta_{\mathrm{Na}}^2 \gamma_0^2 [\mathrm{Ca}^+]} = 0.62$$

$$\mathrm{NaCl}_{(aq)} \leftrightarrow \mathrm{Na}^+ + \mathrm{Cl}^- \quad \log K_{\mathrm{NaCl}} = 0.78$$

$$K_{\mathrm{NaCl}} = \frac{\gamma_0^2 [\mathrm{Na}^+][\mathrm{Cl}^-]}{[\mathrm{NaCl}_{(aq)}]} = 6.0$$

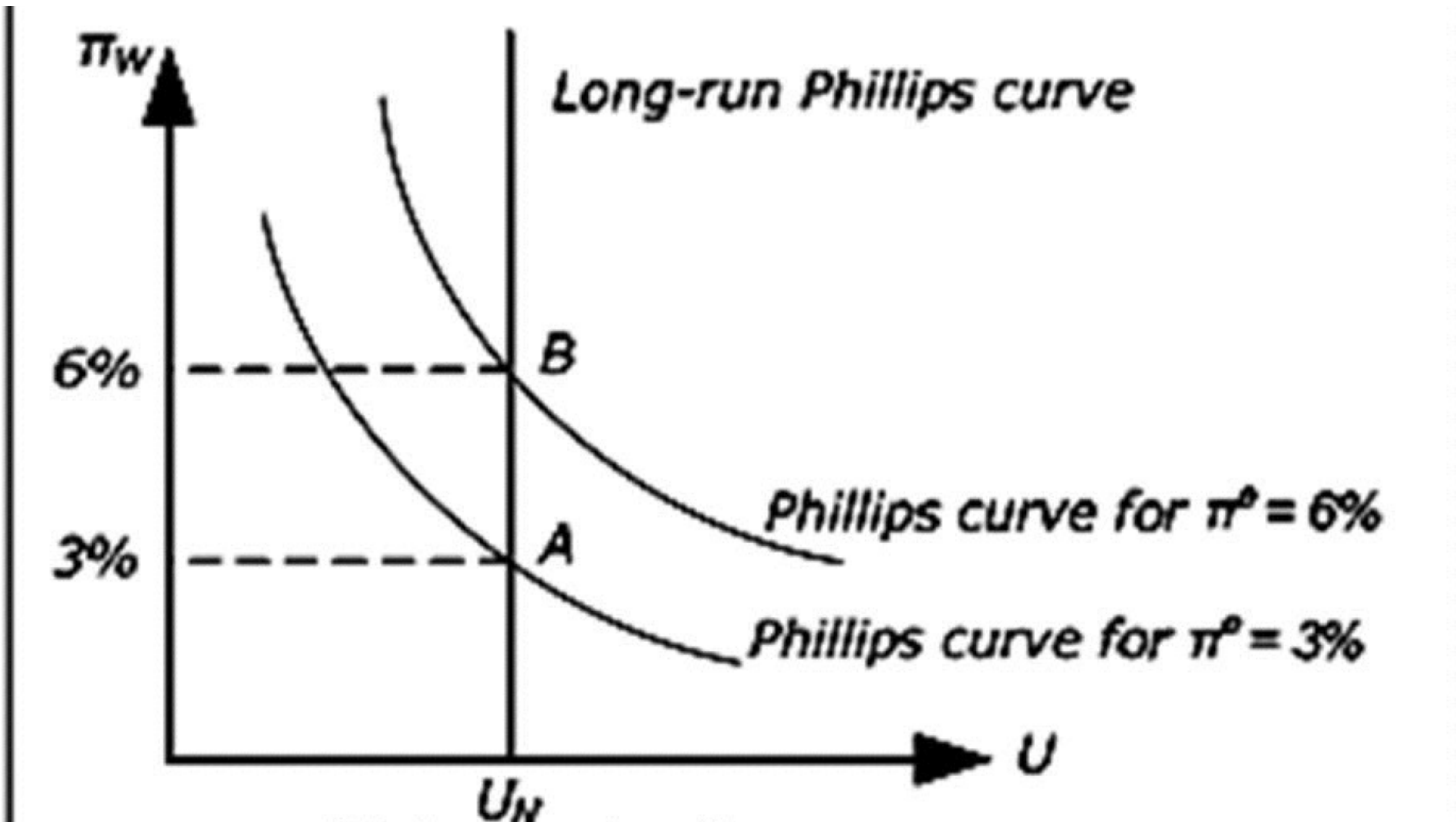
$$\mathrm{CaCl}^+ \leftrightarrow \mathrm{Ca}^{2+} + \mathrm{Cl}^- \quad \log K_{\mathrm{CaCl}} = 0.79$$

$$K_{\mathrm{CaCl}} = \frac{\gamma_0^4 [\mathrm{Ca}^{2+}][\mathrm{Cl}^-]}{[\mathrm{CaCl}^+]} = 5.0$$

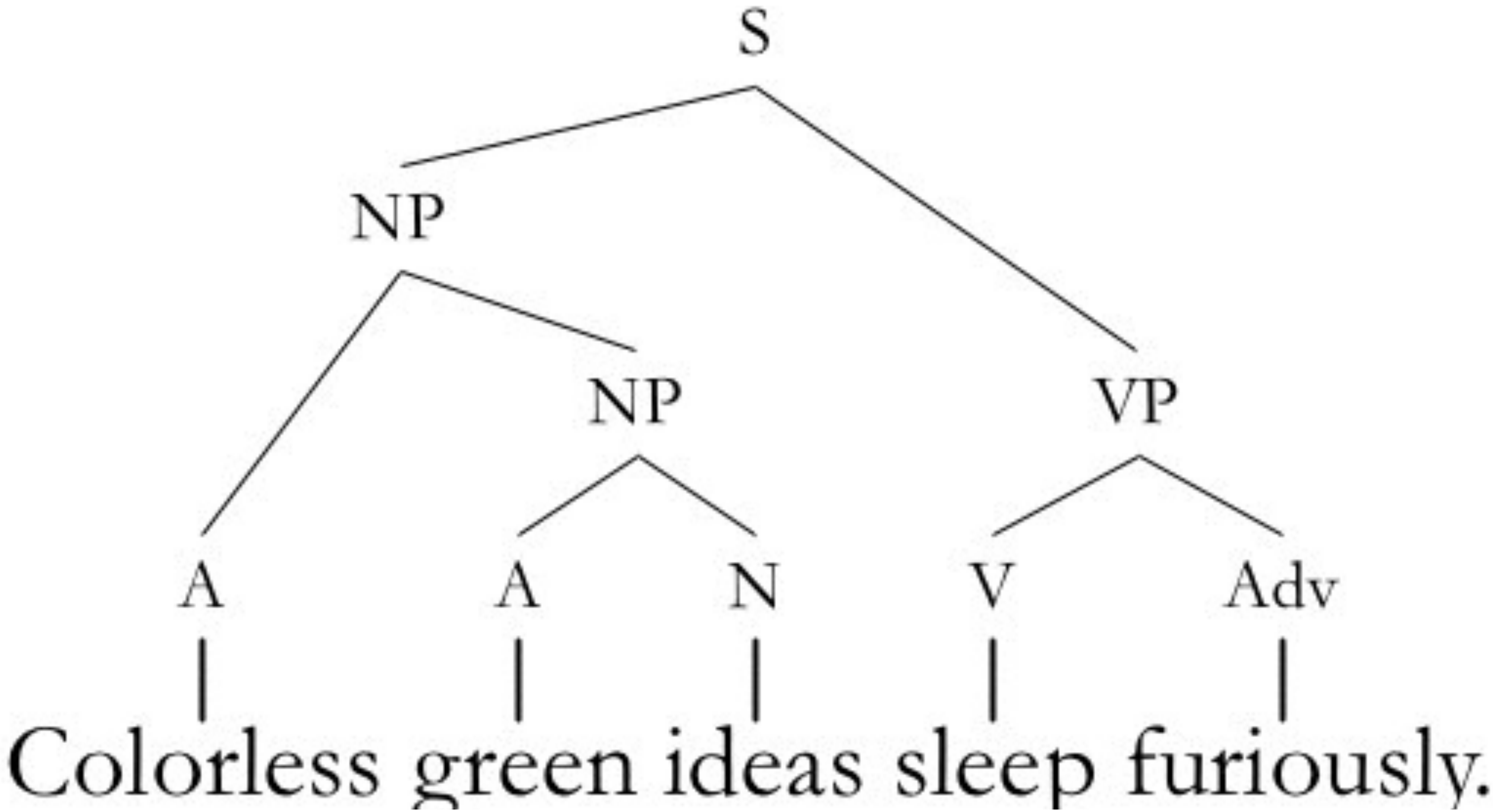
$$c_{\mathrm{Na}} = [\mathrm{Na}^+] + [\mathrm{NaCl}_{(aq)}] + c_{\mathrm{NaX}}$$

$$c_{\mathrm{Ca}} = [\mathrm{Ca}^+] + [\mathrm{CaCl}^+] + c_{\mathrm{CaX}_2}$$

Mathematical Economics



Linguistics



Vocabularies

- In a deductive system the vocabulary is roughly the syntax of the language we use in the system.
- Less formally, we can say that the vocabulary defines the type of expressions you can expect to find in the system.
- For instance, in text on evolutionary theory you would expect to find words like *natural selection* and so on.
- In formal logic the vocabulary is defined in a very precise way.

Deduction rules

- All deduction systems have some set of formal and informal rules which tells us what conclusions we can prove from other statements.
- In physics the rules are somewhat informal and established by praxis.
- In formal logic the deduction rules are where precisely defined.
- In mathematics it can happen that the deduction rules are implicitly understood. They can, however, be exactly stated (one would hope?)

Axioms

- The main idea is that the axioms are basic truths (intuitive truths maybe).
- Starting with axioms and using the deduction rules we create theorems.
- The axioms and theorems are the only truths in the system.
- In formal systems we divide the axioms into logical and non-logical axioms.
- In some systems with very strong deduction rules we have no logical axioms at all. Natural deduction is one example.

Do the axioms have to be true?

- The classic idea was that the axioms should be basic and fundamental truths.
- But later mathematicians realized that we could regard the axioms as assumptions and deduce consequences of these assumption.
- And important example of this is Non-Euclidian Geometries, developed in the 19th century.

Methodology?

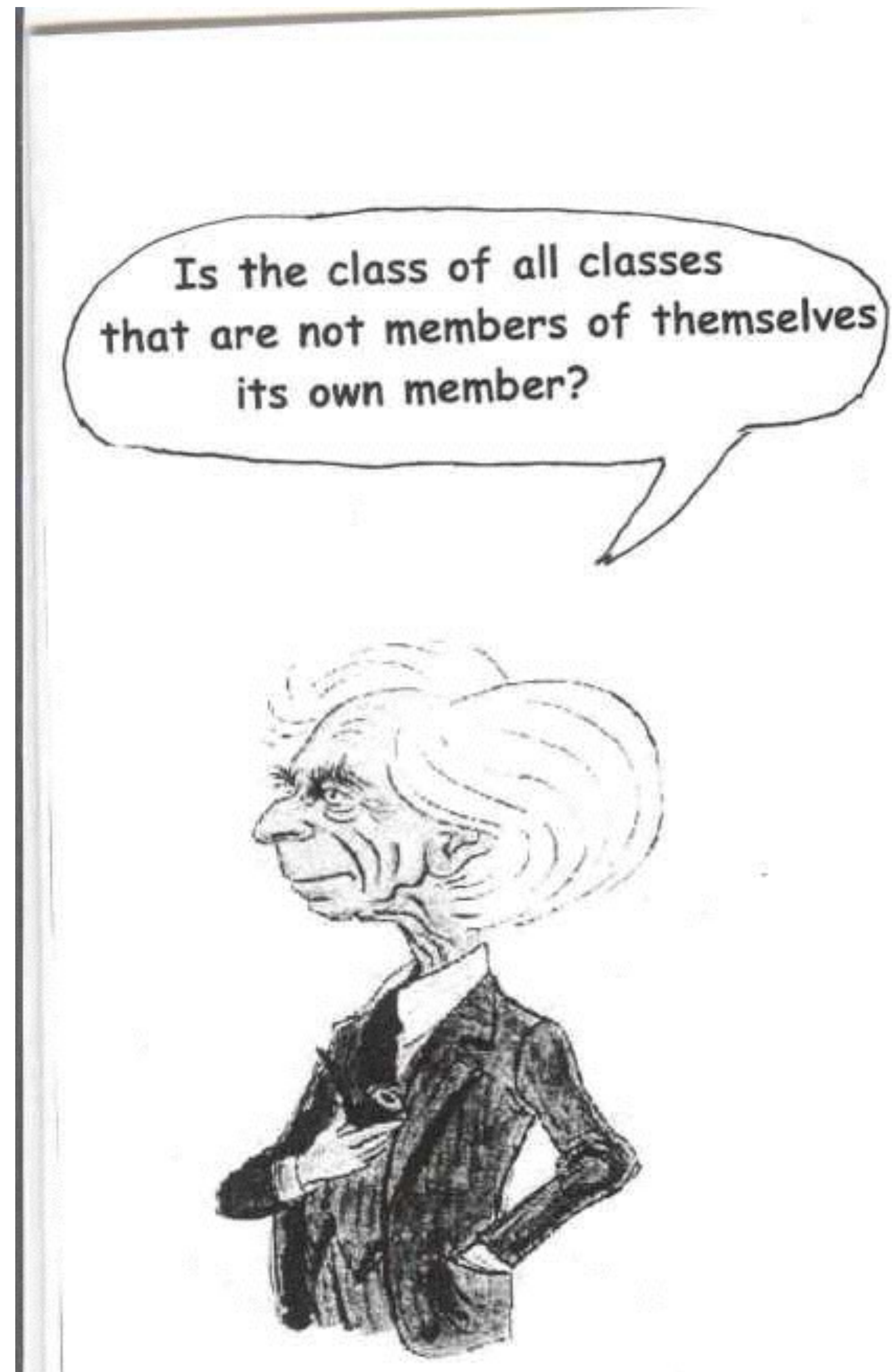
- It seems to be very hard to give prescriptions for how research with deductive methods should be done.
- Its not that hard to learn techniques for checking if proofs are correct. The difficult thing is to find good theorems and theories.
- This is essentially a *creative* activity. And there are no recipes for creativity.
- Or are there? The best way of learning how to find proofs is to imitate existing proofs.
- Some other tricks will be described in a later lecture.

Paradoxes and impossibility theorems

- We will give a brief discussion of some problems and paradoxes related to deductive systems and mathematics.
- We will describe two great crisis in the history of logic and mathematics

Russell's paradox

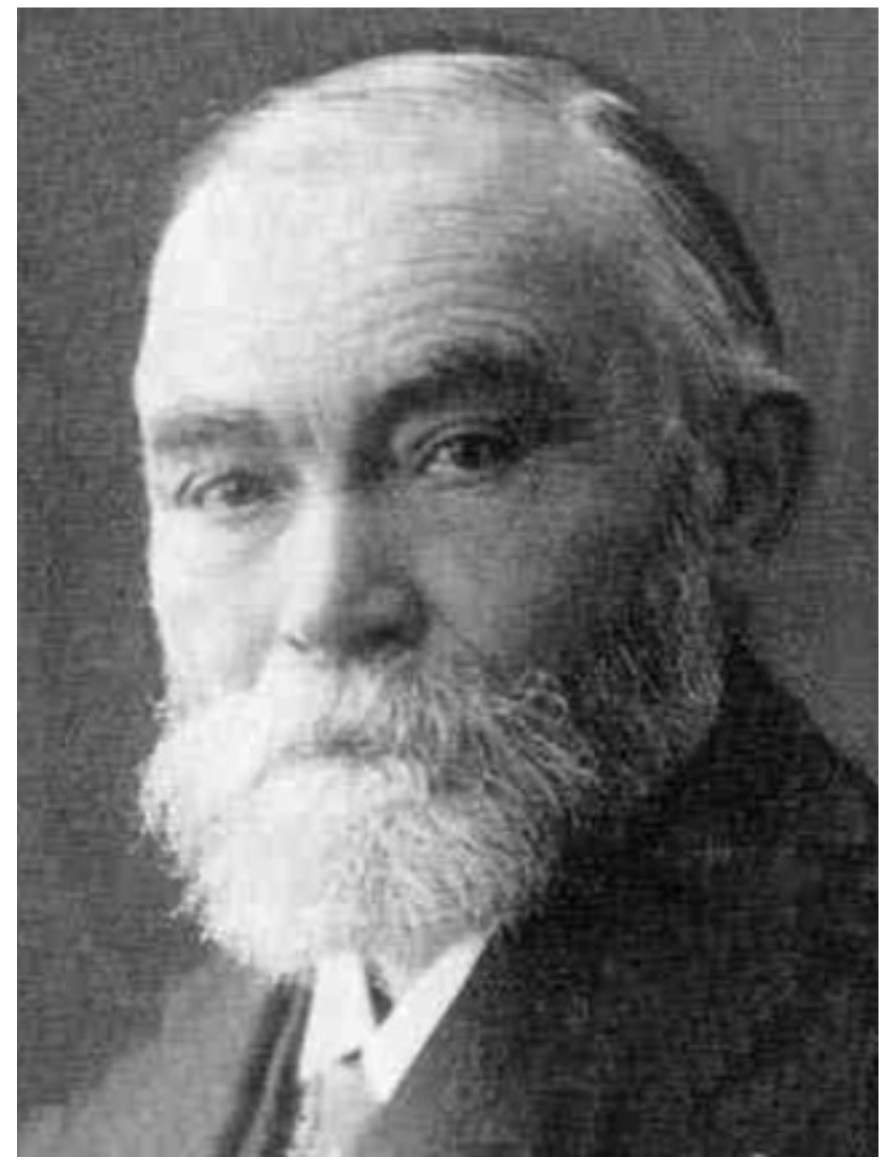
- The first crisis was in the early 20th century.
- We will start with some history.



Frege and mathematical logic

- Gottlob Frege created the modern mathematical logic at the end of the 19th century.
- He tried to construct all mathematics with logic.
- The starting point was a formalized version of set theory.
- Among other things Frege postulated that if P(x) is any predicate there always exists a set of all objects x such that P(x) is true:

$$\{x{:}P(x)\}$$



Bertrand Russell

- In the beginning of the 20th century Russell showed that Frege's axiom leads to contradictions.

- If we define

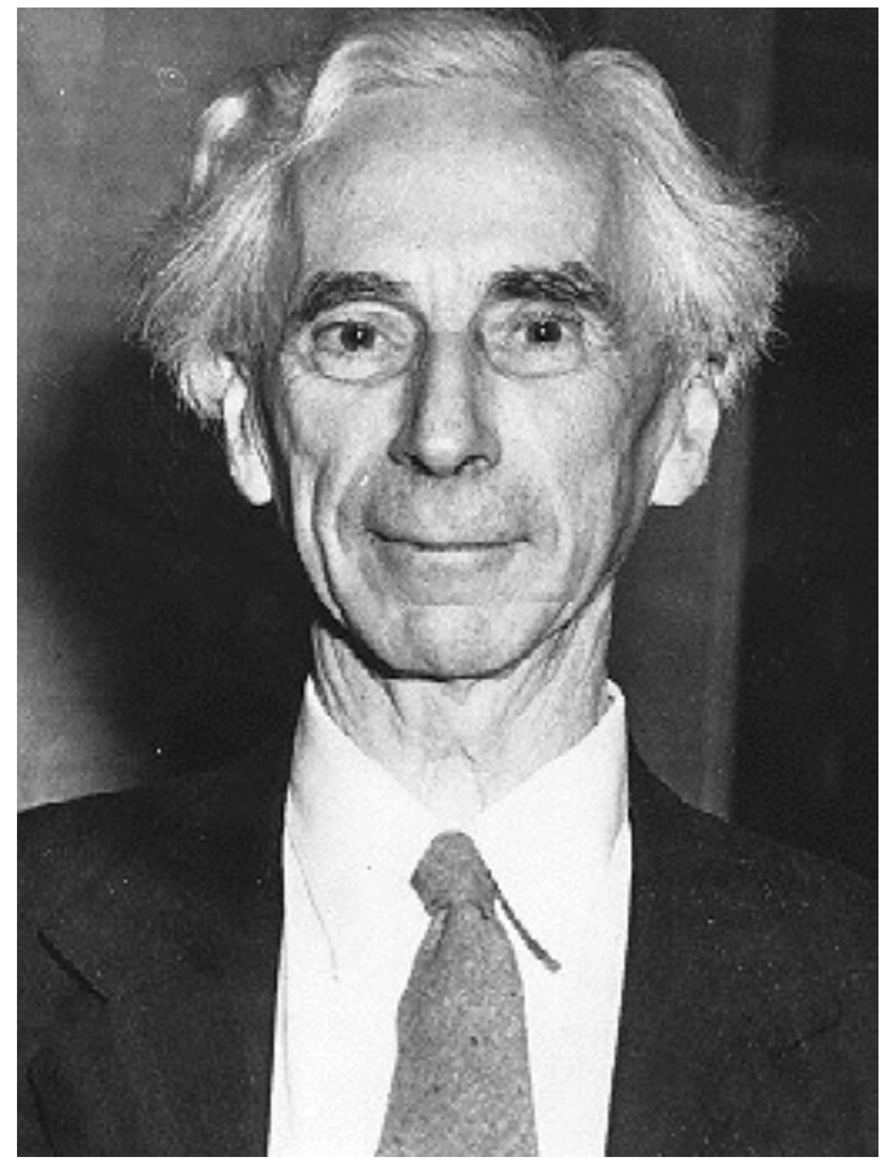
$$P(x): x \notin x$$

And $M = \{x: x \notin x\}$

What happens then? Is $M \in M$

or $M \notin M$

true?



Some related paradoxes

- The liar paradox - 'I am lying'. True or false?
- Grelling paradox - Among English adjectives there are some, such as 'short', 'polysyllable', 'English', which apply to themselves. Let us call such adjectives *autological;* all others are *heterological.* Thus 'long', 'monosyllable', 'green' are heterological. But what about 'heterological'? Is it heterological or not?
- Berry paradox - Consider the phrase "The smallest positive integer not definable in under eleven words". There must be such an integer (why?). But this integer is definable in ten words!

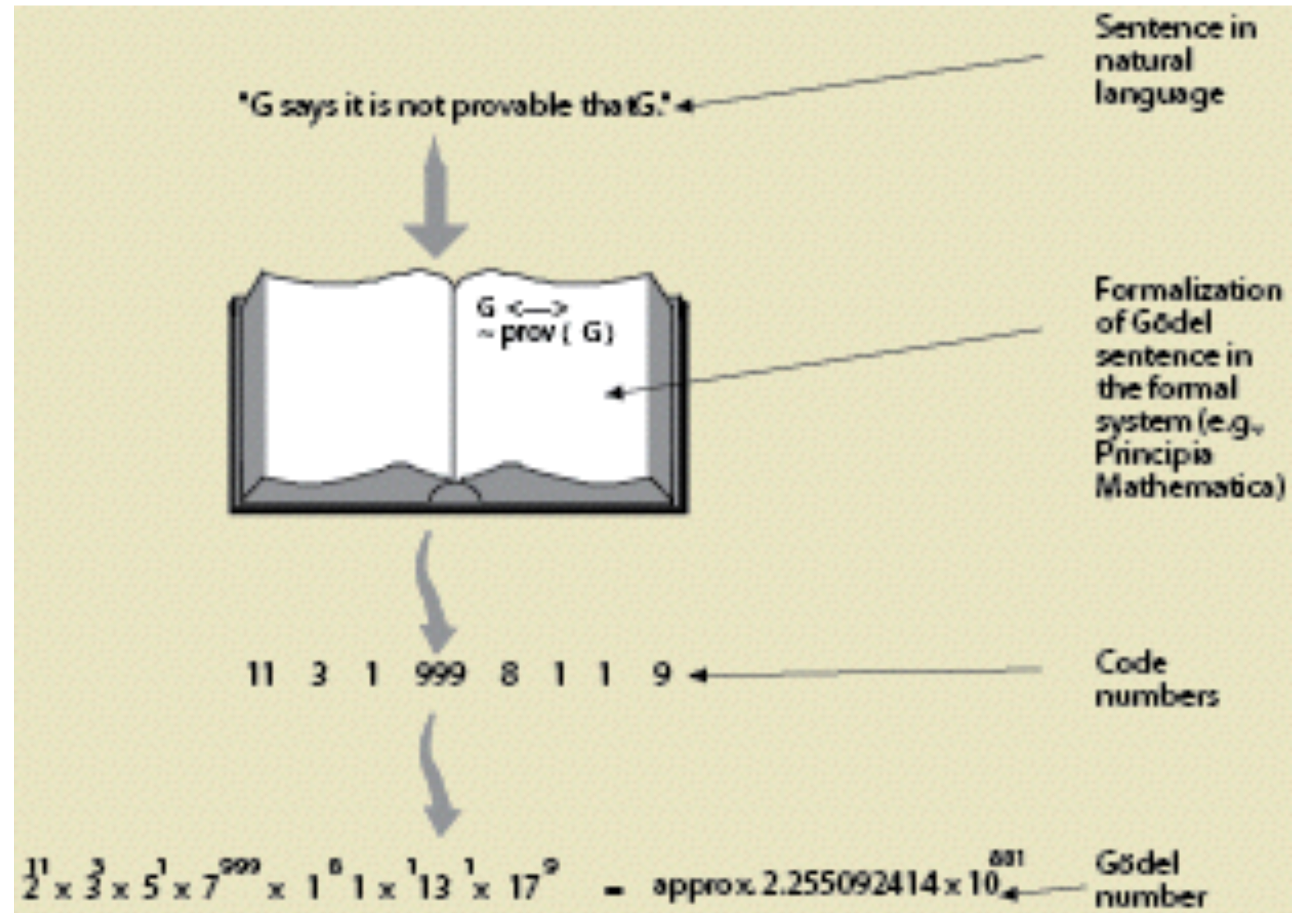
Russel's solution

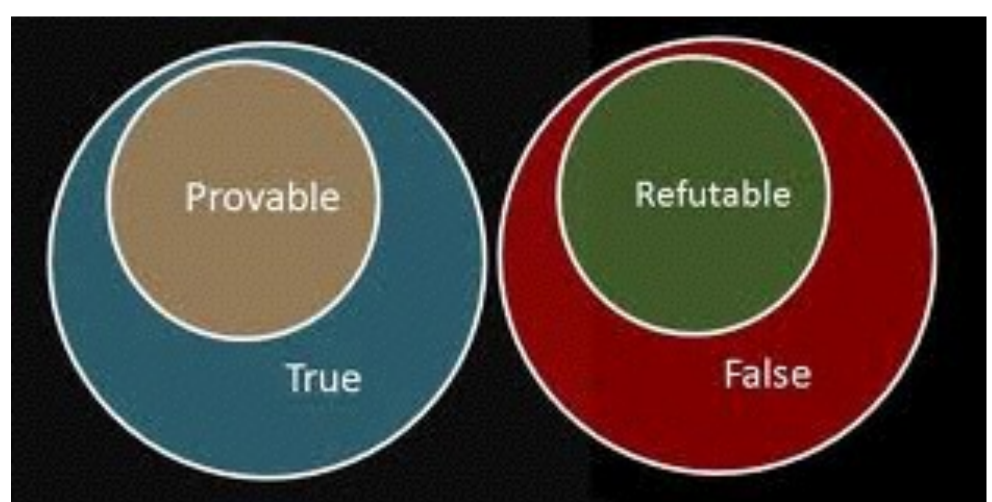
- Russell found that Frege's axiom must be restricted in some way.
- His idea was to block the possibility that a set could be a member of itself.
- In order to do that he developed the so called *type theory* of sets.
- Other solutions came soon. The paradox is not considered a problem any more.
- But a disturbing fact is that Frege was one of the greatest logicians ever and he felt that his axiom was (intuitively) obvious. If he could make such a mistake, can we ever be certain that we don't make similar logical mistakes?

The ghost of self-reference

- Frege's problem was that an unexpected self-reference occurred.
- An analysis of the other paradoxes seem to show that the also are the victims of self-reference.
- Conjecture (the lecturer's): All 'paradoxes' are in some sense caused by self-reference.
- So if we just somehow can *block* all self-references there will be no paradoxes. Or ... ?

Gödel's Theorem





Gödel

- Kurt Gödel studied formal deductive systems of a special kind.
- He showed that all formulas in such a system can be given a so called *Gödel number.*
- He also showed that it is possible to construct a predicate that represents *provability.*
- Then he showed that there are sentences that cannot be proved in the system but still, in some more general sense, are true.

More details

- The Gödel Sentence: $G \leftrightarrow \neg Pr[S](\mathbf{G})$
- Gödel's theorem can be stated in at least two different forms.
- One form is that a sufficiently strong and (efficiently) decidable formal system must contain 'true' sentences which cannot be proved inside the system.
- Another form is that such a system must contain sentences which cannot be proved or disproved inside the system.
- To make things more complicated, there is a *Gödel's second incompleteness theorem* which says that the system cannot be proved to be consistent with methods inside the system.

Implications

- One thing Gödel's proof shows is that self-reference cannot actually be blocked. It is in a certain sense unavoidable.
- It also shows that the powers of formal systems are limited.
- We could of course accept these facts.
- Or we could just give up the idea of using formal systems.
- There are however some related theorems which are even more disturbing.

Tarski

- Alfred Tarski showed that the definition of truth is much more complicated than expected.
- The Tarski type of truth definition is like this: 'Snow is white' if and only if snow is white.
- This type of definition requires a *meta-level.* Truth comes in *layers,* so to say.
- And there is no way to define truth in any effectively decidable way.

Turing

- As we all know, Alan Turing defined the Turing Machine.
- He proved that there are natural problems which cannot be solved in an 'mechanical' way.
- An example is the halting problem.
- Another is the problem of finding proofs in first order logic.

So what are the conclusions of all this?

- Obviously, deductive systems cannot generate truths all in a mechanical way.
- We must sometime rely on other methods for finding truths. (But maybe still work inside a formal system?)
- Truth cannot be defined in a mechanical way.
- There are problems which cannot be solved in a mechanical way.
- But still, we will continue to use deductive systems.

Quantitative Data Analysis

We briefly describe some statistical methods for analysis of data. The methods are parametrical, i.e. we make assumptions about the distributions of the stochastic variables we measure. Two methods you should know are:

- Hypothesis Testing
- Maximum Likelihood-Method
- Linear regression and correlation

Hypothesis Testing

Let us assume that we have a hypothesis we want to test. We compare it to a zero-hypothesis H0. We design a test which gives us a value t. We define a set C such that we reject H0 if t is a member of C. (That means that we accept H.) In that case we say that the test is significant at level a if the probability that t belongs to C is less than or equal to a, given the assumption that H0 is true. The probability a is usually small, like 0.05, 0.01 or 0.001.

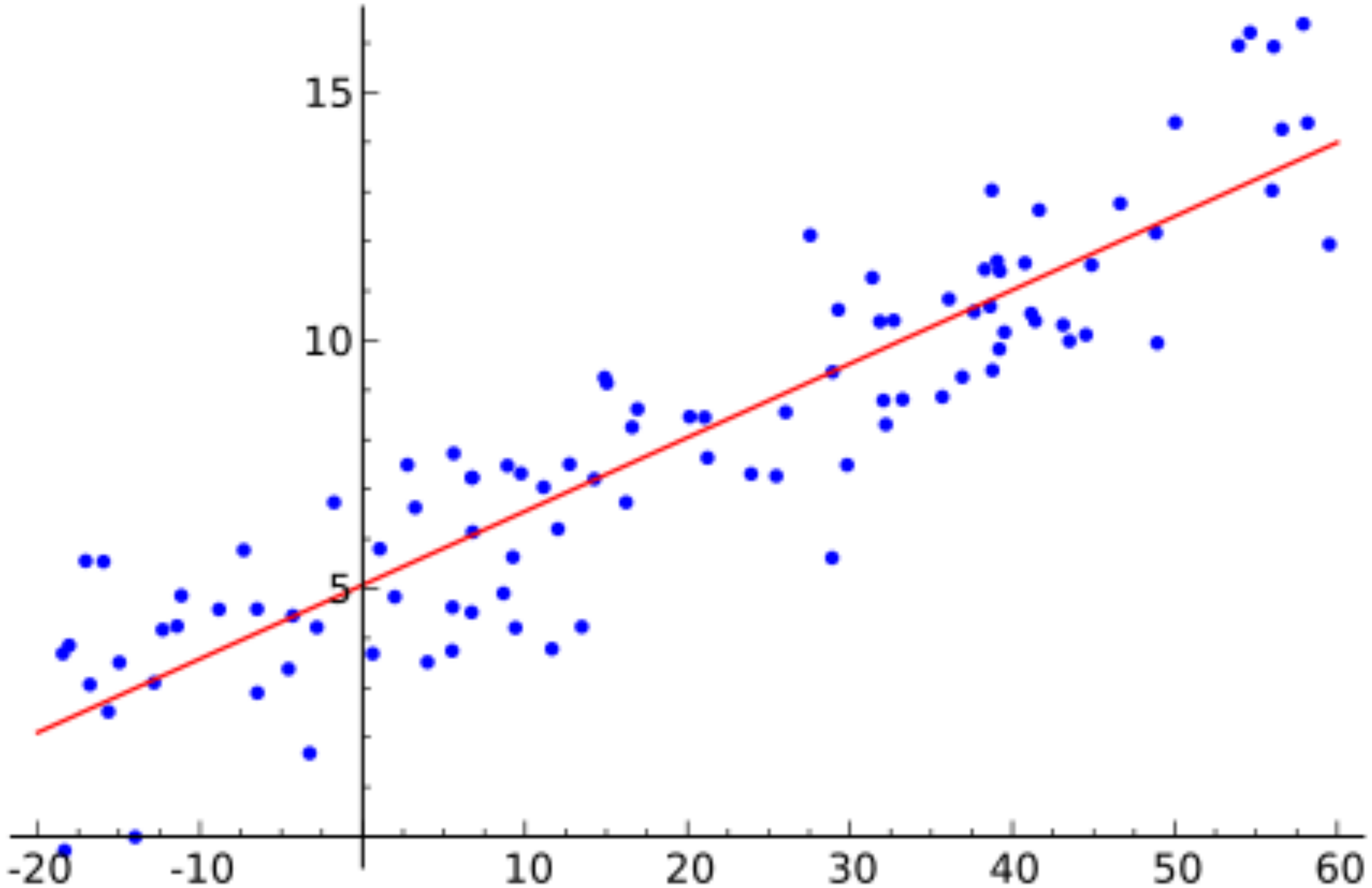
The Maximum Likelihood-method

The method can be illustrated with two special cases:

1. Let us assume that we want to find the value of a parameter f. We do an experiment which gives us a value t. We then (analytically) find the value f0 for the parameter which maximizes the probability P(t | f = f0). We then say that f has the value f0.

2. Let us assume that we have made an observation E. We have different hypotheses H1, H2, ... , Hn which could be possible causes for E. We then chose the hypothesis Hi which maximizes P(E | Hi) and say that Hi is the cause for E.

Linear regression



Correlation

When data look like these in the graph they are obviously correlated. The regression line can be found with the Least Square-Method. If we want a measure to tell us how strong the correlation is we can use the *correlation coefficient*. It is computed in the following way:

$$C_{xy} = \frac{1}{n-1}(\sum x_i y_i - \frac{1}{n}(\sum x_i)(\sum y_i))$$

$$s_x = \sqrt{\frac{1}{n-1}(\sum x_i^2 - \frac{1}{n}\sum x_i)}$$

$$s_y = \sqrt{\frac{1}{n-1}(\sum y_i^2 - \frac{1}{n}\sum y_i)}$$

$$r = \frac{C_{xy}}{s_x s_y}$$