KTH Computer Science
and Communication

Homework VI, Complexity Theory Fall 2011

Due on October 14 at 13.15, i.e. at the beginning of the lecture. The general rules on homework solutions available at the course home-page apply. In particular, discussions of ideas in groups of up to at most two people are allowed but solutions should be written down individually.

Some of the problems below are "classical" and hence their solutions is probably posted on the Internet. It is not allowed to use such solutions in any way. The order of the problems is "random" and hence do not expect that the lowest numbered problems are the easiest.

Any corrections or clarifications on this problem set will be posted under "homework" on the course home-page http://www.csc.kth.se/utbildning/kth/kurser/DD2446/kplx11/uppgifter.

1 (10p) In class I sketched the proof that the evaluation of straight line programs is P-complete. The point, which is both a detail but at the same time essential, that the reduction runs in space $O(\log n)$ was essentially swept under the rug. Please make a detailed account of this reduction and in particular check that the reduction can be implemented in logarithmic space.

2 (10p) Prove that multiplication of two n-bit numbers can be done by circuits of size $O(n^2)$ and depth $O(\log n)$.

Hint: You want to sum n numbers each with n bits. There are at least two different ways to do this in logarithmic depth. The key trick in one is to, given three numbers, in constant depth, produce two numbers with the same sum. The key trick in the other method is to use a redundant but binary representation that allows you to add two numbers in $O(1)$ depth.

3 (15p) Consider the fan-in 2 version of Boolean circuits. Thus we have $\wedge$-gates, $\vee$-gates and negations. The size of the circuit is given by the total number of $\wedge$-gates and $\vee$-gates while negations and inputs are for free. Prove that a circuit that computes the parity (i.e. exclusive-or) of n bits needs to be of size at least $2n - 2$. This is not a very impressive bound but please supply a mathematically correct argument.
Hint: There are a few possible ways of attack this problem but here is one suggestion. If you set an input bit to a constant, your circuit simplifies and the result should compute parity of $n - 1$ bits.