Exercises 1

1. Given homogeneous coordinates of a point, $x = (x_1, x_2, x_3)^T$ and a line $l = (l_1, l_2, l_3)^T$ in a plane, the equation of the line can be written as $l^T x = 0$. Show that this notation can be used to represent:
(a) intersection x between two lines l and l_0 as $x = l \times l_0$,
(b) line l defined by two points x and x_0 as $l = x \times x_0$. where "$\times$" denotes cross product.

2. Projective transformations between two planes (that also include perspective transformation etween a plane in a world an the image plane) can be represented by the following expressions:

$$y = Ax$$

where $x = (x_1, x_2, x_3)^T$ and $y = (y_1, y_2, y_3)^T$ represent homogeneous coordinates in these two planes respectively. Here, A is non-singular 3×3-matrix. Estimate how are the following geometric entities transformed under this specific transformation:
(i) a line $l^T x = 0$, where l reprsents a vector of length 3,
(ii) an ellipse $x^T C x = 0$, where C represents a positive definite 3×3-matrix.

3. A grey level image defined as $f : \Omega \to [0, z_{max}]$ has a histogram of the following form:

$$p(z) = \frac{\pi}{2 z_{max}} \sin(\frac{\pi}{2} \frac{z}{z_{max}}).$$

Define a montonic grey level transformation function $T : [0, z_{max}] \to [-\frac{z_{max}}{2}, \frac{z_{max}}{2}]$ such that grey levels in the transformed image $g(x, y) = T(f(x, y))$ (where $(x, y) \in \Omega$) are uniformly distributed in teh intervall $[-\frac{z_{max}}{2}, \frac{z_{max}}{2}]$. For which $z \in [0, z_{max}]$ will the estimated transformation result in stretching of the grey level values?

4. Estimate the result of convolution $h_i * f$ with $i = 1, 2, 3$ where

$$f = (\ldots, 0, 1, 2, 3, 11, 4, 0, \ldots)$$

and

$$h1 = (1, 2, 4),$$
$$h2 = (+1, 0, -1)$$
$$h3 = (1, -2, 1).$$

Estimate Fourier transform of vector $(1, 2, 3, 5)$.

5. If the mirroring of a function is defined as $f_-(x) = f(-x)$, show that $h_- * f_- = (h * f)_-$.

6. Estimate Fourier tarnsform of a triangle shaped filter, that along each coordinate direction has the following skape:

$$f(x_k) = \begin{cases} 1 + x_k & \text{om } -1 < x_k < 0, \\ 1 - x_k & \text{om } 0 < x_k < 1, \\ 0 & \text{annars} \end{cases}$$

Draw how the Fourier transform looks like and explain what are the unwanted effects of such a filter when used on images.

Excercise 2, Exam 11/03/2003

(a) Describe the basics and draw figures of perspective and ortographic camera models. Given a set of parallel lines in 3D - explain what is the difference in their image projections for both models.
(b) Under what conditions will a set of parallel lines viewed with a pinhole camera have its vanishing point (in the image) at infinity?
(c) If the area of a planar square in the scene is A, what is its area in an image under ortographic projection? Give your answer in terms of any parameters neccessary to define this relationship, specifying what each parameter means.
(d) What is the difference between an affine and perspective camera model?

Excercise 2 (d), Exam 17/04/2004

A scene point at coordinates $(400, 600, 1200)$ is projected in a pinhole camera to coordinates $(24, 36)$ where both are given in millimeters in the camera's reference frame and the image coordinates have their origin at the camera's principal point.
- Assuming the aspect ratio of the pixels is one, what is the focal length of the camera? (the aspect ratio is defined as the ratio between the width and the height of a pixel)
- Assuming the aspect ratio is *n*ot 1 and the same scene point projects instead to image coordinates $(24, 24)$, given in pixels, what is the aspect ratio of the pixels in this camera?