Reinforcement Learning

Belöningsbaserad Inlärning

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explicit information about correct actions

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  Temporal credit assignment
- The reward does not say *what* was good
  Structural credit assignment

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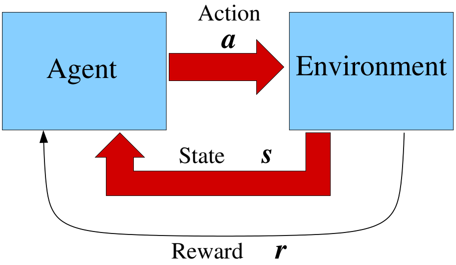

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- Infinite Horizon

$$\max \left[\sum_{t=0}^{\infty} \gamma^t r_t \right]$$

Requires discount (nedskrivning) of future reward $(0 < \gamma < 1)$

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- Find a short/fast/cheap path to a goal
  Reward -1 at each step

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- Environment is a stationary *Markov Decision Process*
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- Deterministic or non-deterministic environment

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- *Policy*
  The action chosen by the agent for each state
  $$\pi(s) \mapsto a$$
- *Value Function*
  Expected total future reward from s when following policy π
  $$V^{\pi}(s) \mapsto \Re$$

Classical model problem: *Grid World*

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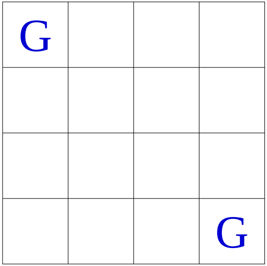

Trivial labyrinth

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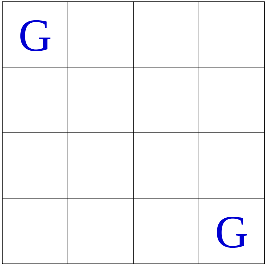

Trivial labyrinth

Reward: -1 at each step until a goal state (G) is reached

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V with an optimal policy

0	−14	−20	−22
−14	−18	−22	−20
−20	−22	−18	−14
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V with a random policy

1. Defining the Problem
   - Framework
   - Role of Reward
   - Simplifying Assumptions
   - Central Concepts
2. Known Environment
   - Bellmans Equation
   - Solving Techniques
3. Unknown Environment
   - Monte-Carlo Method
   - Temporal-Difference
   - Q-Learning
   - Sarsa-Learning
4. Improvements
   - Importance of Making Mistakes
   - Eligibility Trace

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Bellmans Equation:

$$V^\pi(s) = r(s, \pi(s)) + \gamma \cdot V^\pi(\delta(s, \pi(s)))$$

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- Iteratively (*value iteration*)

$$V^\pi_{k+1}(s) \leftarrow r(s, \pi(s)) + \gamma \cdot V^\pi_k(\delta(s, \pi(s)))$$

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Policy iteration:

Interleaved calculation of policy and values

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Very slow convergence

Temporal Difference

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Learns *considerably faster* that the Monte-Carlo method

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$$\pi(s) = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

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Small problem: max-operation requires a search through all possible actions

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Almost the same as Q-learning, but one uses the current policy to select a':

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The name comes from the experience-tuples structure:

$$< s, a, r, s', a' >$$

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- The environment is not fully observable
- There are way too many states
- The states are not discrete
- The agent is acting in continuous time

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- ϵ-greedy
  Sometimes (with probability ϵ) make a random action instead of the one that seems best (greedy)
- Softmax
  Assign a probability to choose each action depending on how good they seem

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Often denoted TD(λ) where λ is the time constant of the trace e