Decision Trees

Basic Idea: Test the attributes sequentially

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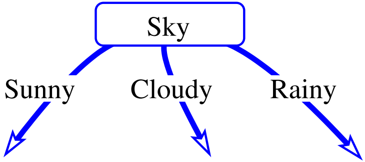

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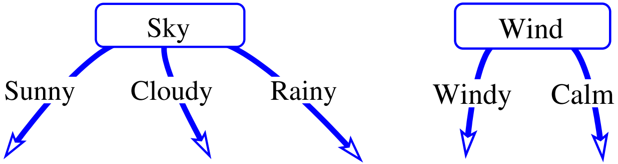
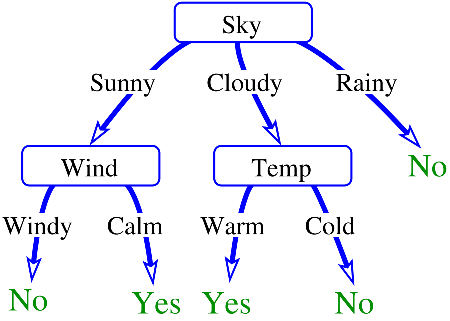


The whole analysis strategy can be seen as a tree.



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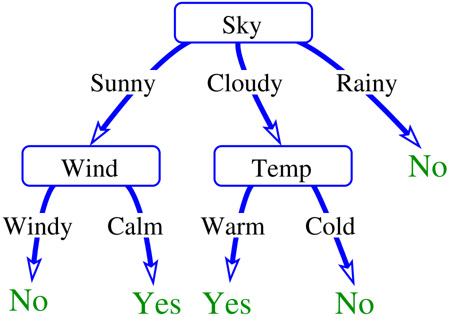
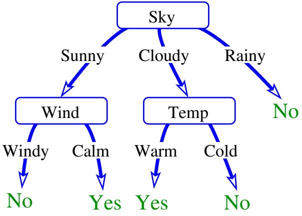
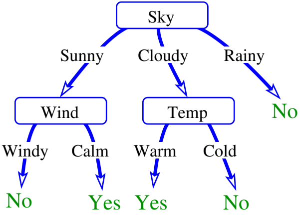


The results (classifications) are coded by the *leaves*

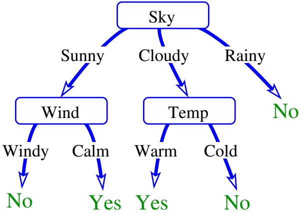


What does the tree encode?



What does the tree encode?

$$(\text{Sunny} \wedge \text{Calm}) \vee (\text{Cloudy} \wedge \text{Warm})$$



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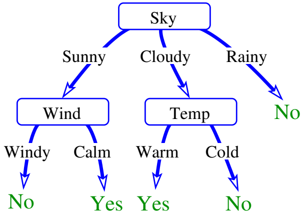
Works as a *disjunction of conjunctions*



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$$(\text{Sunny} \wedge \text{Calm}) \vee (\text{Cloudy} \wedge \text{Warm})$$

Works as a *disjunction of conjunctions*

Normal Form for boolean functions
Arbitrary boolean functions can be represented!

How can a decision tree be constructed automatically?

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1. Choose an attribute to test
2. Branches with a unique class become leaves
3. Other branches are extended recursively

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3. Other branches are extended recursively

Remaining question: how do we choose attributes?

Greedy approach:

Choose the attribute which *tells us most* about the answer

Entropy

Entropy — measure of unpredictability

$$\text{Entropy} = \sum_i -p_i \log_2 p_i$$

p_i probability for event i

Entropy

Example: tossing a coin

Entropy

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$p_{\text{head}} = 0.5; \qquad p_{\text{tail}} = 0.5$

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$$\text{Entropy} = \sum_i -p_i \log_2 p_i =$$
$$= -0.5 \log_2 0.5 + -0.5 \log_2 0.5$$

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The result of a coin-toss has 1 bit of information

Entropy

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$p_1 = \frac{1}{6}; \quad p_2 = \frac{1}{6}; \ldots \quad p_6 = \frac{1}{6}$

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$$= -\log_2 \frac{1}{6} = \log_2 6 \approx 2.58$$

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$$= 6 \times -\frac{1}{6} \log_2 \frac{1}{6} =$$
$$= -\log_2 \frac{1}{6} = \log_2 6 \approx 2.58$$

The result of a dice-roll has 2.58 bit of information

Entropi

Example: rolling a fake dice

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$p_1 = 0.1; \ldots \quad p_5 = 0.1; \quad p_6 = 0.5$

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$p_1 = 0.1; \ldots \quad p_5 = 0.1; \quad p_6 = 0.5$

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$$= -5 \cdot 0.1 \log_2 0.1 - 0.5 \log_2 0.5$$

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$$\text{Entropy} = \sum_i -p_i \log_2 p_i =$$
$$= -5 \cdot 0.1 \log_2 0.1 - 0.5 \log_2 0.5 =$$
$$\approx 2.16$$

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A real dice is more unpredictable (2.58 bit) than a fake (2.16 bit)

Entropy

Unpredictability of a dataset

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- 100 examples, 42 positive

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$$-\frac{58}{100}\log_2\frac{58}{100}-\frac{42}{100}\log_2\frac{42}{100}=0.981$$

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- 100 examples, 3 positive

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Unpredictability of a dataset

- 100 examples, 42 positive

$$-\frac{58}{100}\log_2\frac{58}{100}-\frac{42}{100}\log_2\frac{42}{100}=0.981$$

- 100 examples, 3 positive

$$-\frac{97}{100}\log_2\frac{97}{100}-\frac{3}{100}\log_2\frac{3}{100}=0.194$$

Back to the decision trees

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Smart idea:

Ask about the attribute which maximizes the expected reduction of the entropy.

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Information Gain

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Assume that we ask about attribute A for a dataset S

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$$\text{Gain} = \text{Ent}(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} \text{Ent}(S_v)$$

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What is the entropy for this dataset?

A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	○	○	○	
●	○	○	●	+
○	●	○	○	+
●	○	●	○	
●	●	○	○	+
○	○	○	○	
○	○	●	○	
●	●	○	○	+
○	○	○	●	+
●	○	○	○	
●	●	●	○	+
○	●	○	●	
○	○	○	○	
●	○	○	○	
●	●	○	●	
○	●	○	○	+
○	○	●	○	
●	○	○	○	
○	●	○	○	+
●	○	○	●	+
○	●	●	○	+
○	○	○	○	
●	○	○	○	

What is the entropy for this dataset?

$$\text{Ent} = -\frac{12}{25}\log_2\frac{12}{25} - \frac{13}{25}\log_2\frac{13}{25} \approx \mathbf{0.9988}$$

A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	○	○	○	
●	○	○	●	+
○	●	○	○	+
●	○	●	○	
●	●	○	○	+
○	○	○	○	
○	○	●	○	
●	●	○	○	+
○	○	○	●	+
●	○	○	○	
●	●	●	○	+
○	●	○	●	
○	○	○	○	
●	○	○	○	
●	●	○	●	
○	●	○	○	+
○	○	●	○	
●	○	○	○	
○	●	○	○	+
●	○	○	●	+
○	●	●	○	+
○	○	○	○	
●	○	○	○	

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$A = \bullet$: $\frac{6}{12}$ positive $\rightarrow 1.0$

$A = \circ$: $\frac{6}{13}$ positive $\rightarrow 0.9957$

Expected: $\frac{12}{25} \cdot 1.0 + \frac{13}{25} \cdot 0.9957 \approx \mathbf{0.9977}$

A	B	C	D	
$\bullet$	$\bullet$	$\circ$	$\circ$	+
$\circ$	$\bullet$	$\bullet$	$\circ$	+
$\circ$	$\circ$	$\circ$	$\circ$	
$\bullet$	$\circ$	$\circ$	$\bullet$	+
$\circ$	$\bullet$	$\circ$	$\circ$	+
$\bullet$	$\circ$	$\bullet$	$\circ$	
$\bullet$	$\bullet$	$\circ$	$\circ$	+
$\circ$	$\circ$	$\circ$	$\circ$	
$\circ$	$\circ$	$\bullet$	$\circ$	
$\bullet$	$\bullet$	$\circ$	$\circ$	+
$\circ$	$\circ$	$\circ$	$\bullet$	+
$\bullet$	$\circ$	$\circ$	$\circ$	
$\bullet$	$\bullet$	$\bullet$	$\circ$	+
$\circ$	$\bullet$	$\circ$	$\bullet$	
$\circ$	$\circ$	$\circ$	$\circ$	
$\bullet$	$\circ$	$\circ$	$\circ$	
$\bullet$	$\bullet$	$\circ$	$\bullet$	
$\circ$	$\bullet$	$\circ$	$\circ$	+
$\circ$	$\circ$	$\bullet$	$\circ$	
$\bullet$	$\circ$	$\circ$	$\circ$	
$\circ$	$\bullet$	$\circ$	$\circ$	+
$\bullet$	$\circ$	$\circ$	$\bullet$	+
$\circ$	$\bullet$	$\bullet$	$\circ$	+
$\circ$	$\circ$	$\circ$	$\circ$	
$\bullet$	$\circ$	$\circ$	$\circ$	

What is the entropy for this dataset?

$$\text{Ent} = -\frac{12}{25}\log_2\frac{12}{25} - \frac{13}{25}\log_2\frac{13}{25} \approx \mathbf{0.9988}$$

$A = \bullet$: $\frac{6}{12}$ positive $\rightarrow 1.0$

$A = \circ$: $\frac{6}{13}$ positive $\rightarrow 0.9957$

Expected: $\frac{12}{25} \cdot 1.0 + \frac{13}{25} \cdot 0.9957 \approx \mathbf{0.9977}$

$B = \bullet$: $\frac{9}{11}$ positive $\rightarrow 0.684$

$B = \circ$: $\frac{3}{14}$ positive $\rightarrow 0.750$

Expected: **0.721**

A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	○	○	○	
●	○	○	●	+
○	●	○	○	+
●	○	●	○	
●	●	○	○	+
○	○	○	○	
○	○	●	○	
●	●	○	○	+
○	○	○	●	+
●	○	○	○	
●	●	●	○	+
○	●	○	●	
○	○	○	○	
●	○	○	○	
●	●	○	●	
○	●	○	○	+
○	○	●	○	
●	○	○	○	
○	●	○	○	+
●	○	○	●	+
○	●	●	○	+
○	○	○	○	
●	○	○	○	

What is the entropy for this dataset?

$$\text{Ent} = -\frac{12}{25}\log_2\frac{12}{25} - \frac{13}{25}\log_2\frac{13}{25} \approx \mathbf{0.9988}$$

$A = \bullet$: $\frac{6}{12}$ positive $\rightarrow 1.0$
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Expected: $\frac{12}{25}\cdot 1.0 + \frac{13}{25}\cdot 0.9957 \approx \mathbf{0.9977}$

$B = \bullet$: $\frac{9}{11}$ positive $\rightarrow 0.684$
$B = \circ$: $\frac{3}{14}$ positive $\rightarrow 0.750$
Expected: **0.721**

$C = \bullet$: $\frac{3}{6}$ positive $\rightarrow 1.0$
$C = \circ$: $\frac{9}{19}$ positive $\rightarrow 0.9980$
Expected: **0.9985**

A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	○	○	○	
●	○	○	●	+
○	●	○	○	+
●	○	●	○	
●	●	○	○	+
○	○	○	○	
○	○	●	○	
●	●	○	○	+
○	○	○	●	+
●	○	○	○	
●	●	●	○	+
○	●	○	●	
○	○	○	○	
●	○	○	○	
●	●	○	●	
○	●	○	○	+
○	○	●	○	
●	○	○	○	
○	●	○	○	+
●	○	○	●	+
○	●	●	○	+
○	○	○	○	
●	○	○	○	

What is the entropy for this dataset?

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$A = \bullet$: $\frac{6}{12}$ positive $\rightarrow 1.0$
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$B = \bullet$: $\frac{9}{11}$ positive $\rightarrow 0.684$
$B = \circ$: $\frac{3}{14}$ positive $\rightarrow 0.750$
Expected: **0.721**

$C = \bullet$: $\frac{3}{6}$ positive $\rightarrow 1.0$
$C = \circ$: $\frac{9}{19}$ positive $\rightarrow 0.9980$
Expected: **0.9985**

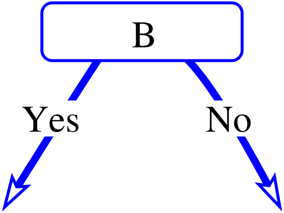
$D = \bullet$: $\frac{3}{5}$ positive $\rightarrow 0.9710$
$D = \circ$: $\frac{9}{20}$ positive $\rightarrow 0.9928$
Expected: **0.9884**

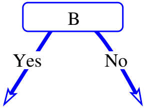
A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	○	○	○	
●	○	○	●	+
○	●	○	○	+
●	○	●	○	
●	●	○	○	+
○	○	○	○	
○	○	●	○	
●	●	○	○	+
○	○	○	●	+
●	○	○	○	
●	●	●	○	+
○	●	○	●	
○	○	○	○	
●	○	○	○	
●	●	○	●	
○	●	○	○	+
○	○	●	○	
●	○	○	○	
○	●	○	○	+
●	○	○	●	+
○	●	●	○	+
○	○	○	○	
●	○	○	○	

$$\text{Gain}(A) = 0.9988 - 0.9977 = \mathbf{0.0011}$$
$$\text{Gain}(B) = 0.9988 - 0.7210 = \mathbf{0.2778}$$
$$\text{Gain}(C) = 0.9988 - 0.9985 = \mathbf{0.0003}$$
$$\text{Gain}(D) = 0.9988 - 0.9884 = \mathbf{0.0104}$$

$$\mathrm{Gain}(A) = 0.9988 - 0.9977 = \mathbf{0.0011}$$
$$\mathrm{Gain}(B) = 0.9988 - 0.7210 = \mathbf{0.2778}$$
$$\mathrm{Gain}(C) = 0.9988 - 0.9985 = \mathbf{0.0003}$$
$$\mathrm{Gain}(D) = 0.9988 - 0.9884 = \mathbf{0.0104}$$

Attribute B gives most information





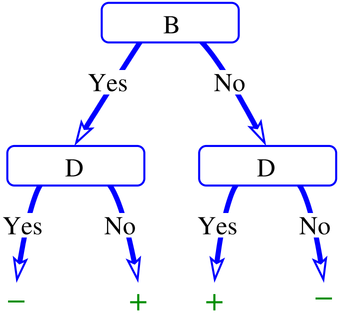
Examples where $B = \bullet$

A	B	C	D	
●	●	○	○	+
○	●	●	○	+
○	●	○	○	+
●	●	○	○	+
●	●	○	○	+
●	●	●	○	+
○	●	○	●	
●	●	○	●	
○	●	○	○	+
○	●	○	○	+
○	●	●	○	+

Examples where $B = \circ$

A	B	C	D	
○	○	○	○	
●	○	○	●	+
●	○	●	○	
○	○	○	○	
○	○	●	○	
○	○	○	●	+
●	○	○	○	
○	○	○	○	
●	○	○	○	
○	○	●	○	
●	○	○	○	
●	○	○	●	+
○	○	○	○	
●	○	○	○	

Information Gain



Which Bias does this learning algorithm have?

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  Yes, some trees are found before others

Which hypotheses (here: trees) are preferred?

- Shallow trees
- "Important attributes" early

Which hypothesis should be preferred when several are compatible with the data?

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Occam's principle (*Occam's razor*, "Occam's rakkniv")

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William from Ockham, Theologian and Philosopher (1288–1348)

"Entia non sunt multiplicanda praeter necessitatem"

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translated:

"Entities should not be multiplied beyond necessity"

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All things being equal,
the simplest explanation tends to be the right one.

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Simple hypotheses tends to generalize better.

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What can be done about it?
Choose a simpler hypothesis and accept some errors for the training examples

Possible ways of improving the decision trees

- Avoid overfitting
  - Limit the tree's height
  - Pruning (Beskärning)
- Attributes with graded values
- Missing attribute values
- Variable cost for different attributes

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