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Theory exam in i Algorithms (Data Structures)and Complexity,
DD2352/DD1352
2013-05-20, 10.00–13.00

No aids are allowed.

For students taking course DD2352 12 points are required for grade E, 15 points for grade D and 18 points for grade C.
You can have up to 4 bonus points.

For students taking course DD1352 14 points are required for grade E, 17 points for grade D and 20 points for grade C.
You can have up to 8 bonus points.

The solutions can be written in Swedish.

1. (8 p)

   Are these statements true or false? For each sub-task a correct answer gives 1 point and an answer with convincing justification gives 2 points.

   a. An algorithm with time complexity $O(\log n!)$ runs in polynomial time.
   b. The problem K-coloring of graphs is in NP.
   c. In a graph G with n nodes and m edges we always have $m \in \Theta(n^2)$.
   d. Kruskal's algorithm is an example of a decomposition algorithm.

2. (3 p)

   If we have a connected, undirected graph G with positive edge weights w, a node s and a tree T in G we can say that T is a *shortest path tree* based at s if T contains all nodes and if, for every node t, the unique path from s to t is a shortest path between s and t in the graph.

   Here is an algorithm:

   F(V, E, w, s)
   $S \leftarrow \{s\}$
   $T \leftarrow \emptyset$
   while $S \neq V$
   Let A be the set of edges (u, v) such that $u \in S$ and $v \notin S$
   Let (x, y) be an edge of smallest weight in A
   $S \leftarrow S \cup \{y\}$, $T \leftarrow T \cup \{(x, y)\}$
   return T

a. Show that this algorithm always returns a tree.

b. Show, by giving a counter example, that this algorithm doesn't always return a shortest path tree.

c. There is another famous problem the algorithm actually solves. What problem?

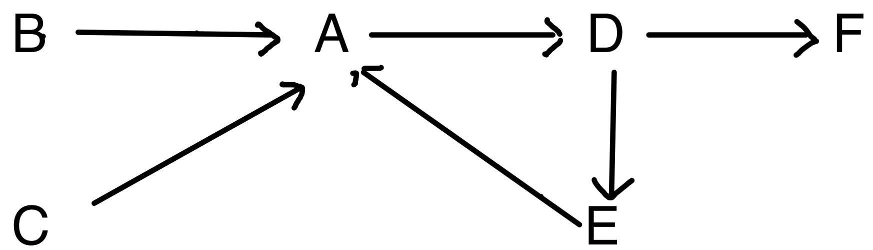
3. (3 p)

In the graph bellow the nodes are problems. An arrow like $A \to B$ indicates that there is a polynomial time reduction (Karp-reduction) from A to B. Let us assume that A is NP-Complete. What do we then know about the rest of the problems? Answer the following:

a. Which problems must be NP-Complete?

b. Which problems must be in NP?

c. *If* we would know that P $\neq$ NP, which problems could then possibly be in P?



4. (3 p)

A propositional logical formula ϕ is on Disjunctive Normal Form (DNF) if

$\phi = c_1 \vee c_2 \vee ... \vee c_k$ where each clause c_i is a variable or a negated variable or a conjunction ($\wedge$) of several variables and/or negated variables. An example is

$$(x_1 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge x_2 \wedge x_4) \vee \bar{x}_3 \vee (x_2 \wedge x_3 \wedge \bar{x}_4 \wedge \bar{x}_3)$$

.

Describe an efficient algorithm that decides if a formula on DNF is satisfiable or not. You don't have to write any code. Just give an informal description of an algorithm and explain why it is efficient.

It is well known that any propositional logical formula can be converted to DNF. It seems that we could decide efficiently if a formula ϕ is satisfiable or not by first converting it to DNF and then run your algorithm. Explain what is wrong with this argument.

5. (3 p)

We now study an optimization problem defined like this:

Input: A list of positive integers $p_1, p_2, ..., p_n$.

Goal: Partition the numbers into two set S_1 and S_2 such that if $Sum(S_i)$ is the sum of the elements in S_i, the $Max(Sum(S_1), Sum(S_2))$ is as small as possible.

a. What important NP-Complete problem corresponds to this optimization problem?

b. We define an algorithm for solving the problem:
Start with S_1 and S_2 empty. Place p_1 in S_1. From now on $p_2, p_3, ..., p_n$ in turn are placed in the S_i with $Sum(S_i)$ smallest (or in S_1 if the sums are equal.).

   Show by an example that this algorithm doesn't always return an optimal solution.

c. Show that the algorithm approximates the problem within $B = 2$.