Algorithms and Complexity
2013
Mästarprov1: Algorithms

Mästarprov 1 should be solved individually in written form and presented orally. No collaboration is allowed.

Written solutions should be handed in latest on **Tuesday, March 5th 17.00,** to Johan, Musard or in the mailbox at the student reception at Lindstedtsvägen 3, 4th floor. Be sure to save a copy of your solutions. Mästarprov 1 is a mandatory and rated part of the course. The test consists of four tasks. The test is roughly graded as follows: Two task correctly solved give an E. Three tasks correctly solved give a C and all tasks correctly solved give an A. You can read more about the grading criteria and the final grade on the course web page. The report should be written either in English or Swedish. (Not all teachers doing the examining will be able to speak Swedish so even if you are a Swedish speaker it could perhaps be nice if you give English a try.)

In all problems you should give an analysis of the time complexity of your algorithm and you should be able to argue for its correctness.

1. Unique paths

Modify Dijkstra's algorithm so that, given two nodes s and t in a directed graph G with positive edge weights, it tells you if there is an unique shortest path from s to t or not.

You don't have to find the unique path. You just have to tell if it exists or not.

2. Guarding an art gallery (Simple)

This problem concerns a fictive problem of an art gallery. To make thing simple we will assume that the gallery has a really simple structure; it is a long corridor in form of a line. On this line we have positions $1, 2, 3, ..., n$ that are 5 meters apart. The gallery has paintings in each positions. Some paintings are more valuable than others. There are k valuable paintings and that they are in positions $p[1] \leq p[2] ... \leq p[k]$ where $p[i] \in [1, 2, ..., n]$. The gallery wants to employ guards to guard the valuable paintings. The guards are supposed to be placed in some of the positions and a guard placed in position i can guard paintings in positions j such that $|j - i| \leq 7$. Design a greedy algorithm that finds a way of guarding all the valuable paintings with as few guards as possible.

3. Guarding an art gallery (Harder)

Now let us assume that we are given values $val[i]$ of all the paintings. The gallery realizes that it can be too expensive to guard all paintings. Instead they decide to employ k guards. But what number k should they choose? They want to guard paintings of as high value as possible. They want to find $best[k]$ = the highest possible sum of values of paintings k guards can guard. Then when they know $best[k]$ they will use the function to decide what k to choose. Your task is to write a Dynamic Programming-algorithm that computes $best[k]$.

4. Surveillance of a network

In this problem we have a computer network in form of a graph G. A set of nodes $c_1, c_2, ..., c_m$ belongs to known criminals. There is one more computer K that is owned by the most dangerous criminal. We have a secret police who wants to wiretap the connections in the network. The police wants to be sure to get all messages sent of from any of the c_i:s to K. But they want to tap as few connections as possible. Design an algorithm that finds this minimal set of connections.