Concept Learning

1. Concepts and Hypotheses
   - Definitions
   - Example
   - Hypotheses
2. Search-based Learning
   - Find-S
   - List-then-Eliminate
   - Candidate Elimination
3. Unbiased Learning
   - Bias
   - Unbiased Learner

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Concept Learning

Concept Learning — Begreppsinlärning

Learning of a boolean function from examples

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Categories

- "Nice weather"
- "Dog"
- "Motor vehicle"
- "Criminal offence"

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Subsets of a superset X

Terminology

c The concept to learn

h Hypothesis

H Hypotheses space (Hypotesrum)

D Set of available training data

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$$c(x) \rightarrow 0/1, \quad x \in X$$

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D Set of available training data

$$D \subseteq X$$

Terminology

Two kinds of training examples

Positive example:

$$x : c(x) = 1, \quad x \in D$$

Negative example:

$$x : c(x) = 0, \quad x \in D$$

Example of a *concept*

"Nice Weather"

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Let each "weather instance" x_i be composed of four attributes:

$$x_1 = \ <\text{Sunny, Warm, Windy, Dry}>$$
$$x_2 = \ <\text{Cloudy, Warm, Calm, Dry}>$$
$$x_3 = \ \cdots$$

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Generally: $Sky \times Temperature \times Wind \times Humidity$

Assume that the attributes can only take on certain discrete values:

$$
\begin{aligned}
\text{Sky} &\in \{ \text{Sunny, Cloudy, Rainy} \} \\
\text{Temp} &\in \{ \text{Warm, Cold} \} \\
\text{Wind} &\in \{ \text{Windy, Calm} \} \\
\text{Humid} &\in \{ \text{Humid, Dry} \}
\end{aligned}
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Number of possible weathers: $|X| = 3 \cdot 2 \cdot 2 \cdot 2 = 24$

Typical training samples

$$\begin{aligned}
x_1 &= \quad <\text{Sunny, Warm, Windy, Dry}> &&\rightarrow \text{Nice}\\
x_2 &= \quad <\text{Sunny, Warm, Windy, Humid}> &&\rightarrow \text{Nice}\\
x_3 &= \quad <\text{Rainy, Cold, Windy, Humid}> &&\rightarrow \text{Bad}\\
x_4 &= \quad <\text{Sunny, Warm, Calm, Humid}> &&\rightarrow \text{Nice}
\end{aligned}$$

What does the hypotheses space H look like?

What does the hypotheses space H look like?

Each hypothesis h corresponds to one subset of X

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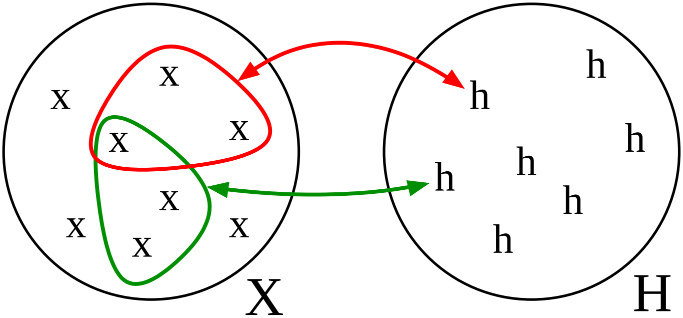

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How many hypotheses can we choose from?

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$$|H| = 2^{24} = 16777216$$

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It is necessary to make restrictions!

Example of a Restriction

Assume that the concept is always a conjunction of attribute values

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Examples of concepts c of this kind

Sunny & Warm
Cold & Calm & Dry

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$4 \cdot 3 \cdot 3 \cdot 3 = 108$

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Learning $\equiv$ search for a hypothesis which matches all examples

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Use the structure of H to search faster

Some hypotheses are more general than others

Partial order between pairs of hypotheses (partiell ordning)

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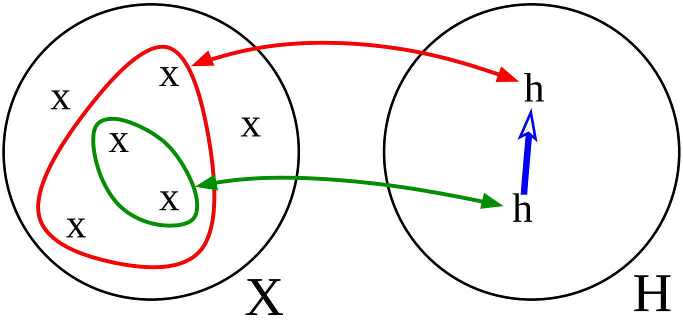
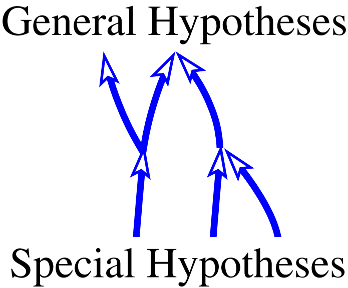
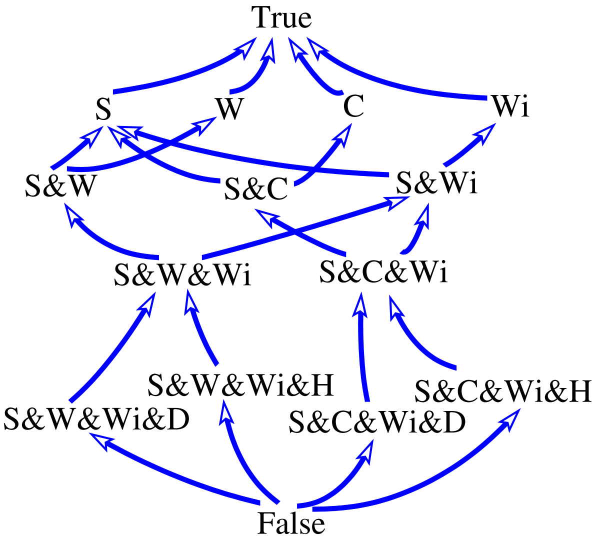




Most General in our example: "All weathers are nice"
Most Special in our example: "No weather is nice" (!)


True
S
W
C
Wi
S&W
S&C
S&Wi
S&W&Wi
S&C&Wi
S&W&Wi&H
S&C&Wi&H
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False

Find-S algorithm

Start from the Most Special hypothesis and generalize when necessary.

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Returns the most special hypothesis which is consistent (konsistent) with all examples.

Concrete example: "Nice Weather" assuming that this concept is a conjunction of attributes.

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Training examples:

$x_1 =$ <Sunny, Warm, Windy, Dry> $\longrightarrow$ Nice

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Final hypothesis: "Nice Weather" $\equiv$ Sunny $\wedge$ Warm

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- Why should we prefer the most specific hypothesis?
- We will not detect inconsistent data since all negative examples are ignored.
- What happens if there are more equally specific hypotheses?

Version Space (VS)

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The set of all hypotheses consistent with the examples seen so far.

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- $|VS| = 1$ One unique solution
- $VS = \emptyset$ Inconsistent examples

The List-then-Eliminate algorithm

Direct representation of the Version Space (VS)

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$VS \leftarrow H$

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Problem: H is normally too large!

Candidate Elimination

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- Efficient representation of the Version Space

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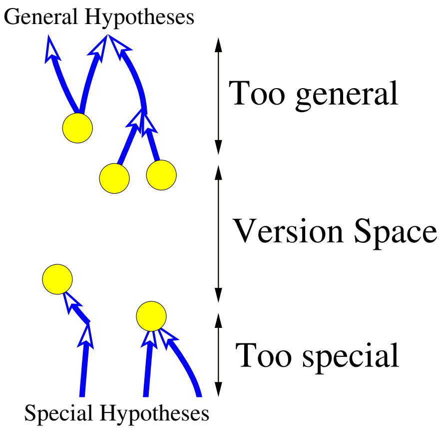
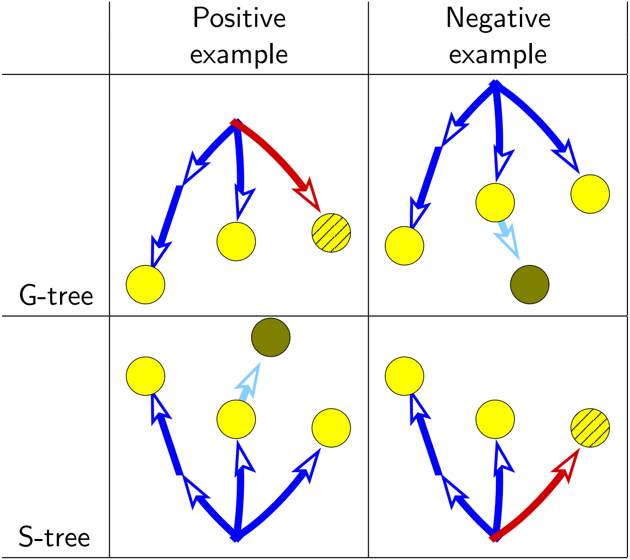
- Efficient representation of the Version Space
- Utilizes the partial ordering between hypotheses.


General Hypotheses
Too general
Version Space
Too special
Special Hypotheses

$G \leftarrow$ most general hypotheses in H
$S \leftarrow$ most special hypotheses in H
for $e \leftarrow$ next example:
    if positive example:
        $G \leftarrow G - \{\text{hypotheses not including } e\}$
        $S \leftarrow \text{generalize } S \text{ to include } e$
        Remove "general duplicates" from S
    else:
        $S \leftarrow S - \{\text{hypotheses including } e\}$
        $G \leftarrow \text{specialize } G \text{ not to include } e$
        Remove "special duplicates" from G
    Clean G from hypotheses not more general than something in S
    Clean S from hypotheses not more special than something in G

Candidate Elimination



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Unbiased Learner A learning algorithm where all hypotheses are treated equally

Restriction Bias Restriction of which hypotheses are allowed

Preference Bias Tendency to prefer certain hypotheses before others

Is an *Unbiased Learner* better?

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Without bias it becomes impossible to generalize to unseen examples $x \notin D$.