

Concept Learning

1 Concepts and Hypotheses

- Definitions
- Example
- Hypotheses

2 Search-based Learning

- Find-S
- List-then-Eliminate
- Candidate Elimination

3 Unbiased Learning

- Bias
- Unbiased Learner

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Concept Learning

Concept Learning — Begreppsinlärning

Learning of a **boolean function** from examples

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Learning of a **boolean function** from examples

Categories

- “Nice weather”
- “Dog”
- “Motor vehicle”
- “Criminal offence”

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Subsets of a superset X

Terminology

c The concept to learn

h Hypothesis

H Hypotheses space (**Hypotesrum**)

D Set of available training data

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$$c(x) \rightarrow 0/1, \quad x \in X$$

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- H Hypotheses space (**Hypotesrum**), All conceivable hypotheses (before data arrives)

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- D Set of available training data

$$D \subseteq X$$

Terminology

Two kinds of training examples

Positive example:

$$x : c(x) = 1, \quad x \in D$$

Negative example:

$$x : c(x) = 0, \quad x \in D$$

Example of a *concept*

"Nice Weather"

Example of a *concept**"Nice Weather"*

Let each "weather instance" x_i be composed of four **attributes**:

 $x_1 = \langle \text{Sunny, Warm, Windy, Dry} \rangle$ $x_2 = \langle \text{Cloudy, Warm, Calm, Dry} \rangle$ $x_3 = \dots$

Example of a *concept**"Nice Weather"*

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Generally: *Sky* \times *Temperature* \times *Wind* \times *Humidity*

Assume that the attributes can only take on certain discrete values:

Sky $\in \{ \text{Sunny, Cloudy, Rainy} \}$

Temp $\in \{ \text{Warm, Cold} \}$

Wind $\in \{ \text{Windy, Calm} \}$

Humid $\in \{ \text{Humid, Dry} \}$

Assume that the attributes can only take on certain discrete values:

Sky $\in \{ \text{Sunny, Cloudy, Rainy} \}$

Temp $\in \{ \text{Warm, Cold} \}$

Wind $\in \{ \text{Windy, Calm} \}$

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Number of possible weathers: $|X| = 3 \cdot 2 \cdot 2 \cdot 2 = 24$

Typical training samples

$x_1 =$ <Sunny, Warm, Windy, Dry> → Nice

$x_2 =$ <Sunny, Warm, Windy, Humid> → Nice

$x_3 =$ <Rainy, Cold, Windy, Humid> → Bad

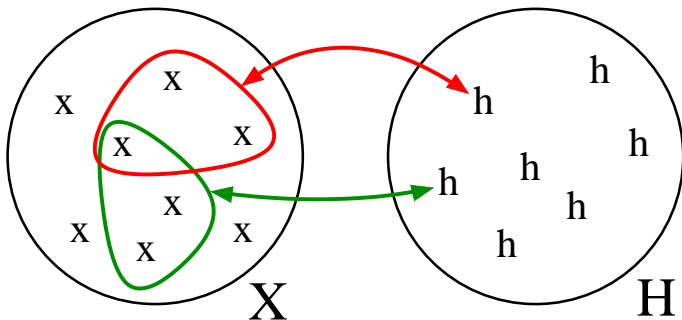
$x_4 =$ <Sunny, Warm, Calm, Humid> → Nice

What does the hypotheses space H look like?

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Each hypothesis h corresponds to one **subset** of X

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How many hypotheses can we choose from?

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$$|H| = 2^{|X|}$$

$$|H| = 2^{24} = 16777216$$

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It is necessary to make restrictions!

Example of a Restriction

Assume that the concept is always a conjunction of attribute values

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Examples of concepts c of this kind

Sunny & Warm
Cold & Calm & Dry

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Sky	Temperature	Wind	Humidity
Sunny			
Cloudy	Warm	Windy	Dry
Rainy	Cold	Calm	Humid
★	★	★	★

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$$4 \cdot 3 \cdot 3 \cdot 3 = 108$$

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Learning \equiv search for a hypothesis which matches all examples

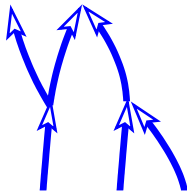
Learning \equiv search for a hypothesis which matches all examples

Use the structure of H to search faster

Some hypotheses are more **general** than others

Partial order between pairs of hypotheses (**partiell ordning**)

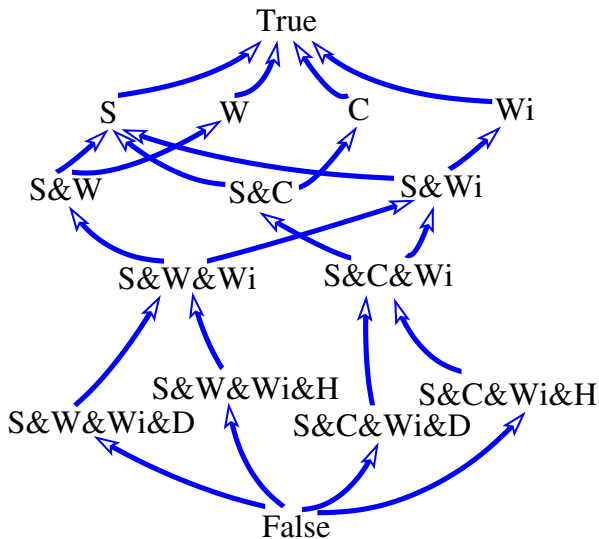
General Hypotheses



Special Hypotheses

Most General in our example: "All weathers are nice"

Most Special in our example: "No weather is nice" (!)



Find-S algorithm

Start from the Most Special hypothesis and generalize when necessary.

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$\hat{h} \leftarrow$ most special hypothesis in H
for $e \leftarrow$ next example:
 if positive example:
 generalize \hat{h} to cover e too

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Returns the most special hypothesis which is **consistent** (**konsistent**) with all examples.

Concrete example: "Nice Weather" assuming that this concept is a conjunction of attributes.

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Initial Hypothesis: $\langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$ (Maximally pessimistic)

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Training examples:

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Current Hypothesis: <Sunny, Warm, Windy, Dry>

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Current Hypothesis: $\langle \text{Sunny, Warm, Windy, Dry} \rangle$

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Final hypothesis: "Nice Weather" $\equiv \text{Sunny} \wedge \text{Warm}$

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- Impossible to know if only one unique hypothesis remains.
- Why should we prefer the most specific hypothesis?
- We will not detect inconsistent data since all negative examples are ignored.
- What happens if there are more equally specific hypotheses?

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The set of all hypotheses consistent with the examples seen so far.

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The set of all hypotheses consistent with the examples seen so far.

- $VS \subseteq H$
- $|VS| = 1$ One unique solution
- $VS = \emptyset$ Inconsistent examples

The List-then-Eliminate algorithm

Direct representation of the Version Space (VS)

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$VS \leftarrow H$

for $e \leftarrow$ next example:

 remove all hypotheses from VS which
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Problem: H is normally too large!

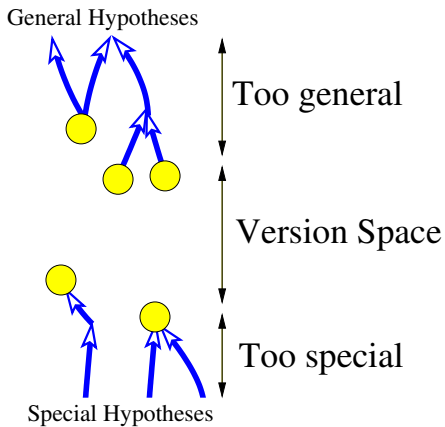
Candidate Elimination

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- Efficient representation of the Version Space

Candidate Elimination

- Efficient representation of the Version Space
- Utilizes the partial ordering between hypotheses.



$G \leftarrow$ most general hypotheses in H

$S \leftarrow$ most special hypotheses in H

for $e \leftarrow$ next example:

if positive example:

$G \leftarrow G - \{\text{hypotheses not including } e\}$

$S \leftarrow$ generalize S to include e

Remove "general duplicates" from S

else:

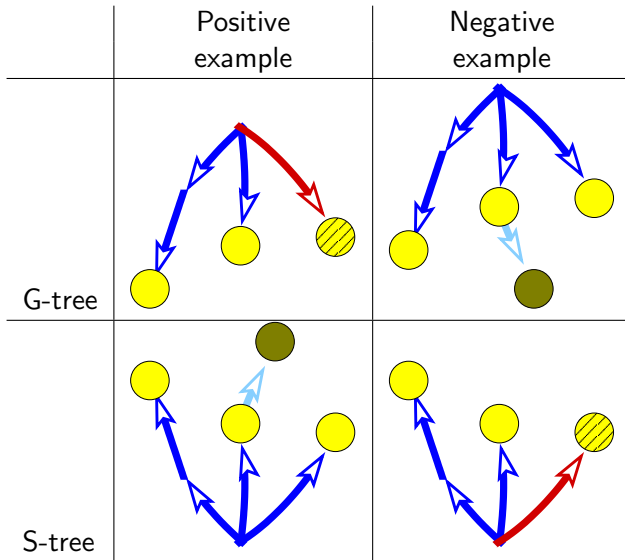
$S \leftarrow S - \{\text{hypotheses including } e\}$

$G \leftarrow$ specialize G not to include e

Remove "special duplicates" from G

Clean G from hypotheses not more general than something in S

Clean S from hypotheses not more special than something in G



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Unbiased Learner A learning algorithm where all hypotheses are treated equally

Restriction Bias Restriction of which hypotheses are allowed

Preference Bias Tendency to prefer certain hypotheses before others

Is an *Unbiased Learner* better?

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Without bias it becomes **impossible to generalize** to unseen examples $x \notin D$.