Algorithms and Complexity. Exercise session 6

NP-problems

Frequency Allocation In mobile telephony, you need to solve the *frequency allocation* problem, which is stated as follows. There are a number of transmitters deployed and each of them can transmit on any of a given set of frequencies. Different transmitters have different frequency sets. Some transmitters are so close that they can not transmit at the same frequency, because then they would interfere with each other. (This is actually not dependent on geographical distance - it can be a mountain, a house or other structure.)
You are given the frequency range of each transmitter and the pairs of transmitters that interfere if they send in the same frequency. The problem is to determine if there is any possible choice of frequencies so that no transmitter interferes with any other. Formulate this problem as a graph problem and prove that it is NP-complete!

Hamiltonian path in a graph Show that the HAMILTONIAN PATH problem is NP-complete. The problem is to determine if there is a simple path that visits each vertex of the graph.

Spanning trees with restricted degrees Show that the following problem is NP-complete: Given an undirected graph $G = (V, E)$ and an integer k, determine if G contains a spanning tree T such that each vertex of the tree has maximum degree k.

Polynomial reduction Construct a polynomial reduction from 3CNF-SAT to EQ-GF[2], satisfiability problem for a system of polynomial equations over GF[2] (ie, integers modulo 2).

Is an Euler graph k-colourable? Many problems of the type *determine whether the graph G has the property e* can be simplified if we assume that the graph has any particular characteristic. We will study a special case. We say that a connected graph is an *Euler graph* if each vertex of the graph has even degree. We now want to determine whether a graph is k-colourable. We have more specifically the following problem:

INPUT: An Euler graph G and an integer k.
OUTPUT: **YES** if the graph is k-colourable. **NO** otherwise.

For $k \leq 2$, there is a polynomial algorithm to determine coloring. Therefore, we assume that $k \geq 3$. Show if there is a polynomial algorithm to solve the above problem, or if the problem is NP-complete.
