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Teoritenta i Algoritmer (datastrukturer) och komplexitet
för KTH DD1352–2354 2009-12-18 klockan 14.00–17.00

No aids are allowed. 10 points are required for grade E, 13 points for grade D and 16 points for grade C.

1. (8 p) Are these statements true or false? For each sub-task a correct answer gives 1 point and an answer with convincing justification gives 2 points.

   a) It is possible to multiply two n-bit numbers in time faster than $O(n^2)$.

   b) Dynamic programming can be used to find shortest paths in graphs.

   c) There are NP-problems that are not in the class PSPACE.

   d) To determine whether a graph can be 2-colored is an NP-complete problem.

2. (3 points)

   (a) Describe Dijkstra's algorithm for finding shortest paths in graphs.
   (b) Describe Kruskal or Prim's algorithm for finding minimal spanning trees in graphs.
   (c) Which of these algorithms can handle both positive and negative edge weights as input.
   Give reasons for your answers.

3. (2p) The problem EQUIVALENT Turing machines is the problem of, given two Turing machines M_1 and M_2, determine if they behave the same on all input x. More accurate we say that the machines are equivalent if, for every input x they both both fail to stop or they stop with $M_1(x) = M_2(x)$ i.e. with the same output.

   Show that this problem is undecidable.

4. (3p)

   The problem PARTITIONING is that given a sequence of integers $x_1, x_2, ..., X_n$ determine whether they can be split into two disjoint parts with equal sum. The problem BIN PACKING is that given real number $y_1, y_2, ..., y_n$, all greater than 0, and an integer K, determine whether the numbers can be packed in K boxes of size 1.

   Show how the problem of PARTITIONING can be reduced to BIN PACKING.

5. (4p)

   (a) Describe what an approximation algorithm is. Give an example of such an algorithm.
   (b) As we have seen TRAVELLING SALESPERSON, in its most general form, cannot to approximated. But suppose that we have the numbers $0 < \alpha < \beta$ and a special class of complete graphs with edge weights w such that $\alpha < w < \beta$. Show that this allows us to construct an approximation algorithm with approximation quotient $B = \frac{\beta}{\alpha}$