FÖRELÄSNING

ALGORITMKONSTRUKTION

- DEKOMPOSITION

KONSTRUKTION AV ALGORITMER

METOD 1: DEKOMPOSITION

(DIVIDE AND CONQUER, SÖNDRA OCH HÄRSKA)

- DELA UPP I MINDRE PROBLEM
- LÖS DELPROBLEMEN REKURSIVT
- KOMBINERA RESULTATEN

ANALYS: ANVÄND EN REKURSIONSRELATION

EXEMPEL: MATRISMULTIPLIKATION

$$C = AB \iff \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

```
MUL(A,B,n) =
   IF n=1 THEN RETURN A·B
   C11 = ADD(MUL(A11, B11, n/2), MUL(A12, B21, n/2), n/2)
   C12 = ADD(MUL(A11, B12, n/2), MUL(A12, B22, n/2), n/2)
   C21 = ADD(MUL(A21, B11, n/2), MUL(A22, B21, n/2), n/2)
   C22 = ADD(MUL(A21, B12, n/2), MUL(A22, B22, n/2), n/2)
   RETURN C
```

FUNGERAR OM $n = 2^k$ FÖR NÅGOT k.

ANALYS: $\left.\begin{array}{l} T(1) = 1 \\ T(n) = 8\,T\left(\frac{n}{2}\right) + 4\cdot\left(\frac{n}{2}\right)^2 \end{array}\right\} \Rightarrow T(n) = O(n^3)$

SNABB MULTIPLIKATION AV 2×2-MATRISER (STRASSEN)

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

KAN BERÄKNAS GENOM

$$m_1 = (a_{12} - a_{22}) \cdot (b_{21} + b_{22})$$
$$m_2 = (a_{11} + a_{22}) \cdot (b_{11} + b_{22})$$
$$m_3 = (a_{11} - a_{21}) \cdot (b_{11} + b_{12})$$
$$m_4 = (a_{11} + a_{12}) \cdot b_{22}$$
$$m_5 = a_{11} \cdot (b_{12} - b_{22})$$
$$m_6 = a_{22} \cdot (b_{21} - b_{11})$$
$$m_7 = (a_{21} + a_{22}) \cdot b_{11}$$

$$c_{11} = m_1 + m_2 - m_4 + m_6$$
$$c_{12} = m_4 + m_5$$
$$c_{21} = m_6 + m_7$$
$$c_{22} = m_2 - m_3 + m_5 - m_7$$

SAMMANLAGT 7 MULTIPLIKATIONER
18 ADDITIONER OCH SUBTRAKTIONER

MULTIPLIKATION AV TVÅ $n \times n$-MATRISER TAR TID

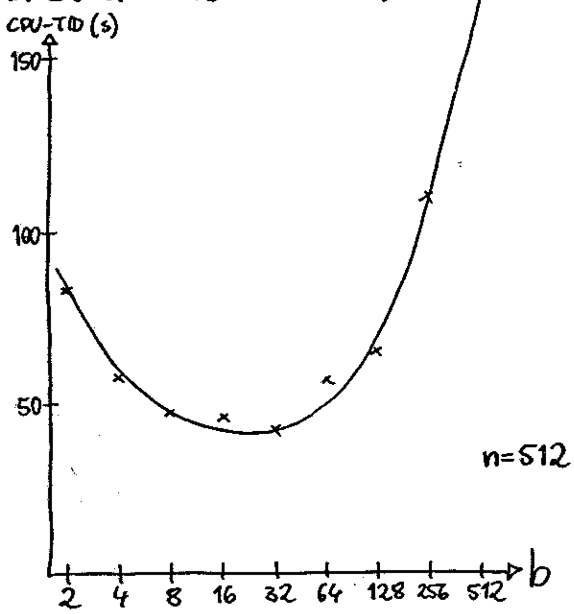
$$T(n) = 7\,T\left(\frac{n}{2}\right) + 18\left(\frac{n}{2}\right)^2, \quad n \geq 2$$
$$T(1) = 1$$
$$\Rightarrow T(n) \sim n^{2.81}$$

Strassen i praktiken

Använd Strassens multiplikationsalgoritm för stora matriser $(n > b)$.

Använd vanlig matrismultiplikation för mindre matriser $(n \leq b)$.

Hur ska brytpunkten b väljas?



Multiplikation av binära tal

$$x = \underbrace{x_{n-1} x_{n-2} \cdots x_{\frac{n}{2}}}_{a} \underbrace{x_{\frac{n}{2}-1} \cdots x_1 x_0}_{b} = a \cdot 2^{\frac{n}{2}} + b$$

$$y = \underbrace{y_{n-1} y_{n-2} \cdots y_{\frac{n}{2}}}_{c} \underbrace{y_{\frac{n}{2}-1} \cdots y_1 y_0}_{d} = c \cdot 2^{\frac{n}{2}} + d$$

$$x \cdot y = a \cdot c \cdot 2^n + (a \cdot d + b \cdot c) 2^{\frac{n}{2}} + b \cdot d$$

Anta att $n = 2^k$

```
mult(x, y, k) =
    IF k=0 THEN RETURN x*y
    ELSE
        [a, b] ← x
        [c, d] ← y
        p1 ← mult(a, c, k-1)
        p2 ← mult(b, d, k-1)
        p3 ← mult(a, d, k-1)
        p4 ← mult(b, c, k-1)
        RETURN p1·2^n + (p3+p4)·2^(n/2) + p2
```

$$\left.\begin{aligned} T(1) &= \Theta(1) \\ T(n) &= 4 \cdot T\left(\tfrac{n}{2}\right) + \Theta(n) \end{aligned}\right\} T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)$$

Smartare multiplikation

$A \leftarrow a \cdot c$

$B \leftarrow b \cdot d$

$C \leftarrow (a+b) \cdot (c+d)$

$D \leftarrow A \cdot 2^n + (C - A - B) \cdot 2^{\frac{n}{2}} + B$

$D = a \cdot c \cdot 2^n + (a \cdot d + b \cdot c) \cdot 2^{\frac{n}{2}} + b \cdot d = x \cdot y$

```
smartmult(x, y, k) =
    IF k≤4 THEN RETURN x*y
    ELSE
        [a, b] ← x
        [c, d] ← y
        A ← smartmult(a, c, k-1)
        B ← smartmult(b, d, k-1)
        C ← smartmult(a+b, c+d, k-1)
        RETURN A·2^n + (C-A-B)·2^(n/2) + B
```

$$\left.\begin{aligned} T(4) &= \Theta(1) \\ T(n) &= 3 \cdot T\left(\tfrac{n}{2}\right) + \Theta(n) \end{aligned}\right\} T(n) = \Theta(n^{\log_2 3}) = \Theta(n^{1.58})$$

(Bästa kända algoritmen $\Theta(n \log n \log\log n)$)