DD2451
Parallel and Distributed Computing

FDD3008
Distributed Algorithms

Lecture 2

Concurrent Objects

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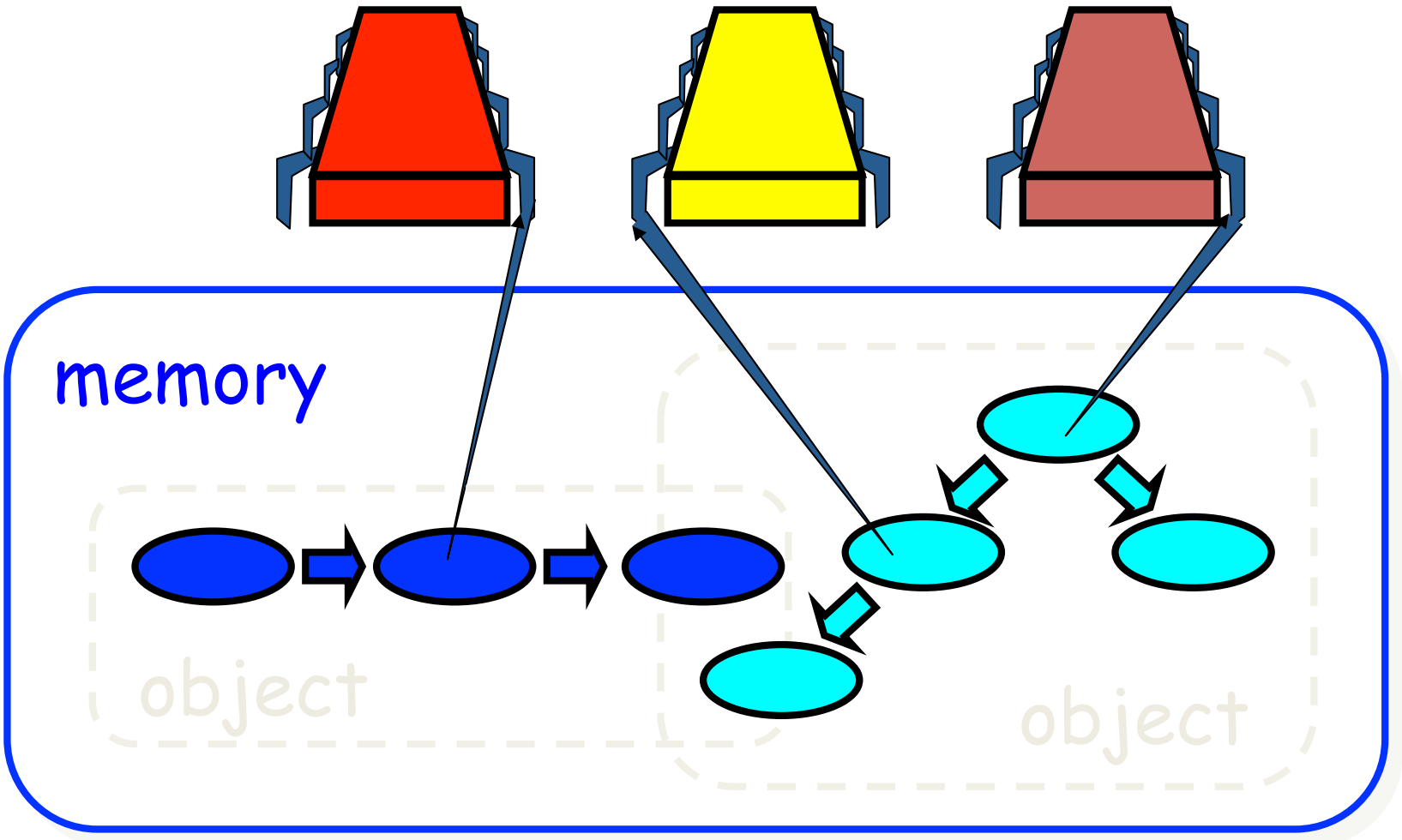
Autumn/Winter 2011

Concurrent Computaton



Objects

Objects have ***state***

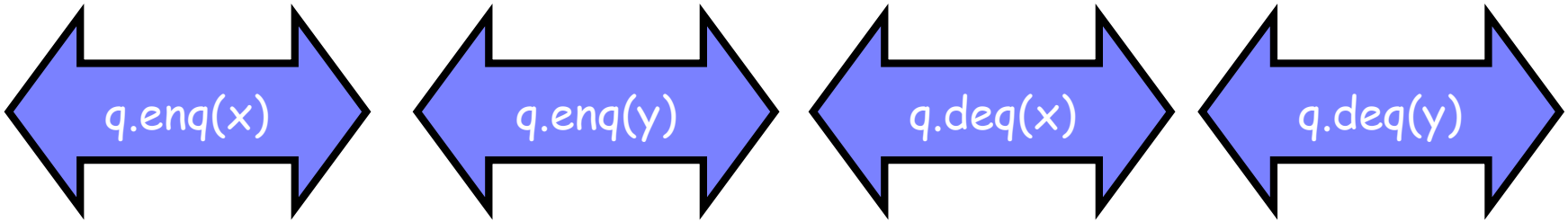
- Usually given by a set of ***fields***

Object have ***methods***

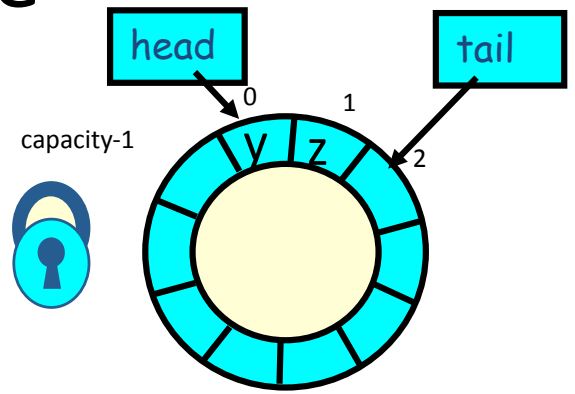
- Only way to manipulate state

Example, a FIFO queue:

- Fields: Sequence of items
- Methods: enq and deq

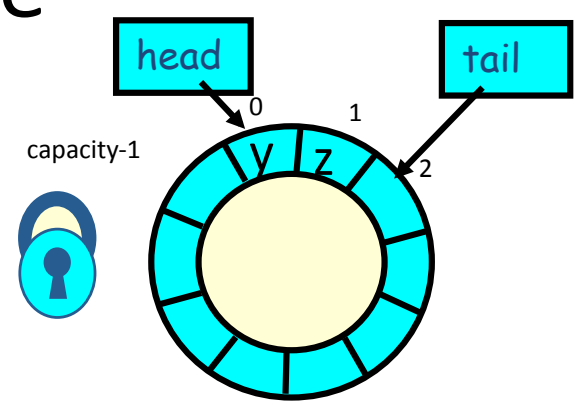


A Concurrent FIFO Queue




```
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock;
  public LockBasedQueue(int capacity) {
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[]) new Object[capacity];
}
```

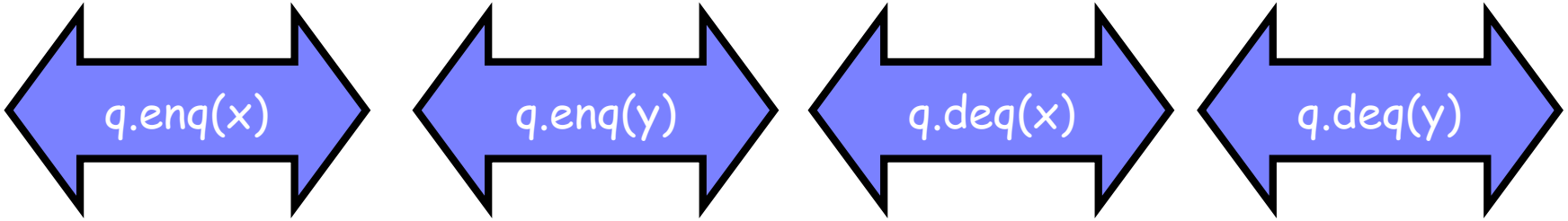
A Concurrent FIFO Queue




```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();    }  }
```

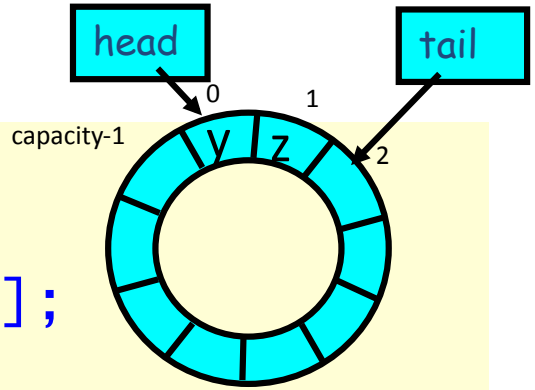
Is The Queue Correct?

- Calls are effectively sequentialized
- Behaviour is familiar:



- (In fact the calls overlap slightly but we ignore that for now)
- Now let's try the same thing without mutual exclusion
- For simplicity, only two threads
  - One thread enq only
    - The producer thread
  - The other deq only
    - The consumer thread

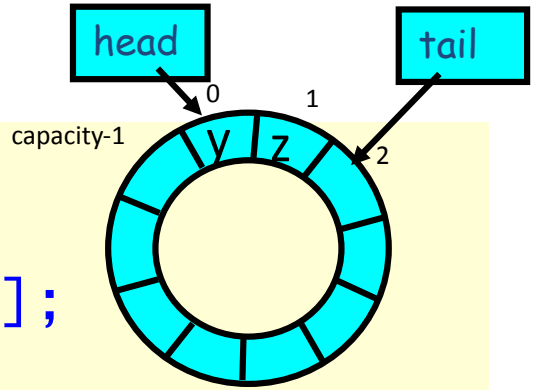
Wait-free 2-Thread Queue




```
public class WaitFreeQueue {
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];

  public void enq(Item x) {
    while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     while (tail == head);     // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```

Wait-free 2-Thread Queue

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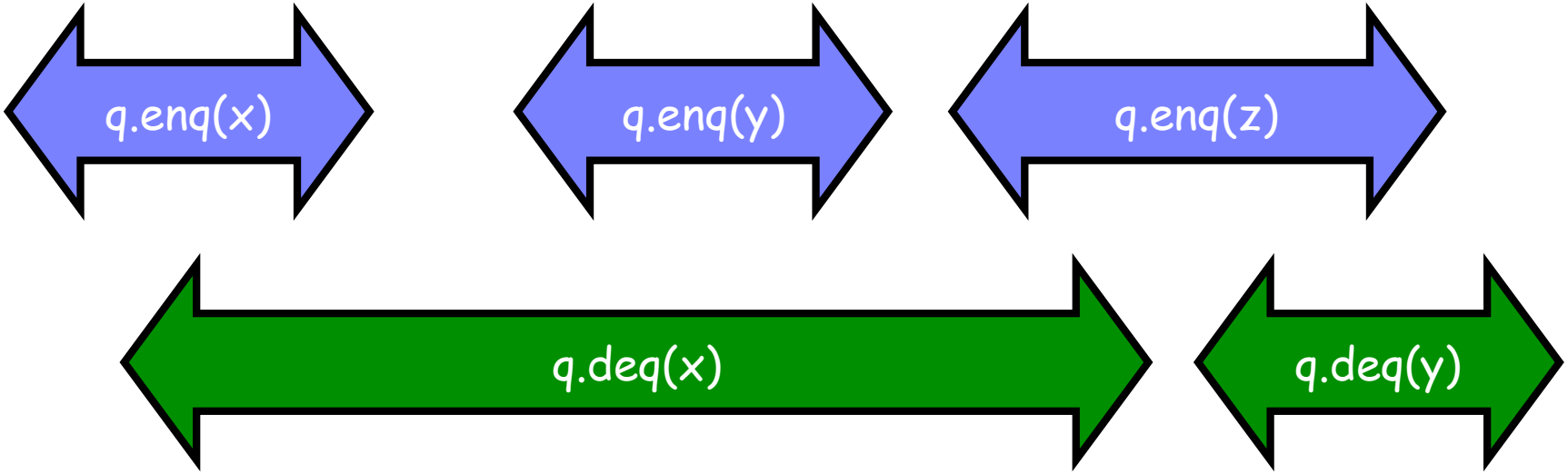
```

Ooops: No locks!

Is the Wait-Free Queue Correct?

The behaviour of the queue may now look something like this:



Do we want this behaviour? Or not? Specify!

Need to be able to answer this question

Need to know when an implementation is correct

Sequential Specifications

- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition)
  - the method will return a particular value
  - or throw a particular exception.
- and (postcondition, cont'd)
  - the object will be in some other state
  - when the method returns.

Pre and PostConditions for Deq

- Precondition:
  - Queue is non-empty
- Postcondition:
  - Returns first item in queue
- Postcondition:
  - Removes first item in queue

- Precondition:
  - Queue is empty
- Postcondition:
  - Throws Empty exception
- Postcondition:
  - Queue state unchanged

Why Sequential Specifications Totally Rock

- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
  - Elsewhere it does not matter
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

What About Concurrent Specifications ?

- Method state potentially matters everywhere
  - Method calls may interfere because of shared variable accesses
  - So intermediate states matter
- Documentation size?
  - In worst case the only meaningful specification is the method itself
- Can add new methods?
  - In worst case only by exploring all possible interactions with existing methods
- Is there any rescue from this morass?

Sequential vs Concurrent

Sequential:

- Methods take time
- Sure, but their *effect* might as well be instantaneous
- Objects need meaningful state only between method calls

Concurrent:

- Method call is not an event
- Method call is an interval
- Since method calls overlap object might never be between method calls

Sequential vs Concurrent

Sequential:

- Each method described in isolation
- Can add new method without affecting old ones

Concurrent:

- Must characterize all possible interactions between concurrent calls
  - What if two deq's overlap? Two enq's? One enq and one deq? Etc? When and how do they overlap?
- Everything can potentially interact with everything else

Sequential vs Concurrent

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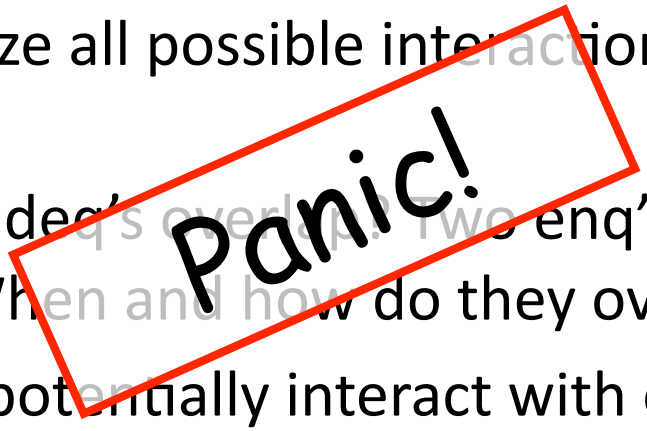

Intuitively…

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  try {
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Intuitively…

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```
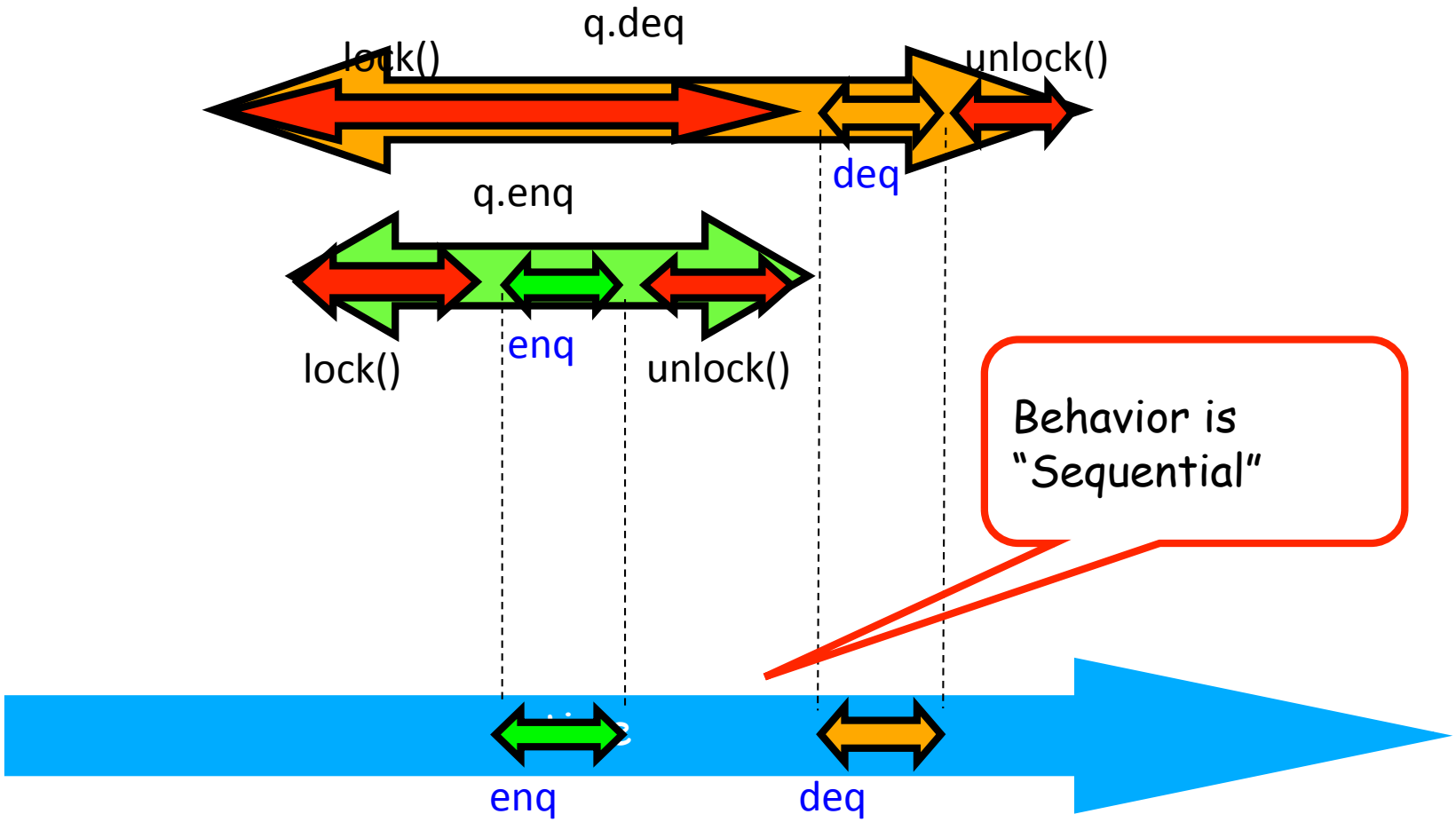

All modifications of queue are done mutually exclusive

Lets capture the idea of describing
the concurrent via the sequential



Linearizability

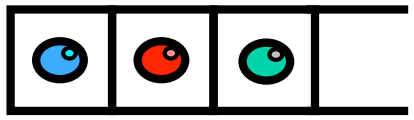
The effect of each method is

- Instantaneous
- Takes place inbetween invocation and response events
- Methods can use locks to achieve this, or not
- Call such an execution linearizable

Linearizable object:

- Each execution is linearizable
- More formal definition to come …

Example



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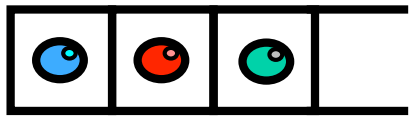

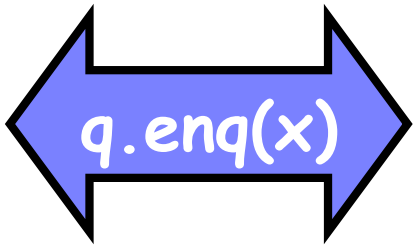

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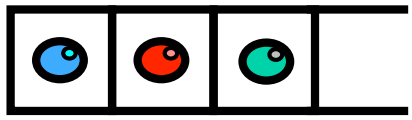

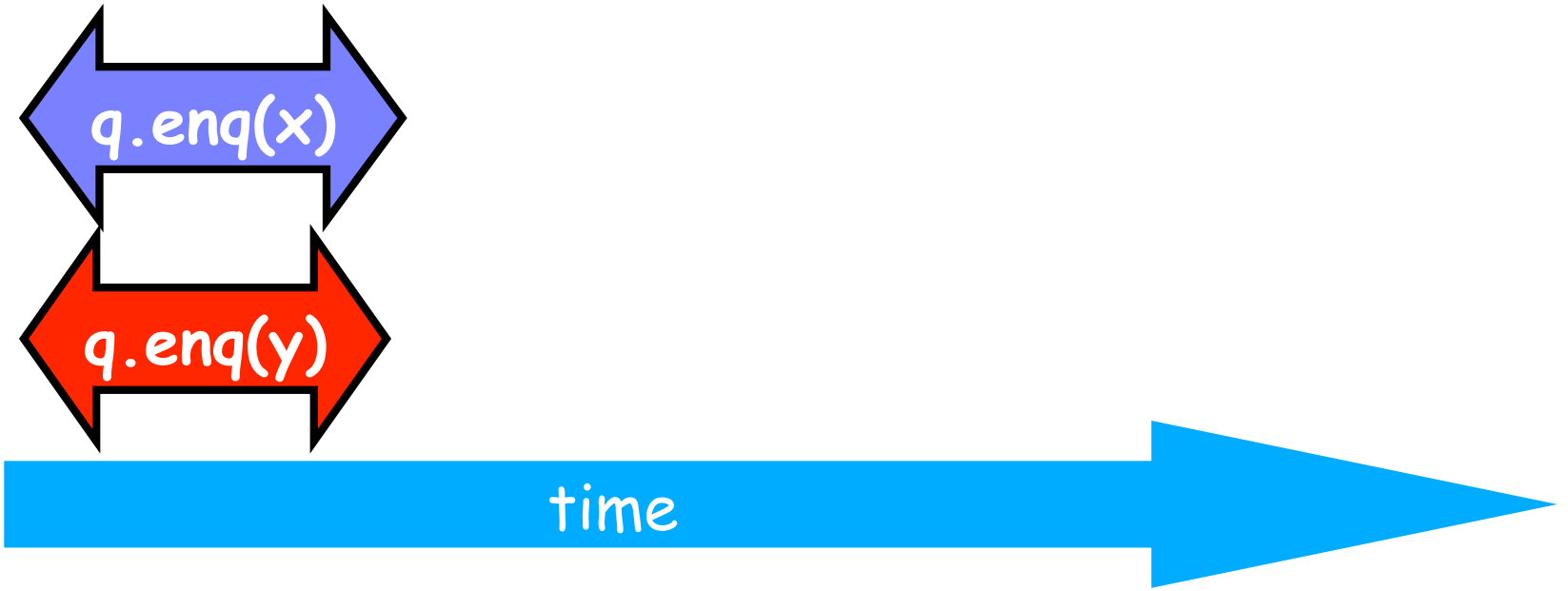

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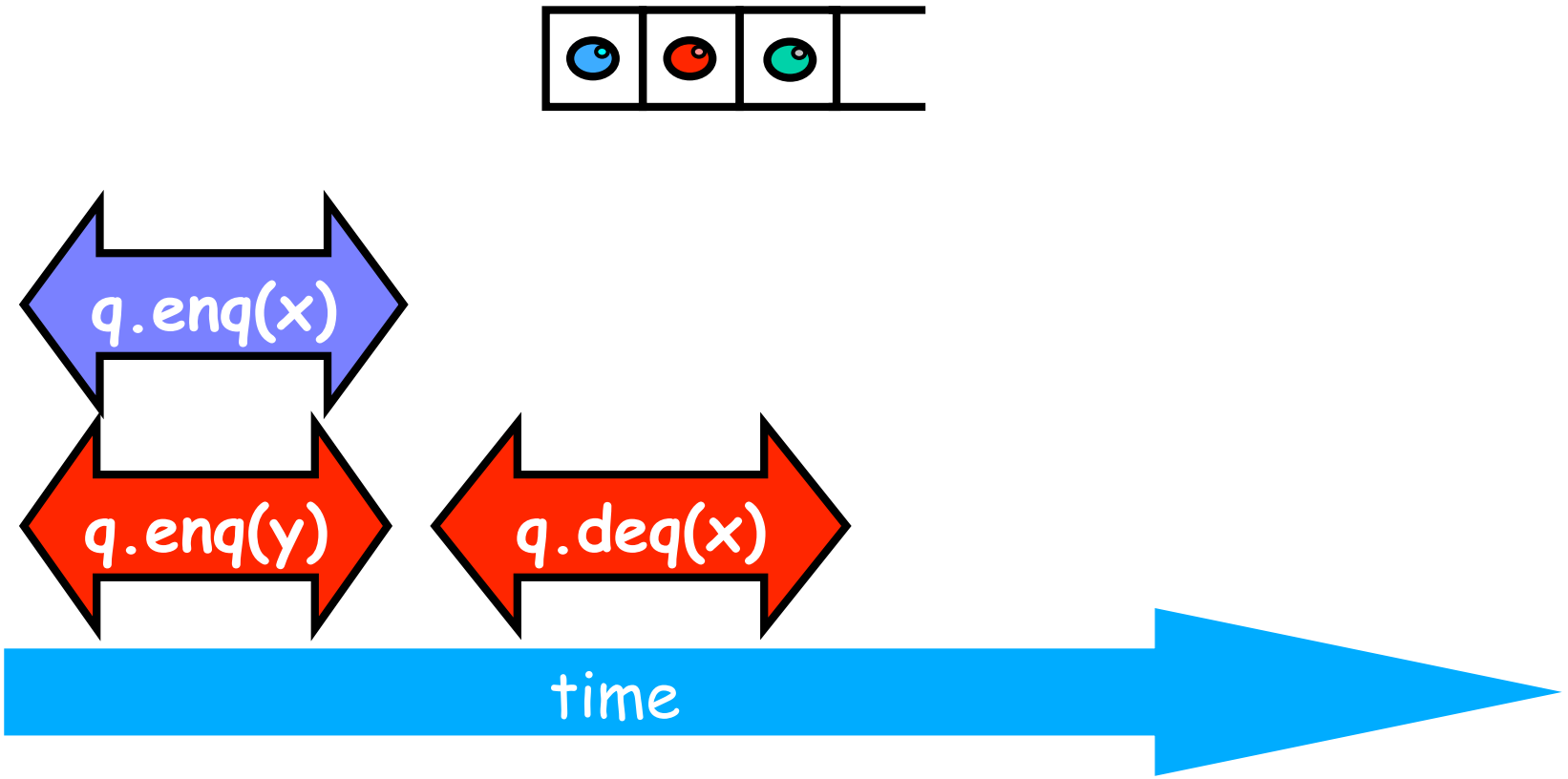

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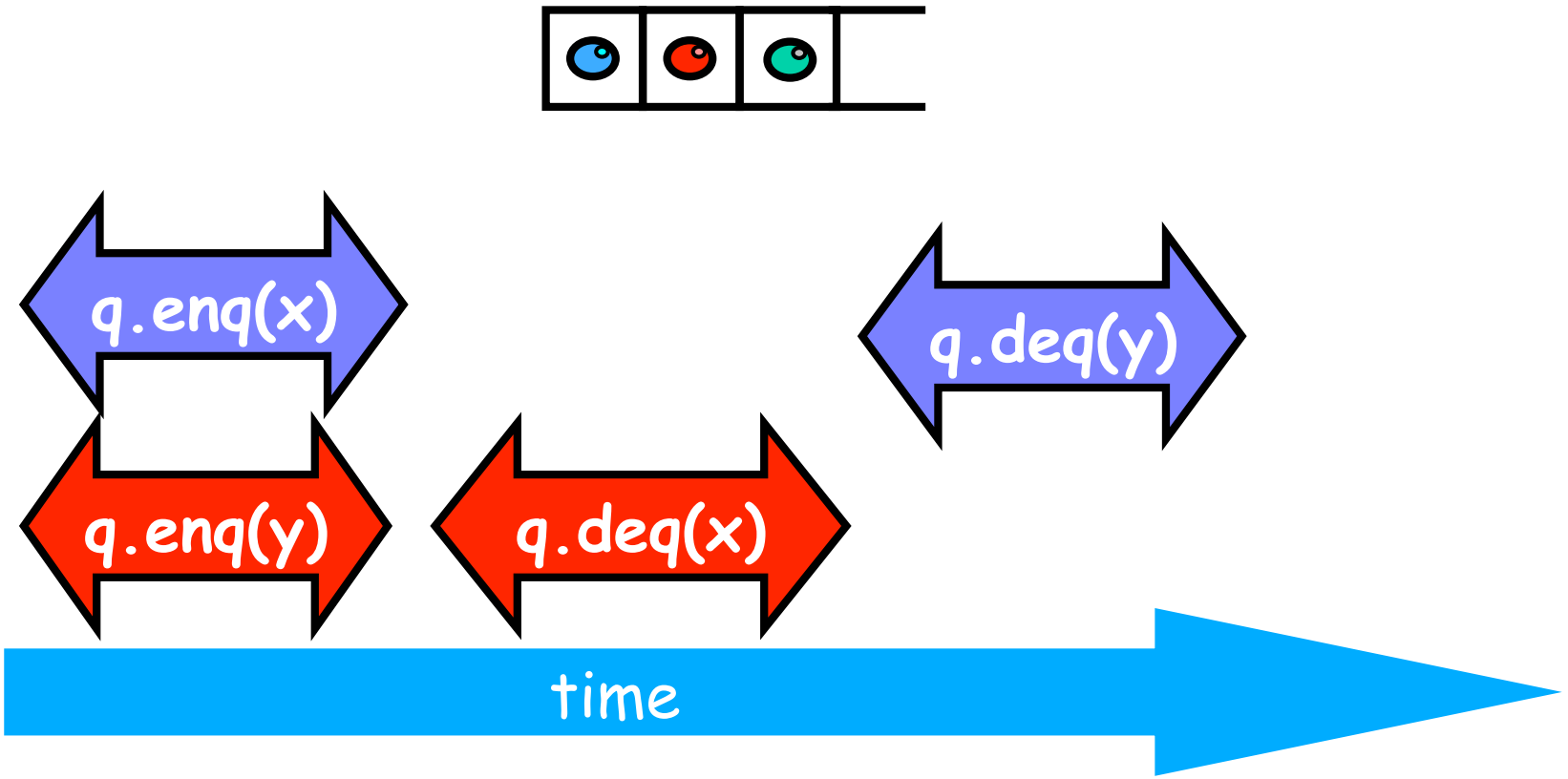

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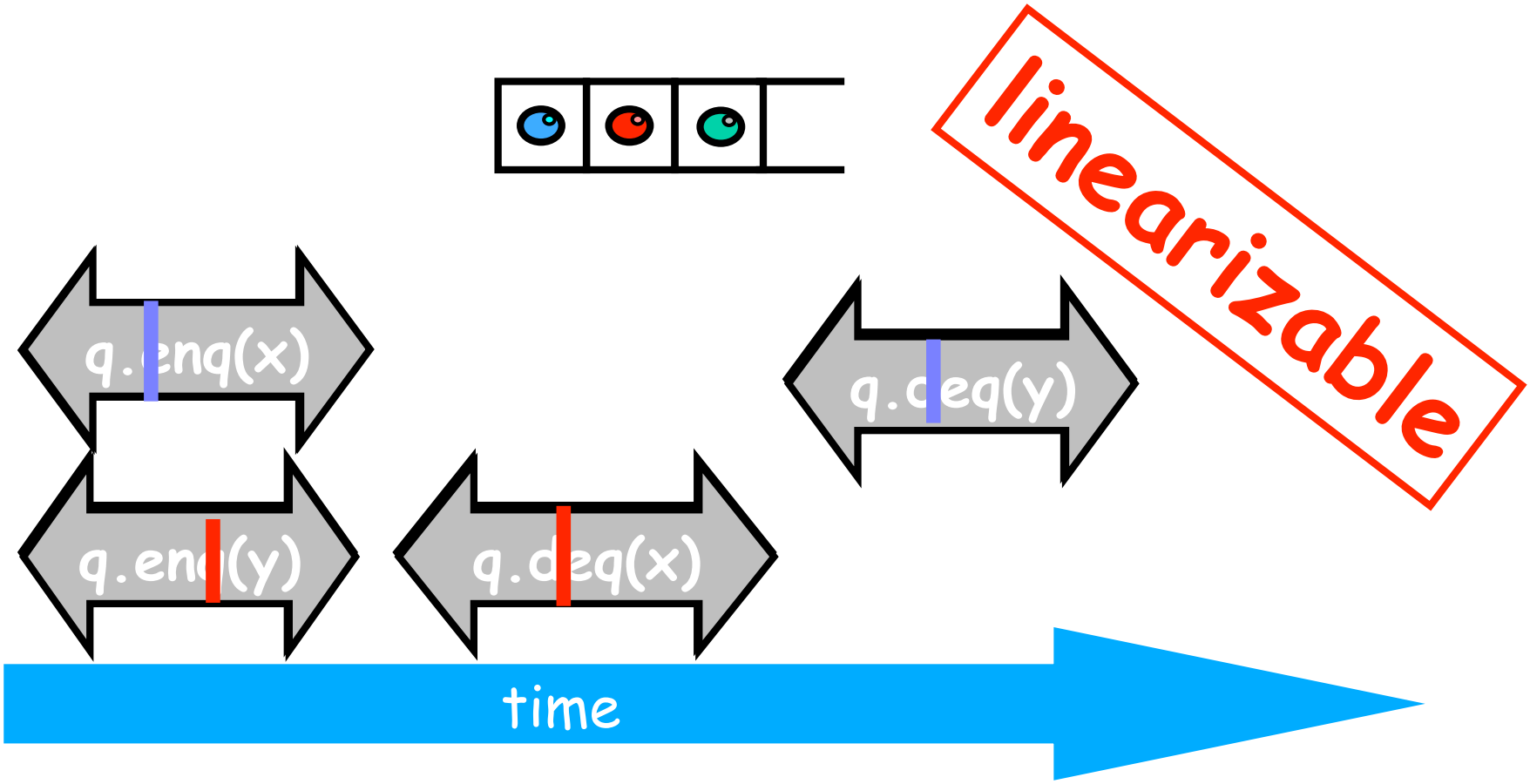

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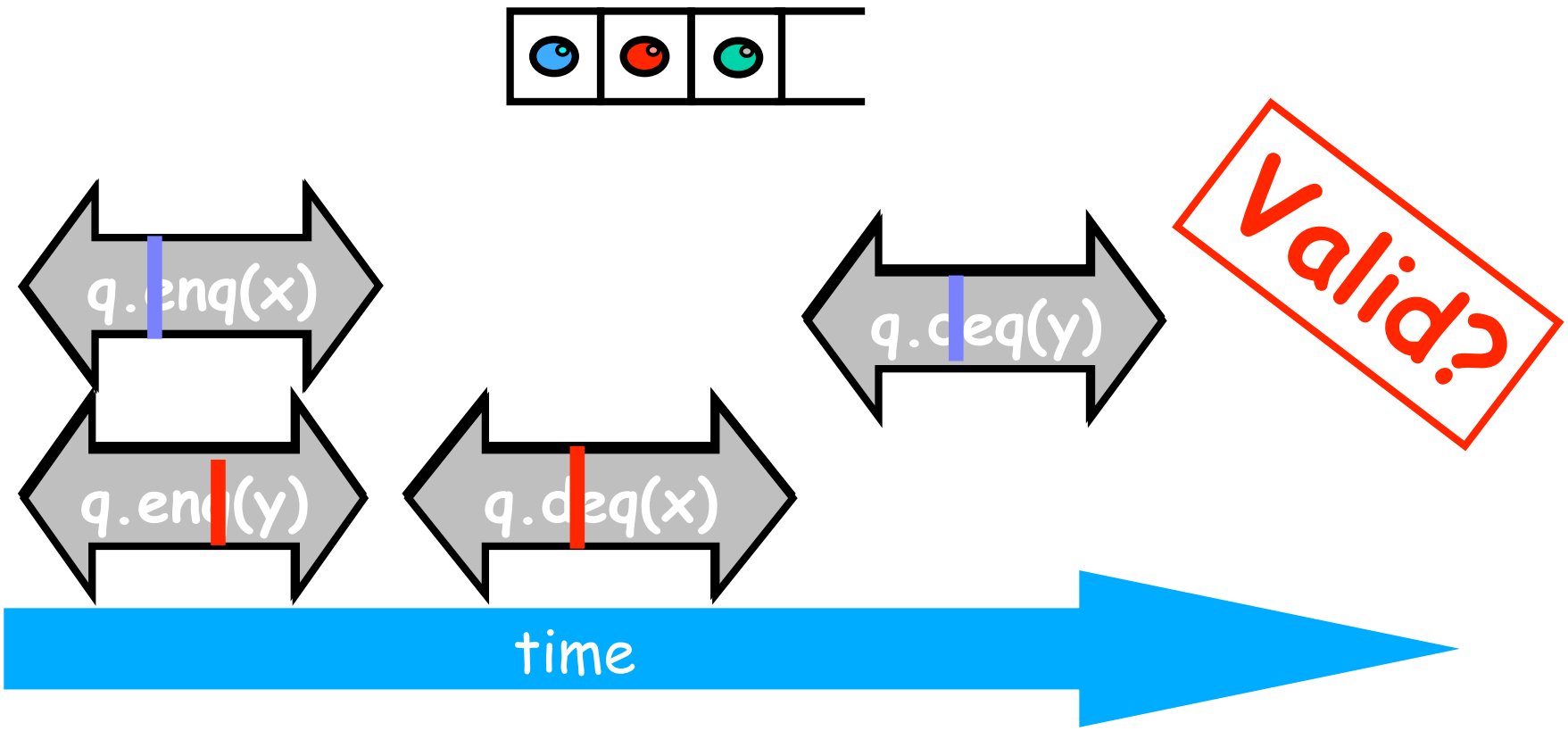

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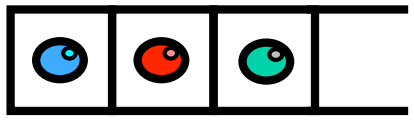

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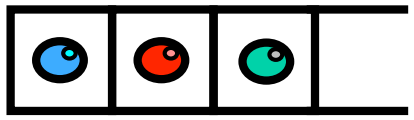

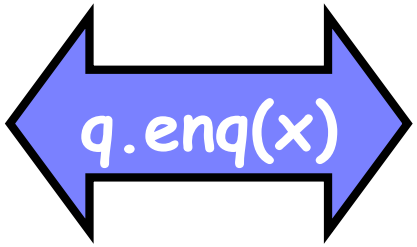

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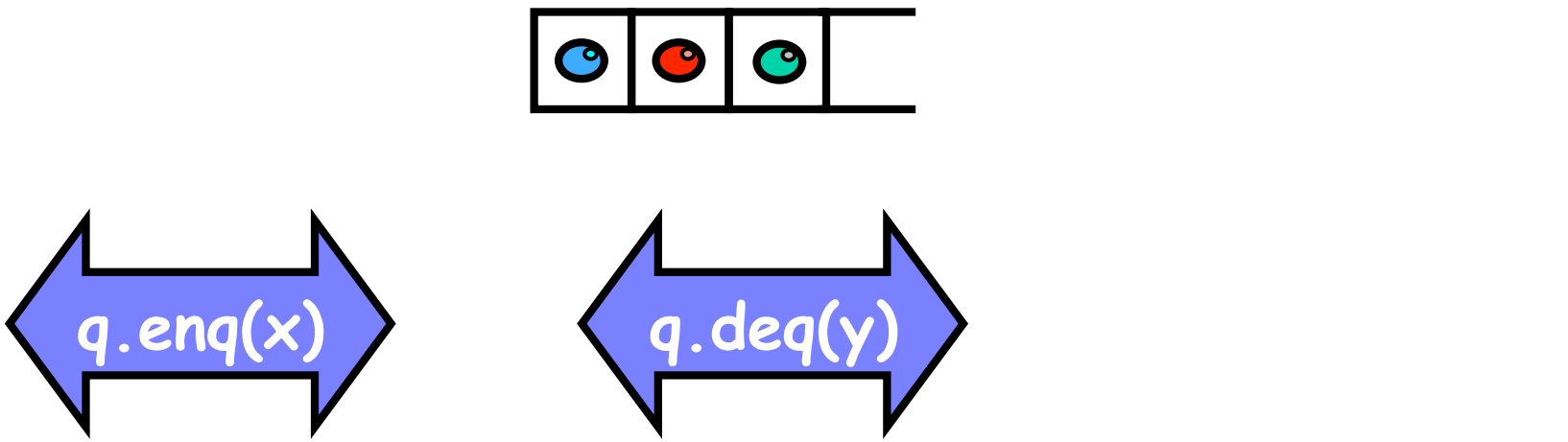

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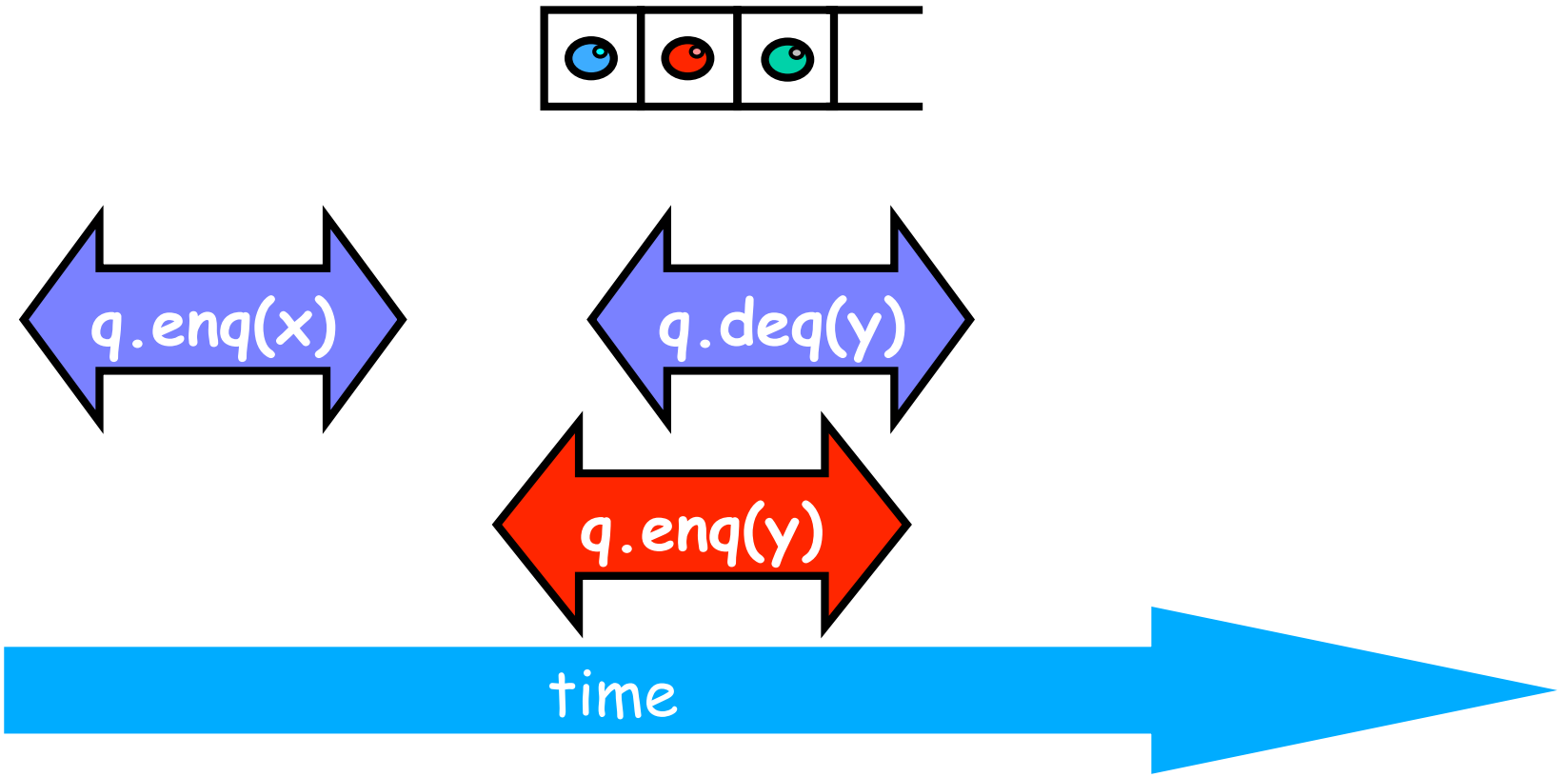

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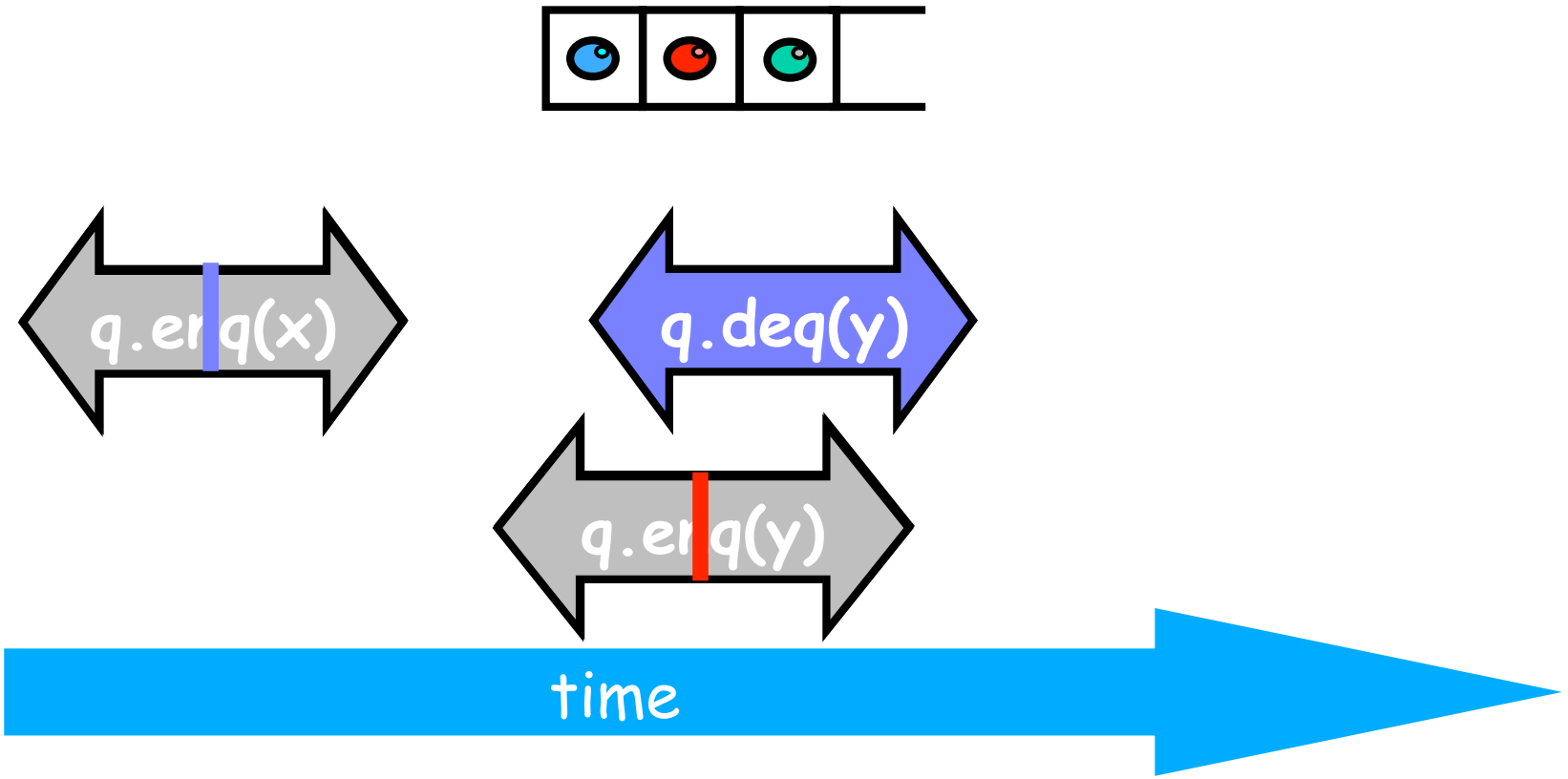

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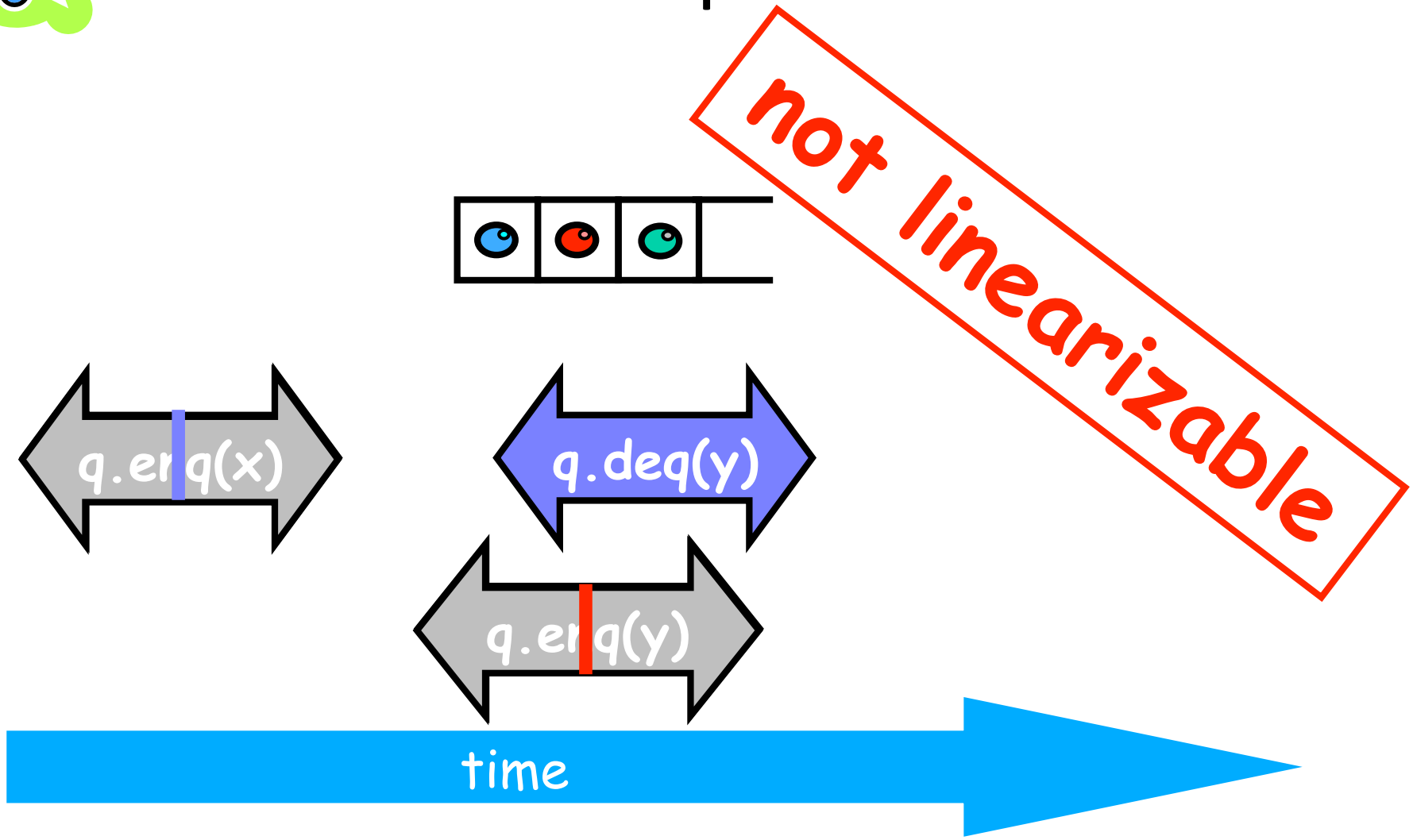

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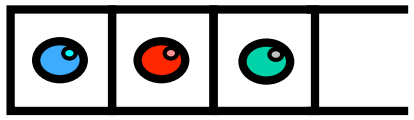

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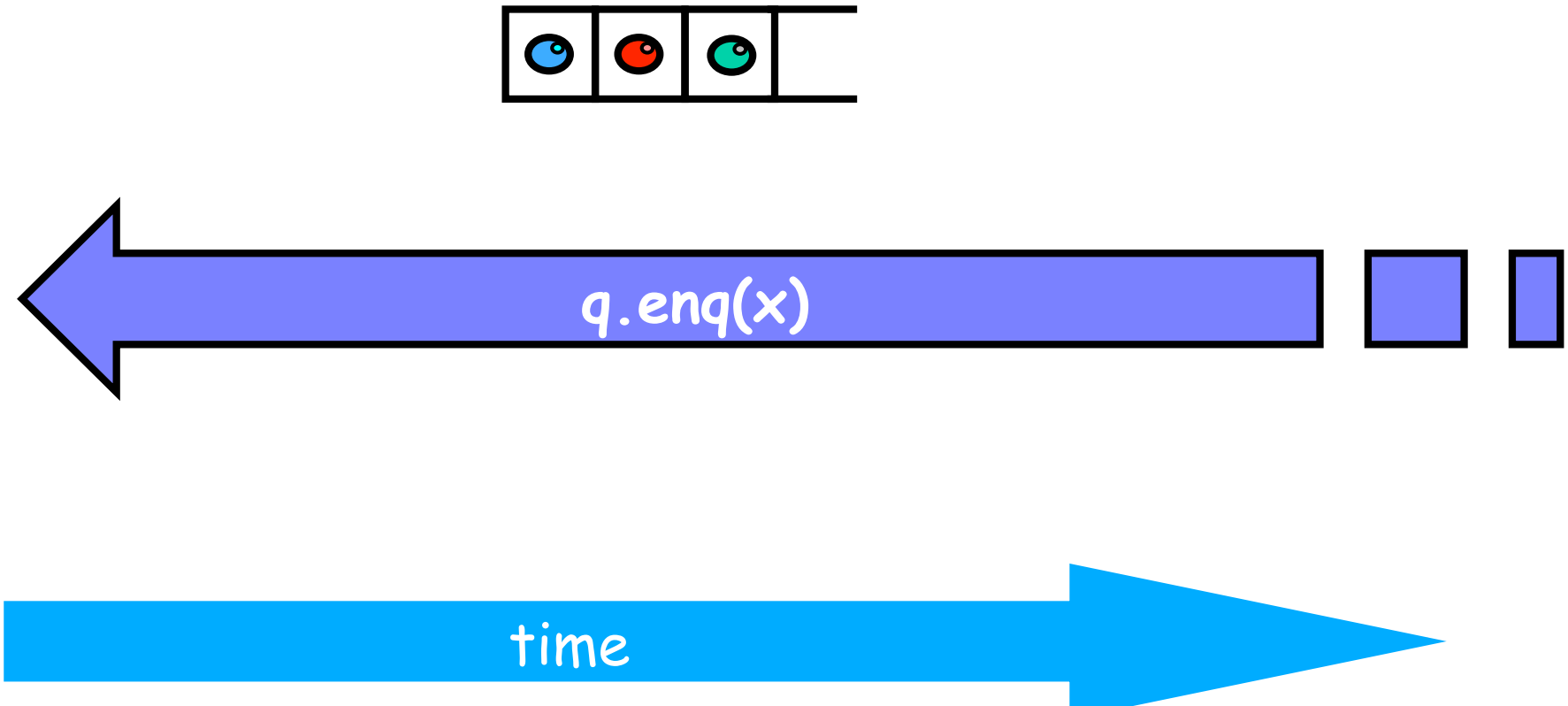

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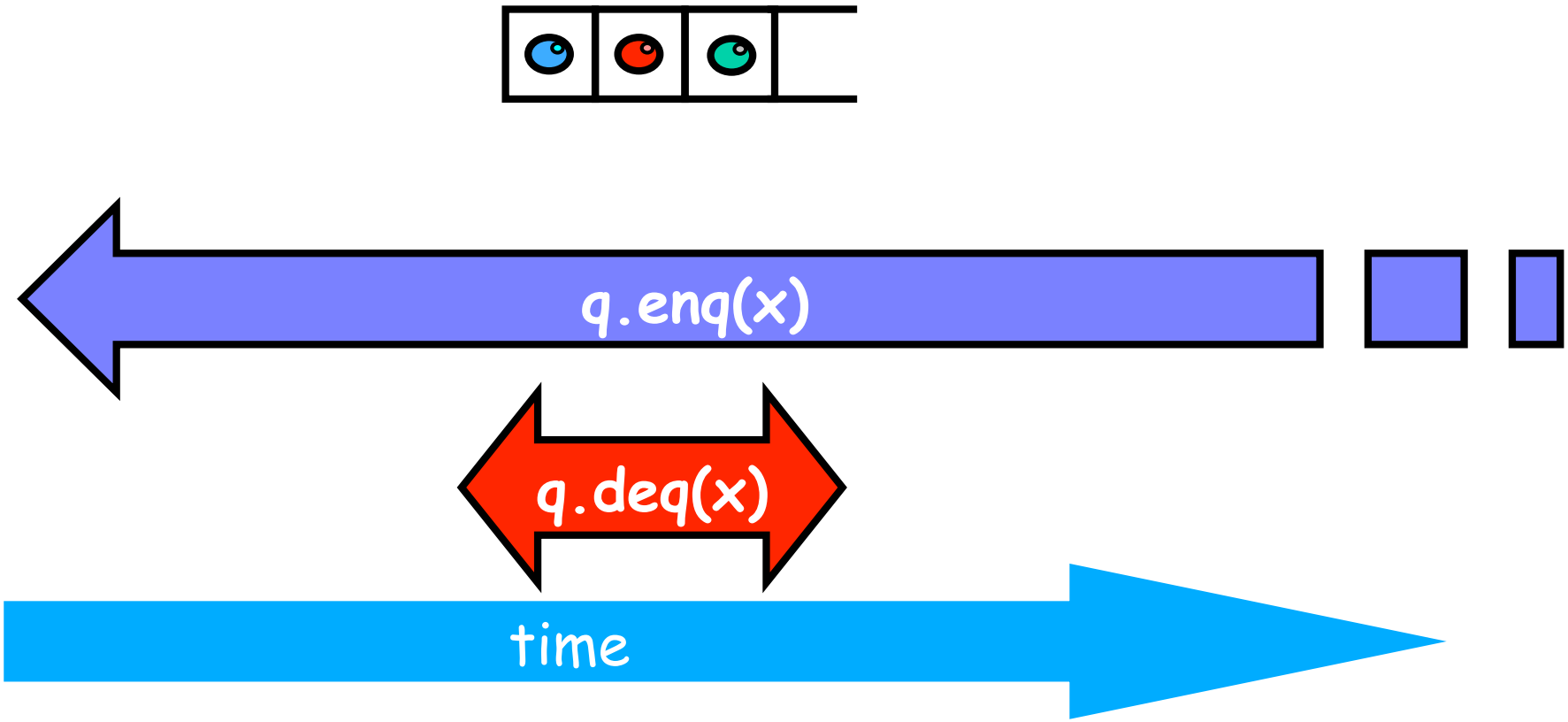

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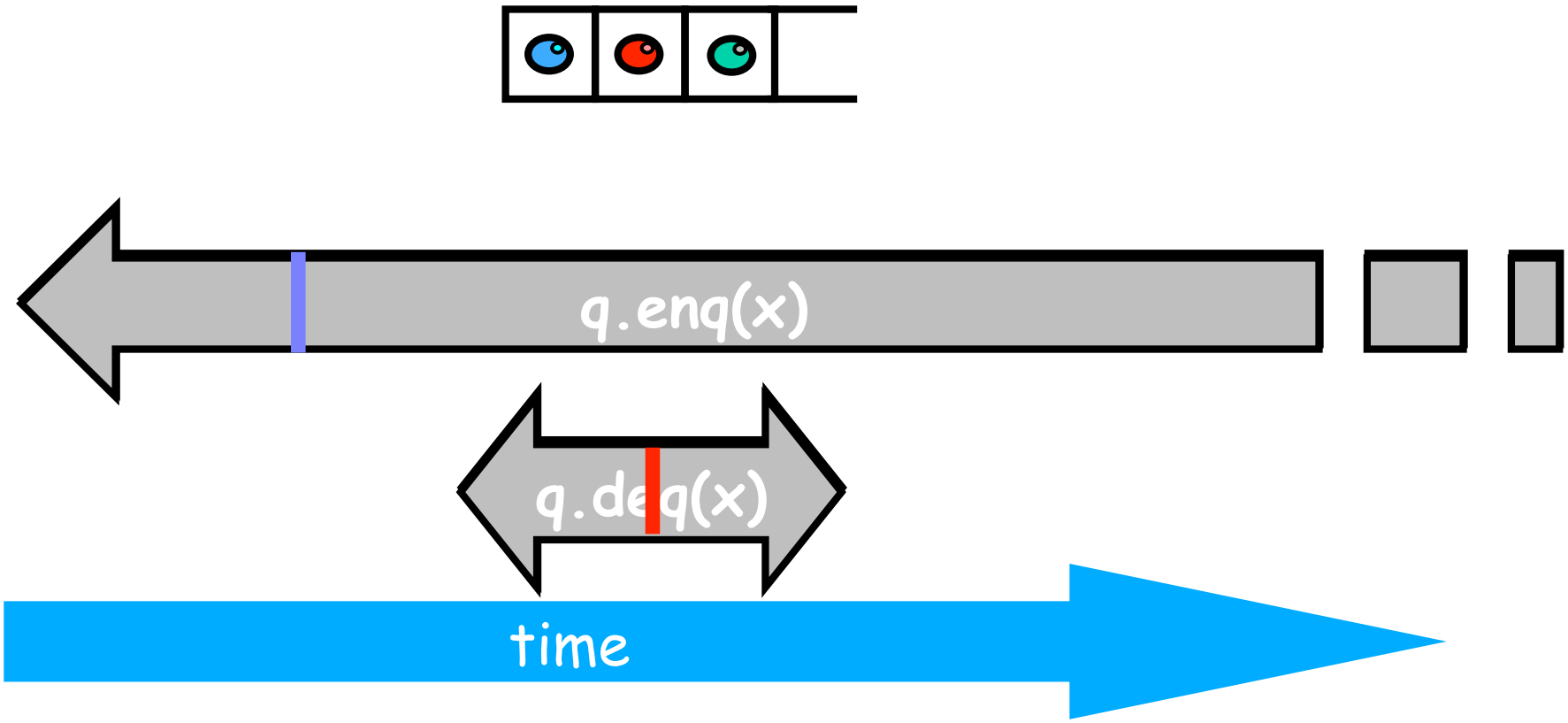

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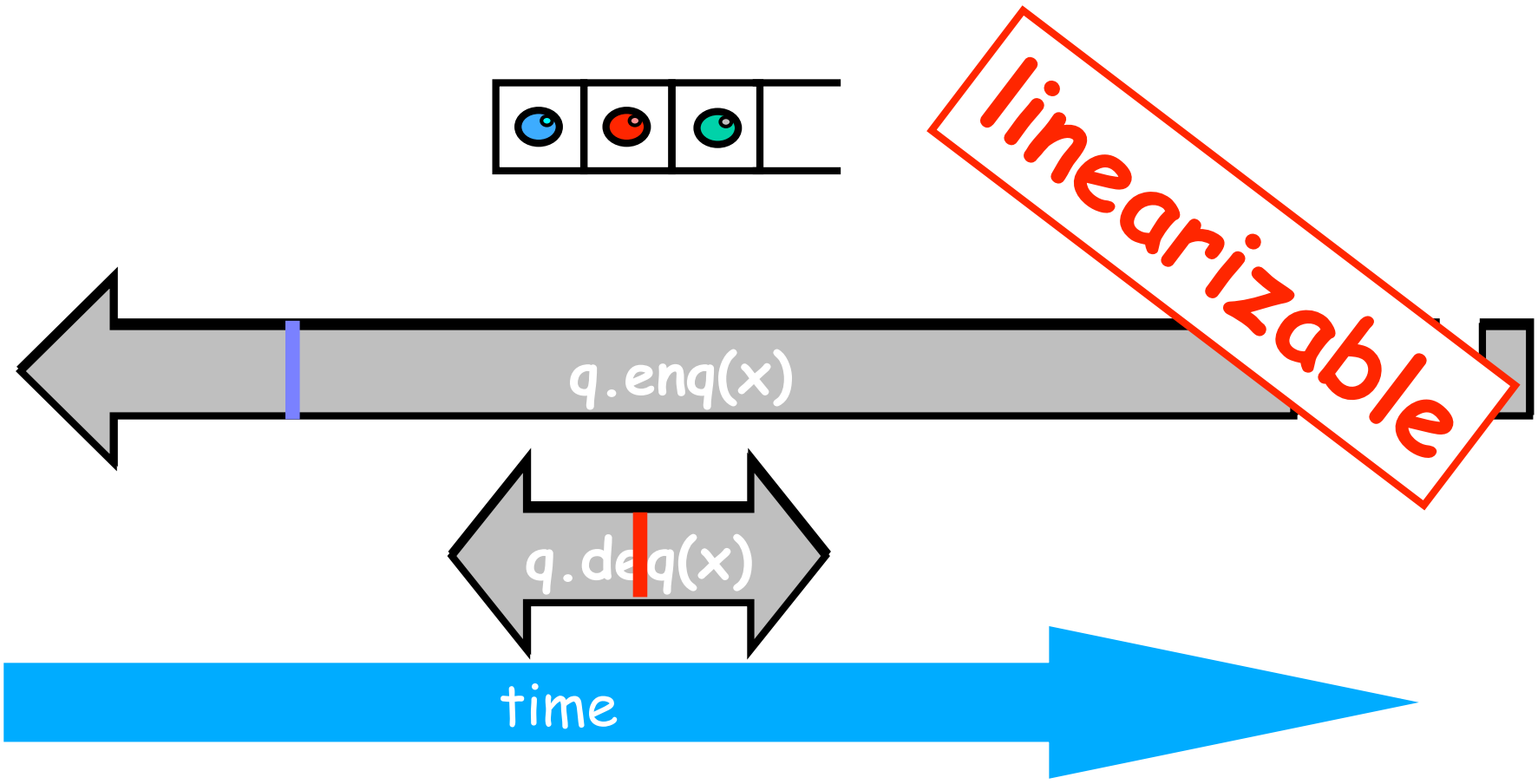

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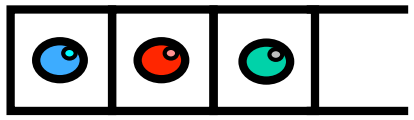

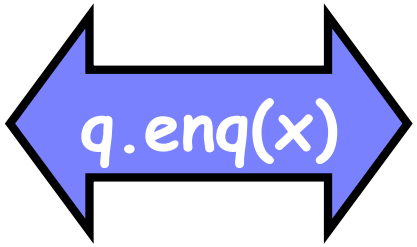

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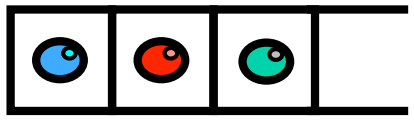

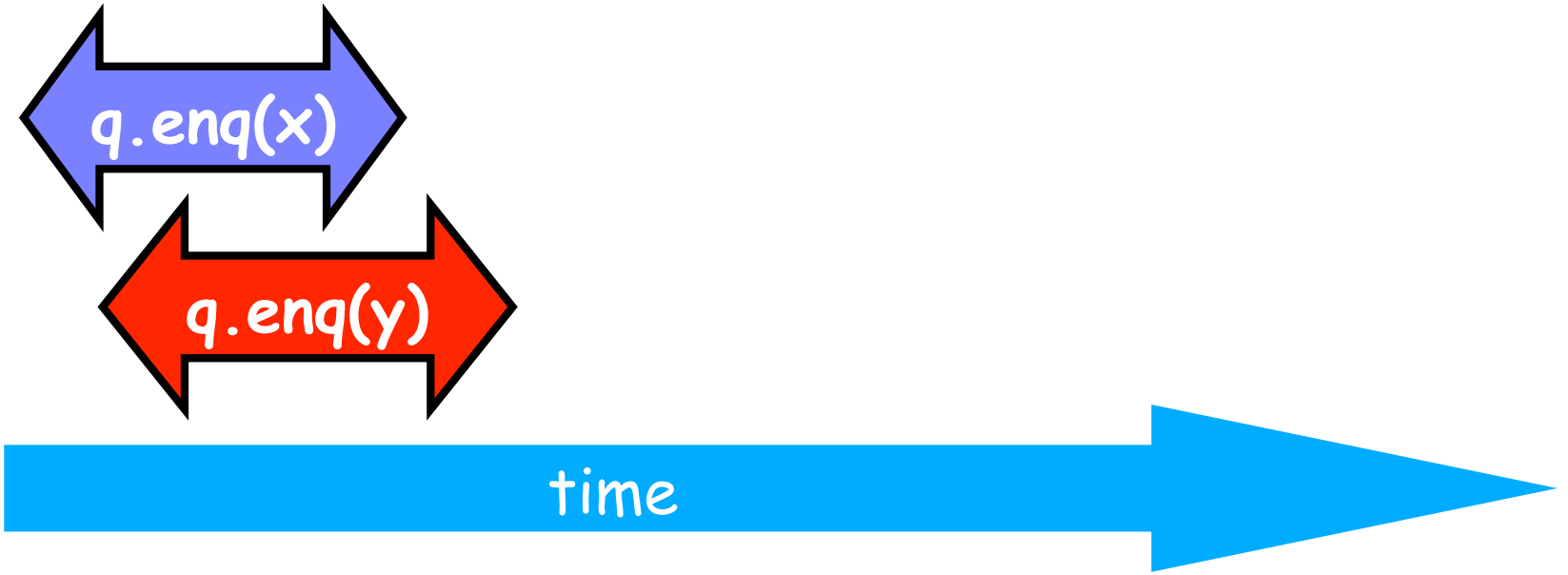

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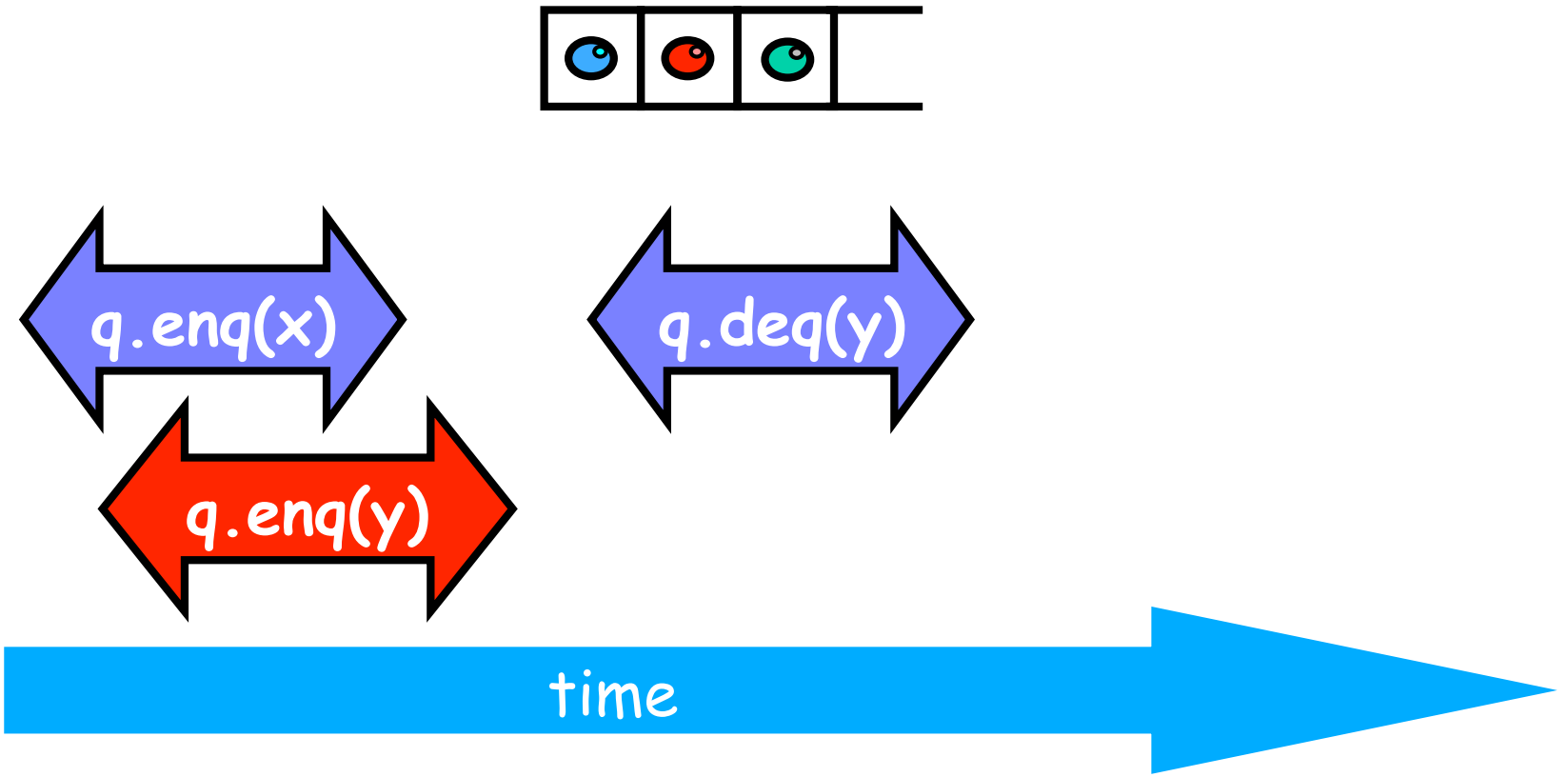

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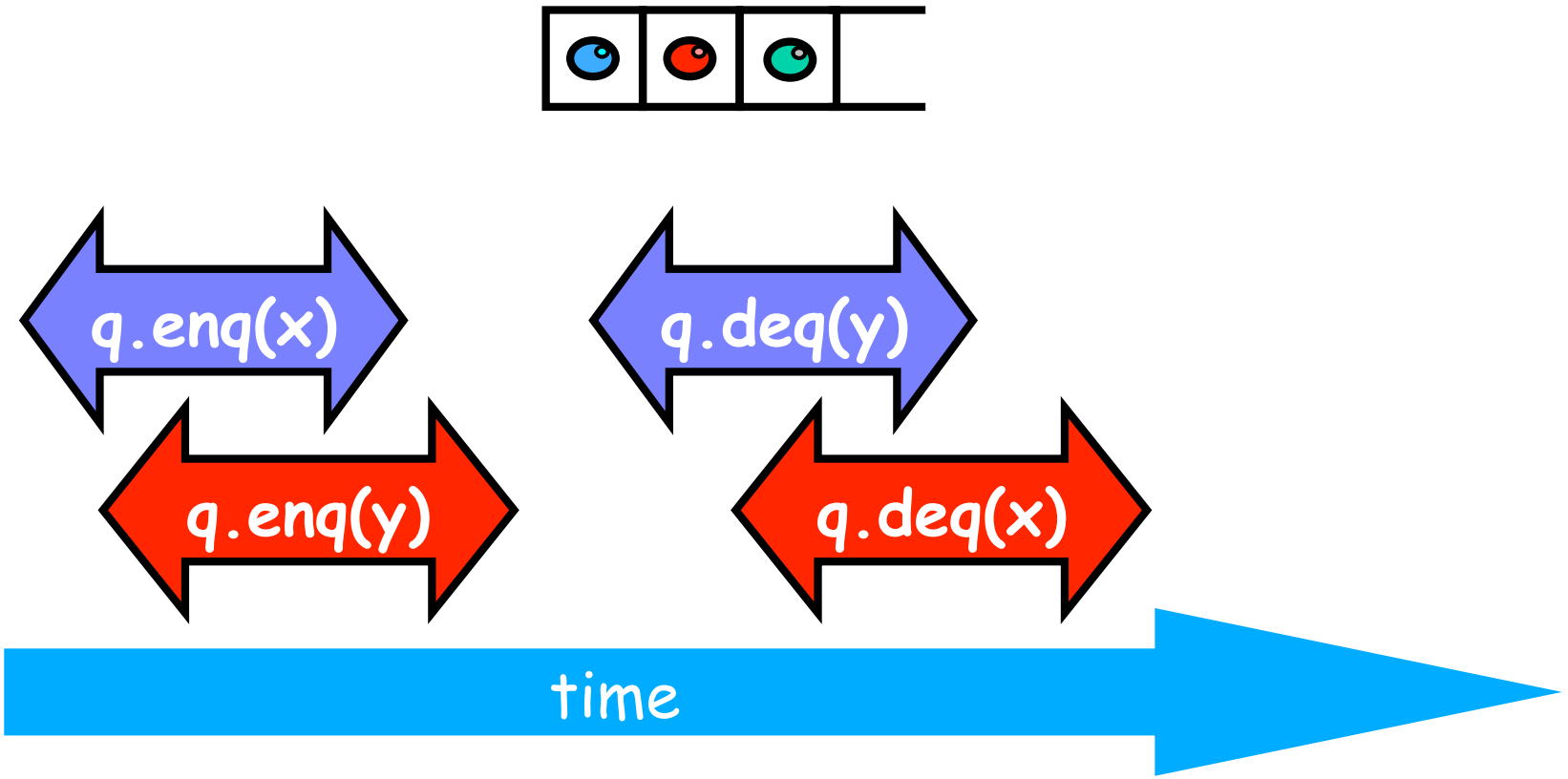

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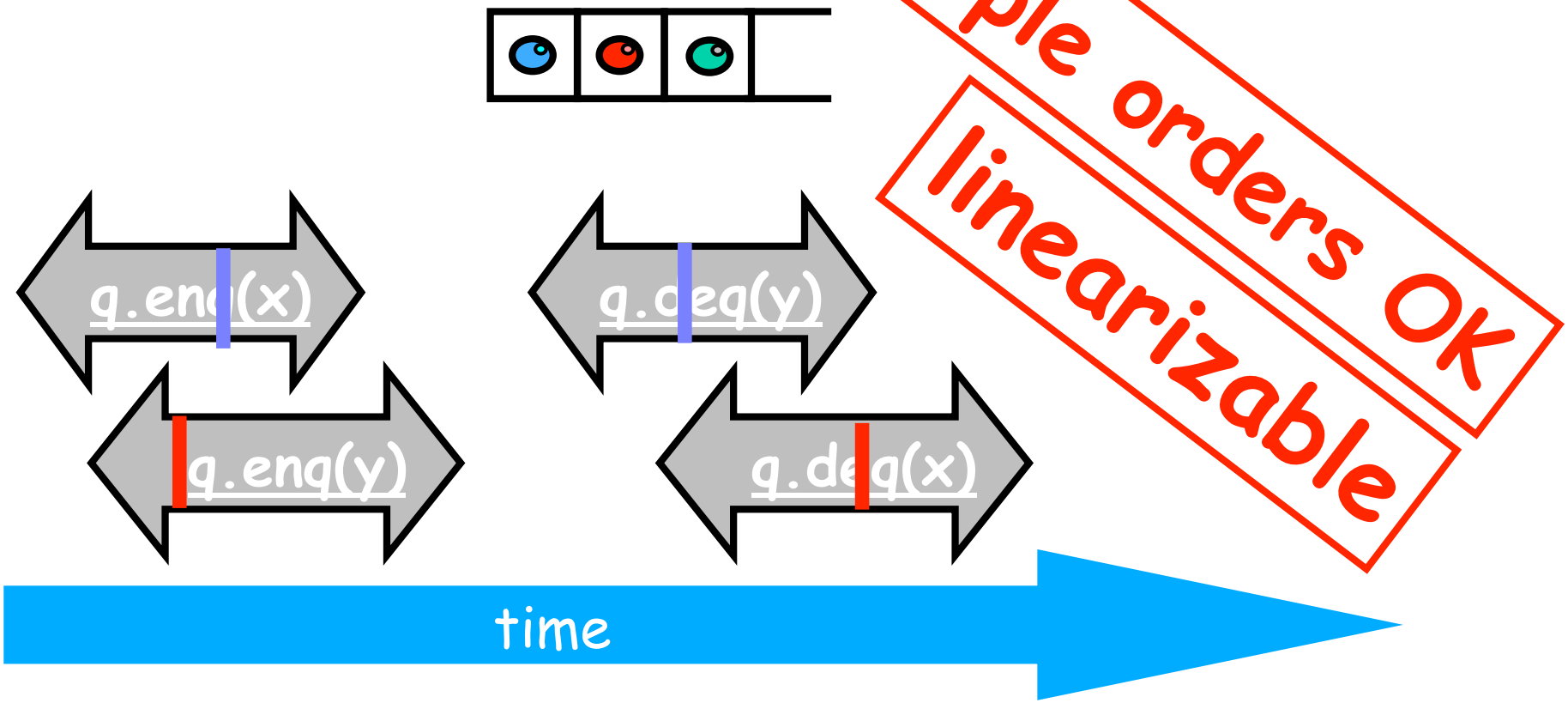

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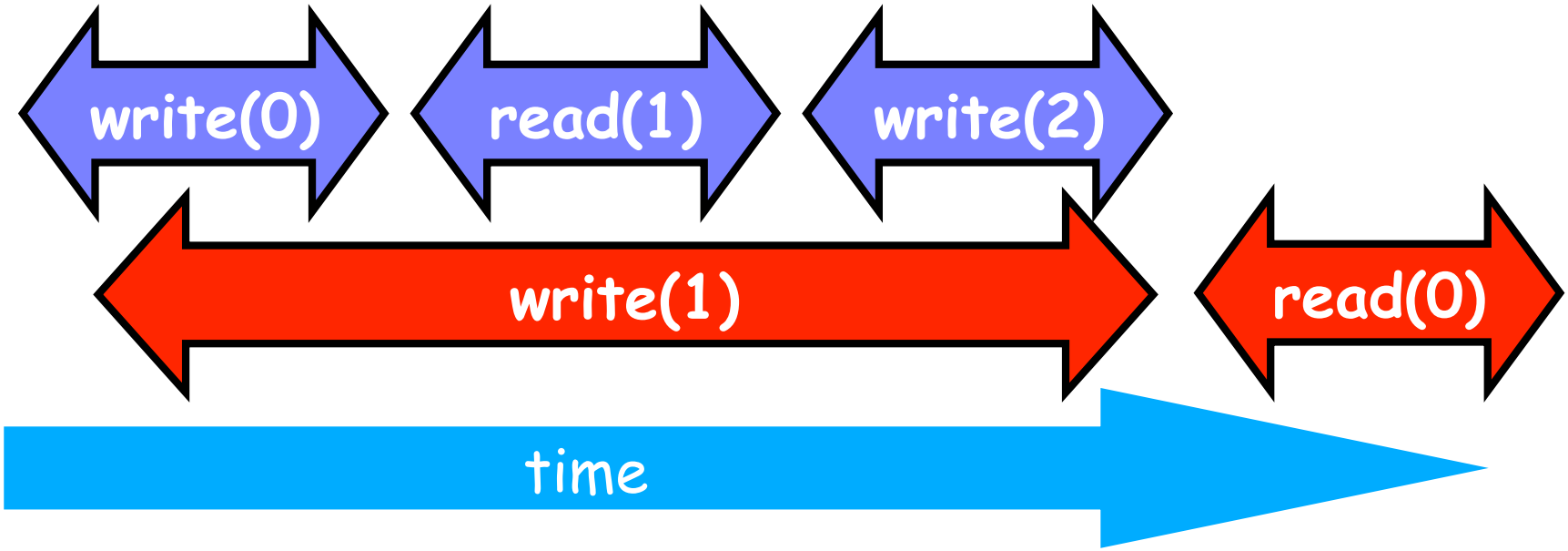

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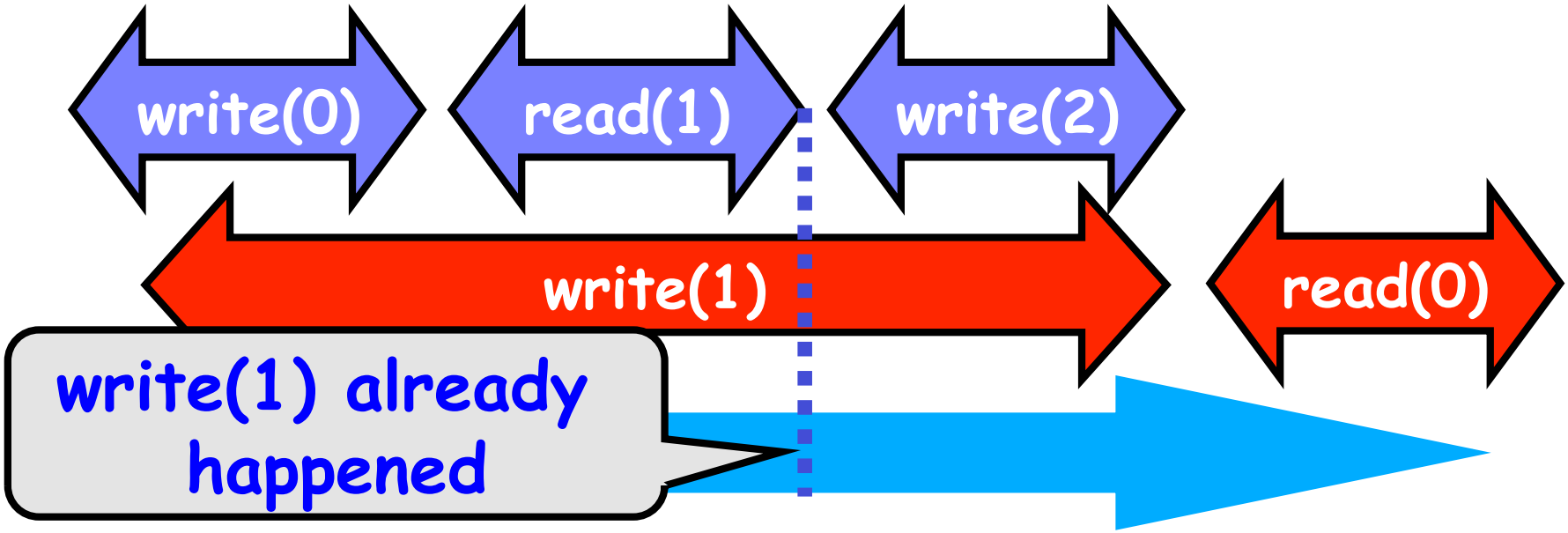

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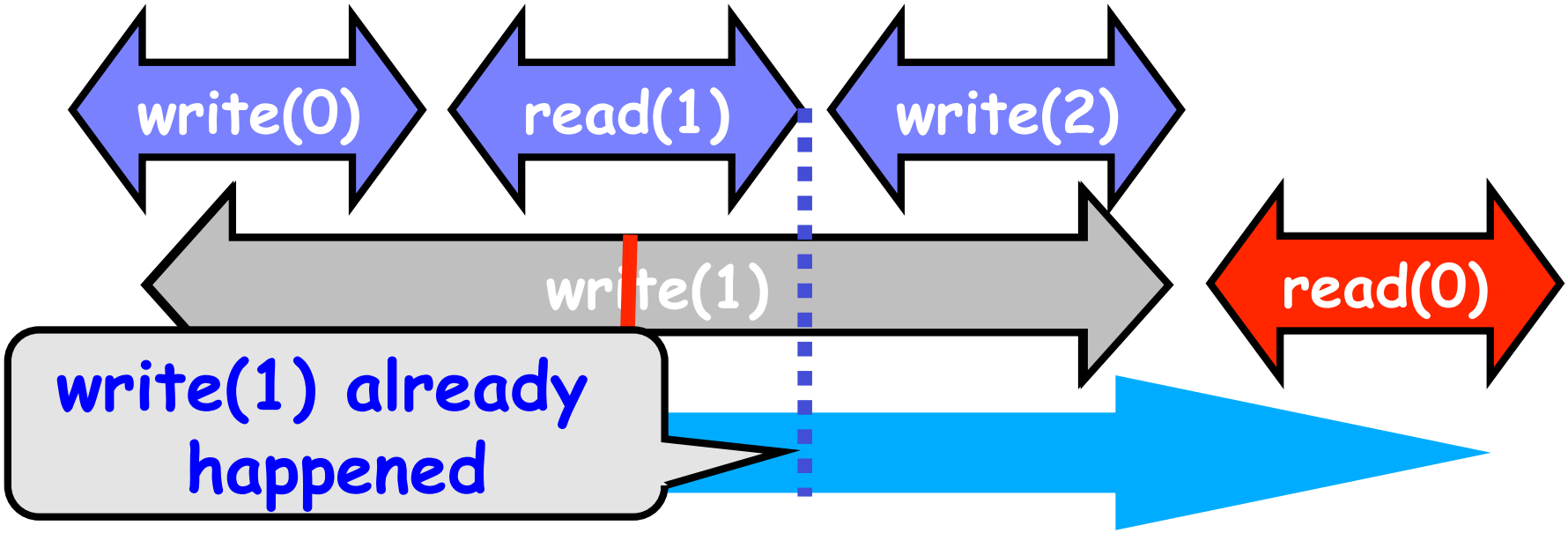

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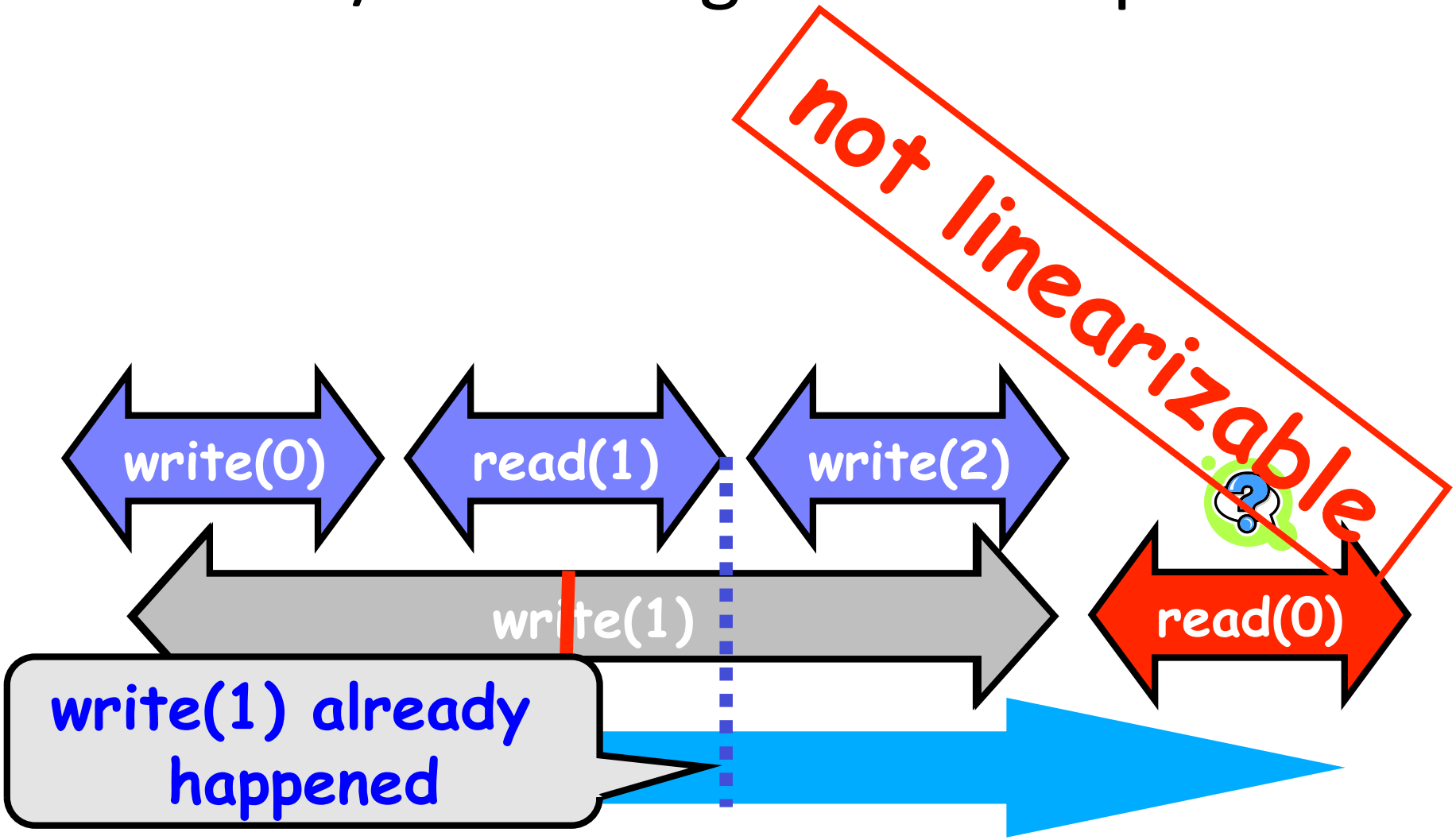

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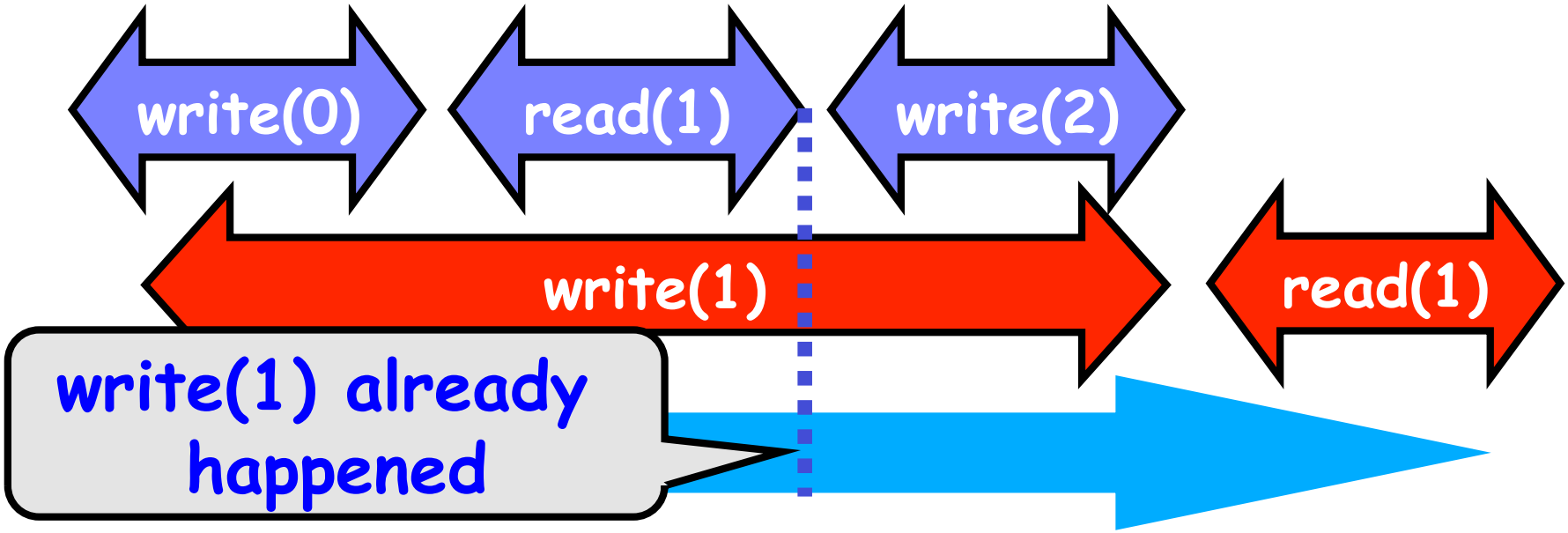

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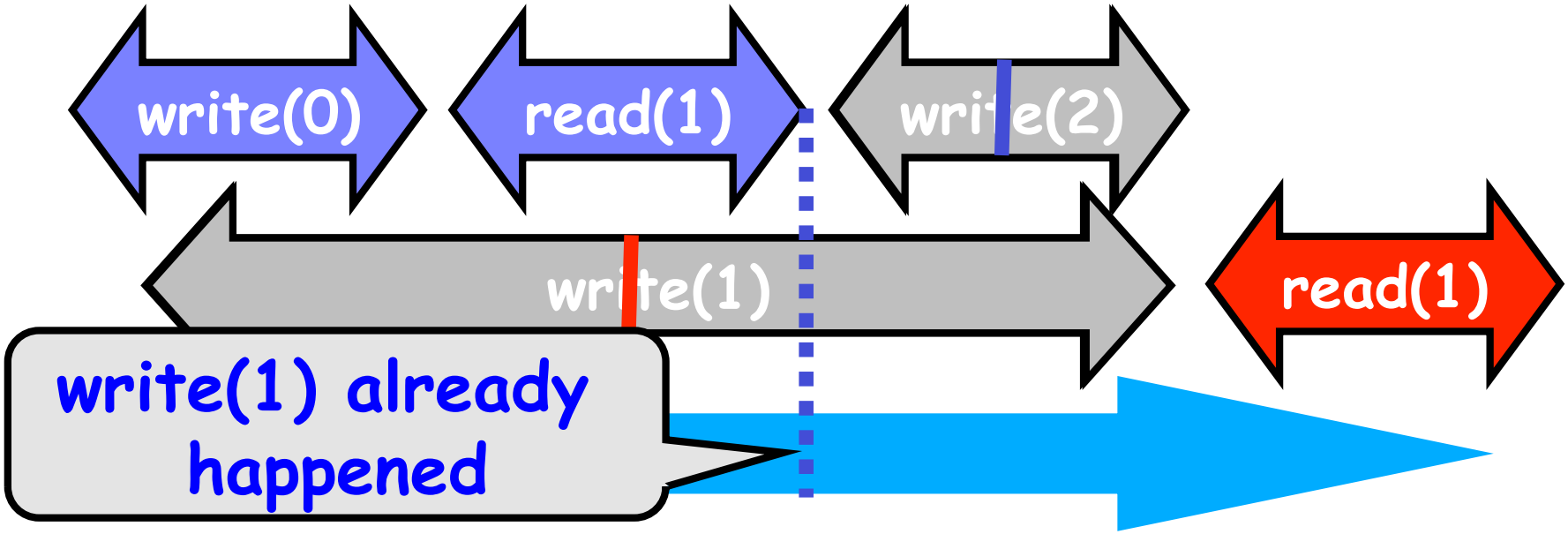

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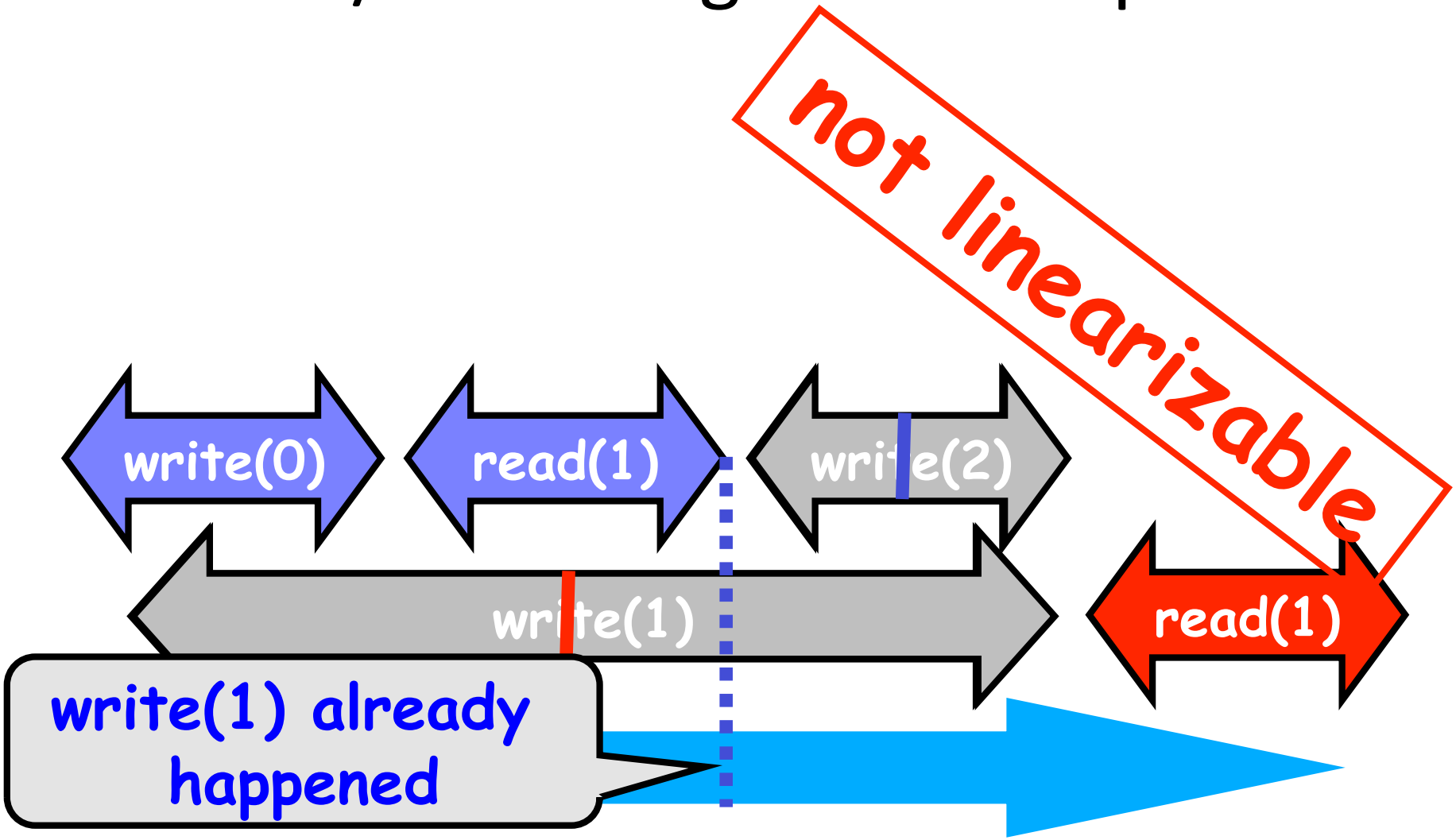

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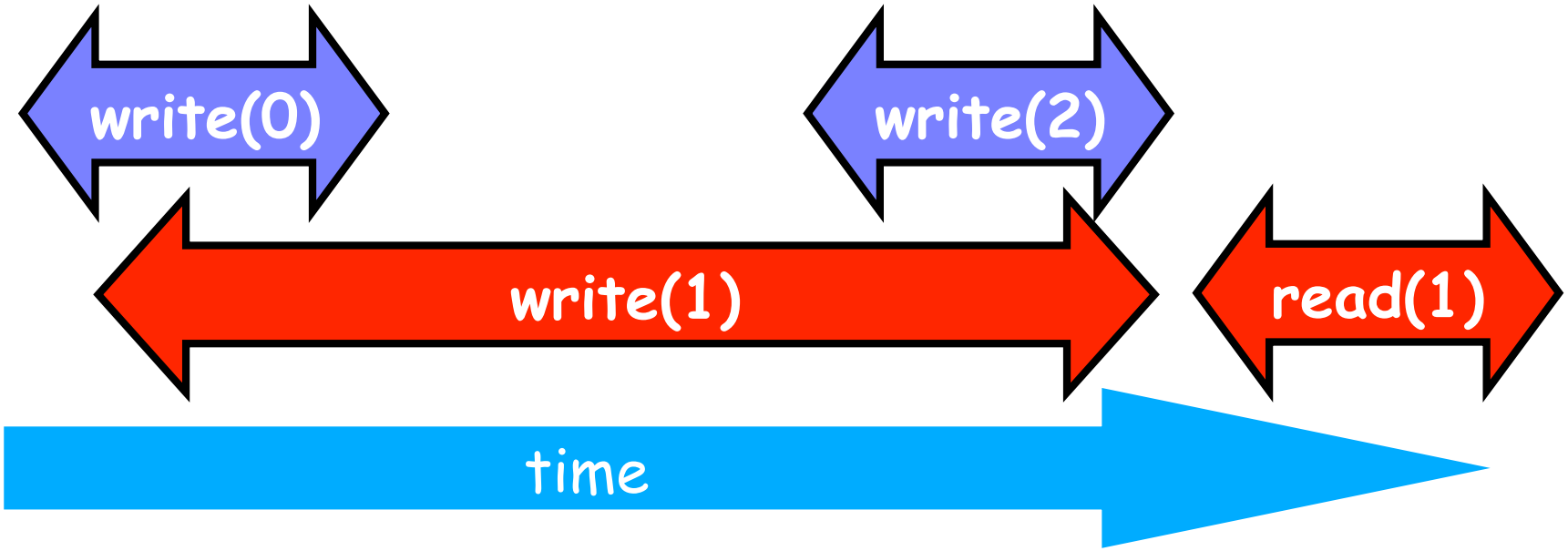

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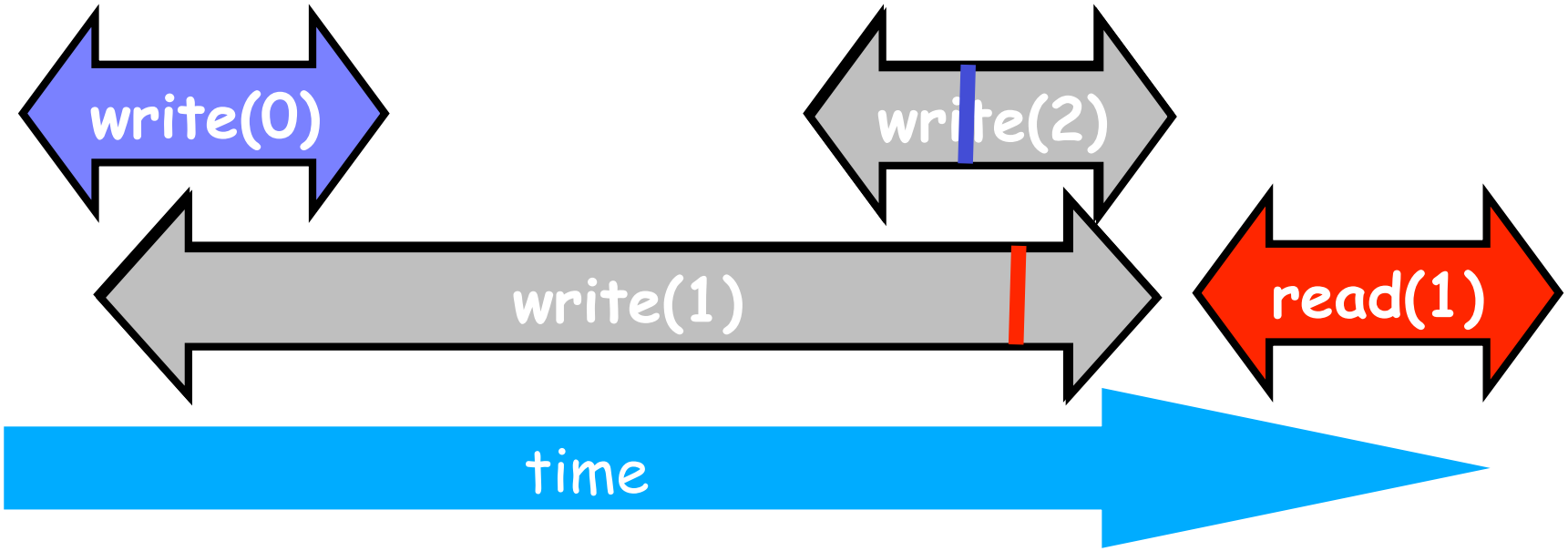

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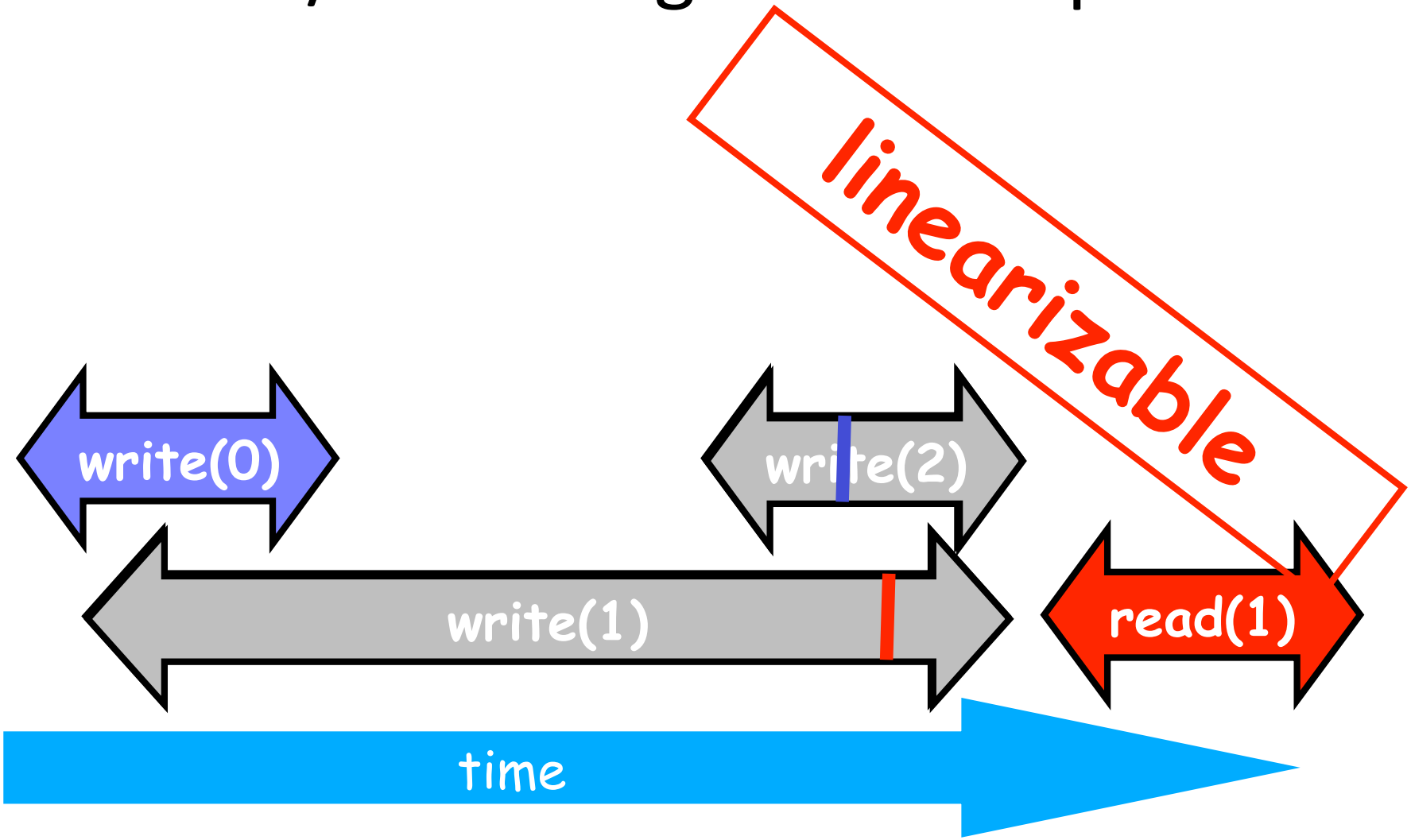
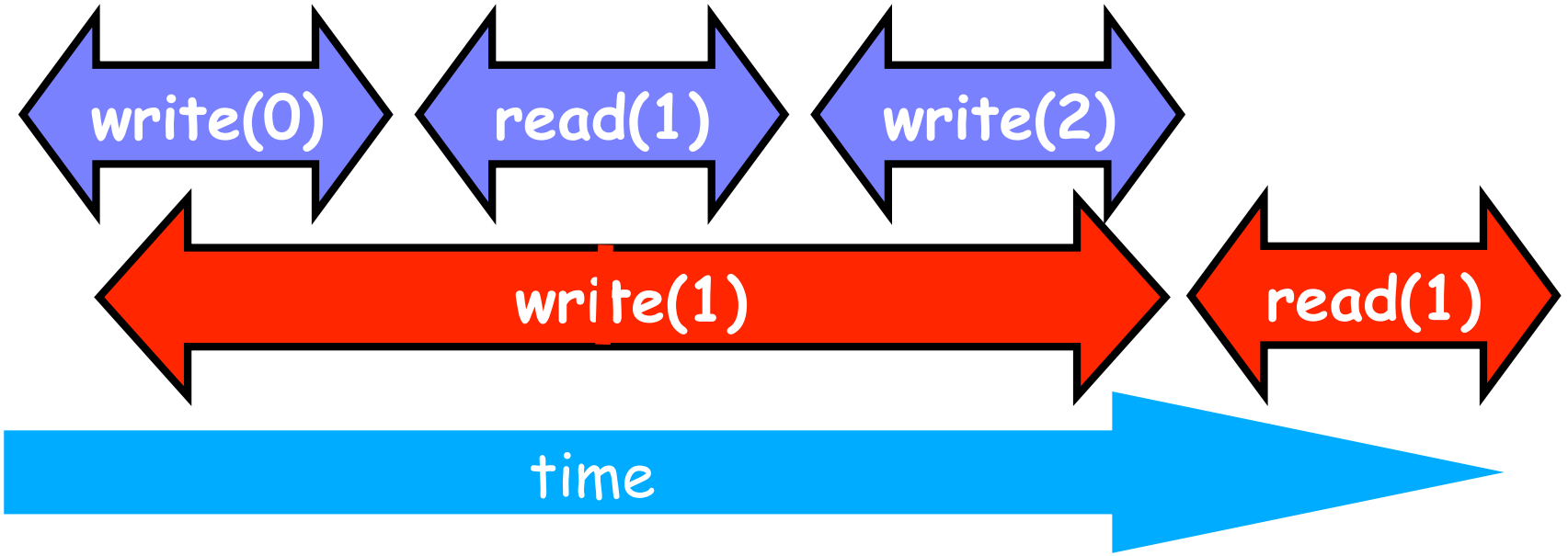
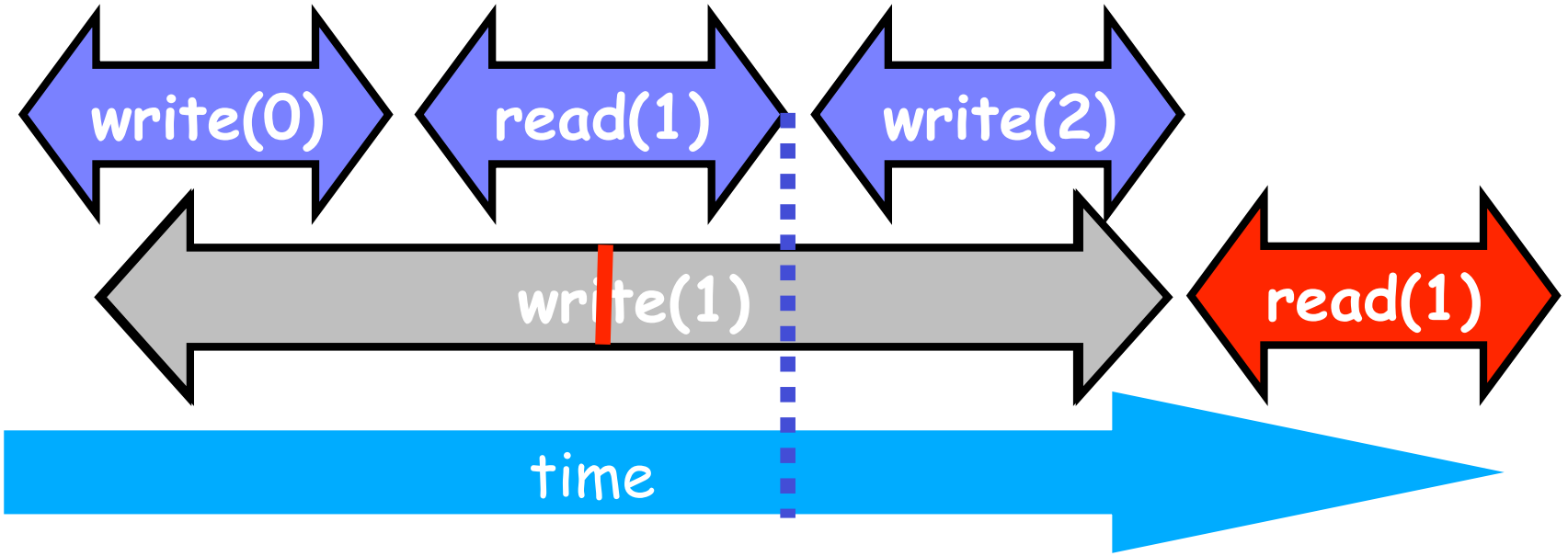


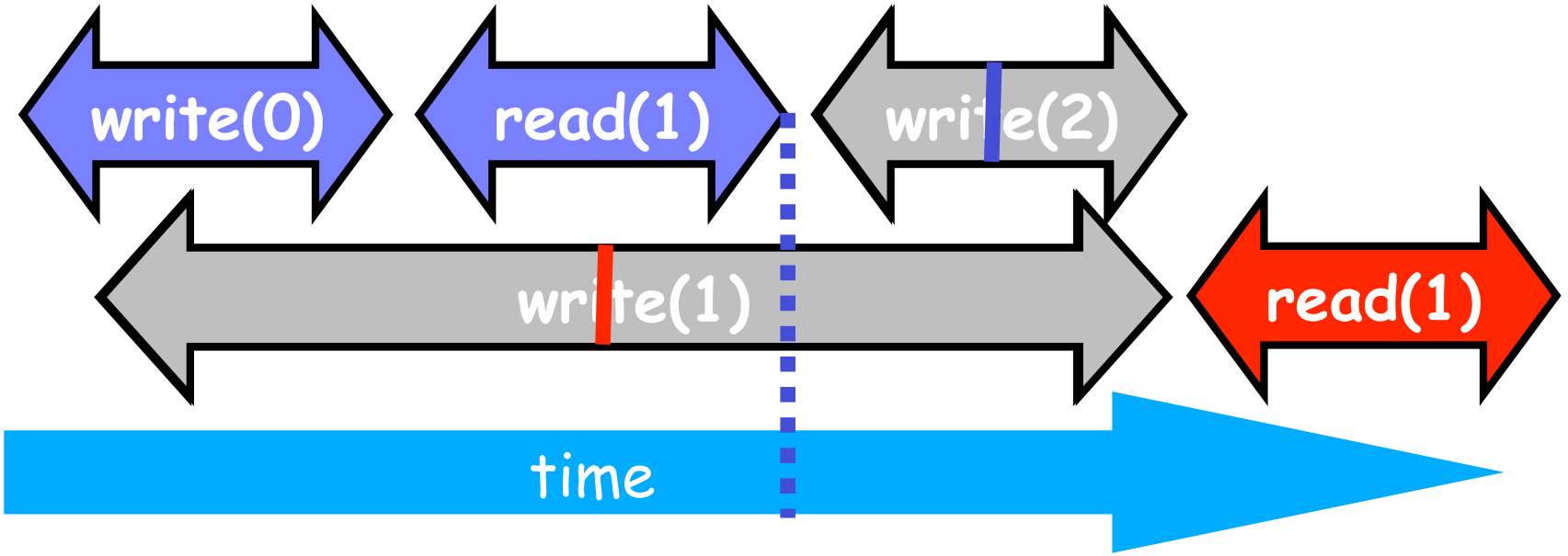
Read/Write Register Example



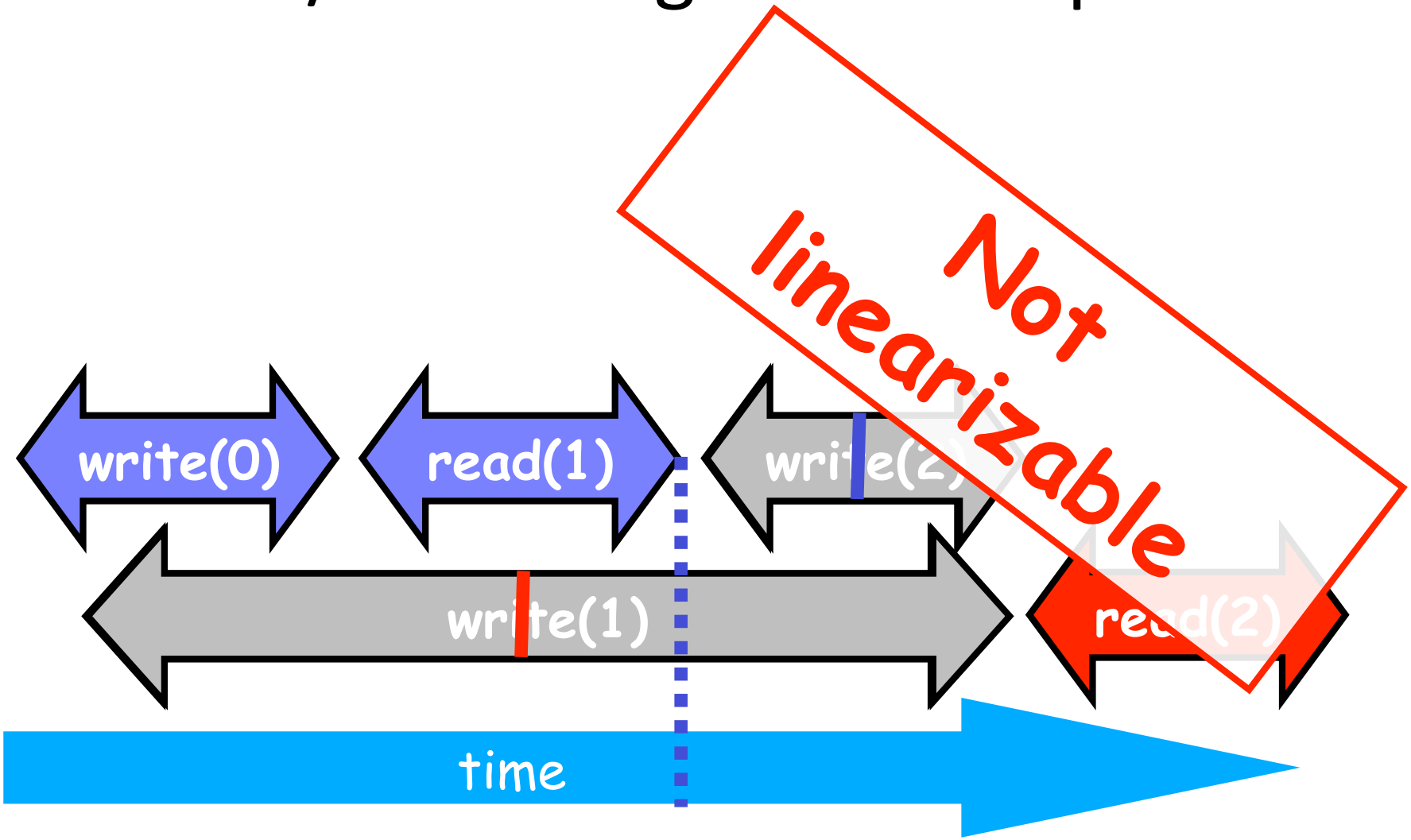
Read/Write Register Example



Read/Write Register Example



Read/Write Register Example



Formal Model

- Goal: Build formal model of computations on shared objects
- So that we pin down concepts precisely
- And can prove properties about them
- Formalization:
  - Can be a goal by itself
  - Here: Tool for understanding
  - Do not go overboard!
- Our terminology slightly different from Herlihy/Shavit's
- You can use either
- Good to be clear about this *when doubt may arise*

States and Executions

Object state:

- Assignment to fields

Thread state:

- Assignment to local variables
- Assignment to program counter, operand stacks and activation record stacks

System state:

- State of all objects and threads

Execution:

- Sequences of system/thread states
- Can be finite or infinite – only finite for now
- Produced by running the program

Example

Initial state: head = 0, tail = 0, items array allocated + initialized

Producer and consumer threads initialized, program counters at initial states

Execution: Sequence of transitions from state to state

- Control in producer or consumer thread
- Inside or outside enq/deq methods

```
public class WaitFreeQueue {
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];

  public void enq(Item x) {
    while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     while (tail == head);      // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```

Histories/Traces

Executions produce histories – or traces – of events

How – next slides

Here:

behaviour(program *P*)

=

{ traces *t* | exists execution of *P* such that

t = trace(execution) }

Specification *Spec* = set of traces

Correctness of program *P*: *R*(behaviour(*P*),*Spec*)

Question is: What is *R*?

Events

This lecture: Events = method calls and returns

Call event – method invocation:

$A: x.m(a_1,\ldots,a_n)$

thread object method arguments

Normal return event:

$A: x.\text{ok}(r_1,\ldots,r_m)$

result values

No thread id given

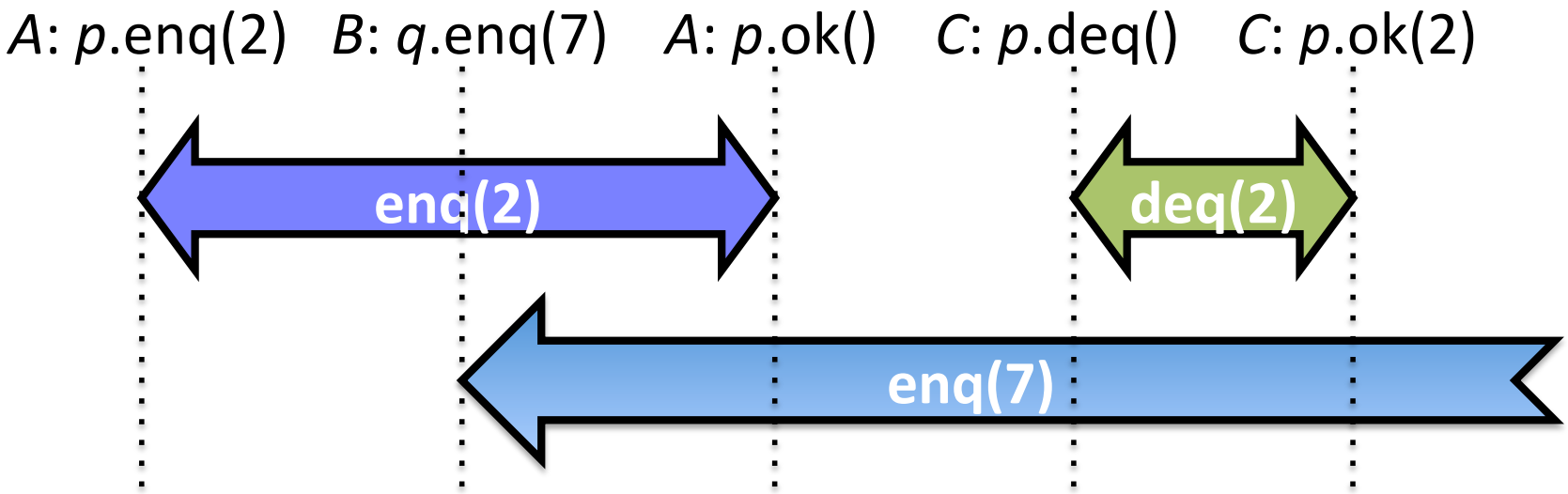
Exceptional return event:

$A: x.e(r_1,\ldots,r_m)$

exception

Traces - Matching Events

Trace: Finite or infinite sequence of events



Matching response:

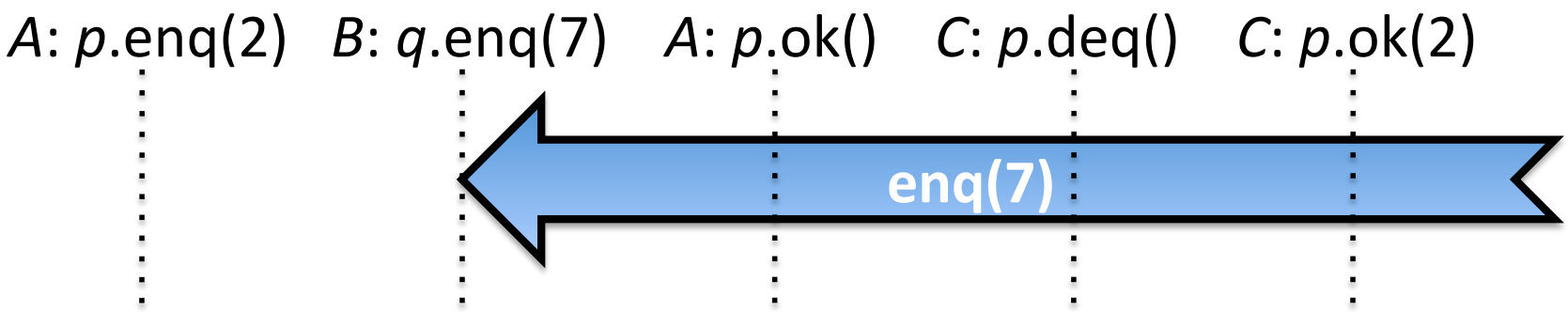
A: $x.m(a_1,\ldots,a_n)$ matches B: $y.\text{ok}(r_1,\ldots,r_m)$ iff

$A = B$ and $x = y$

Method call: Matching call-return pair – interval (lecture 1)

Complication – Incomplete Calls

Pending invocation



Pending: No matching response

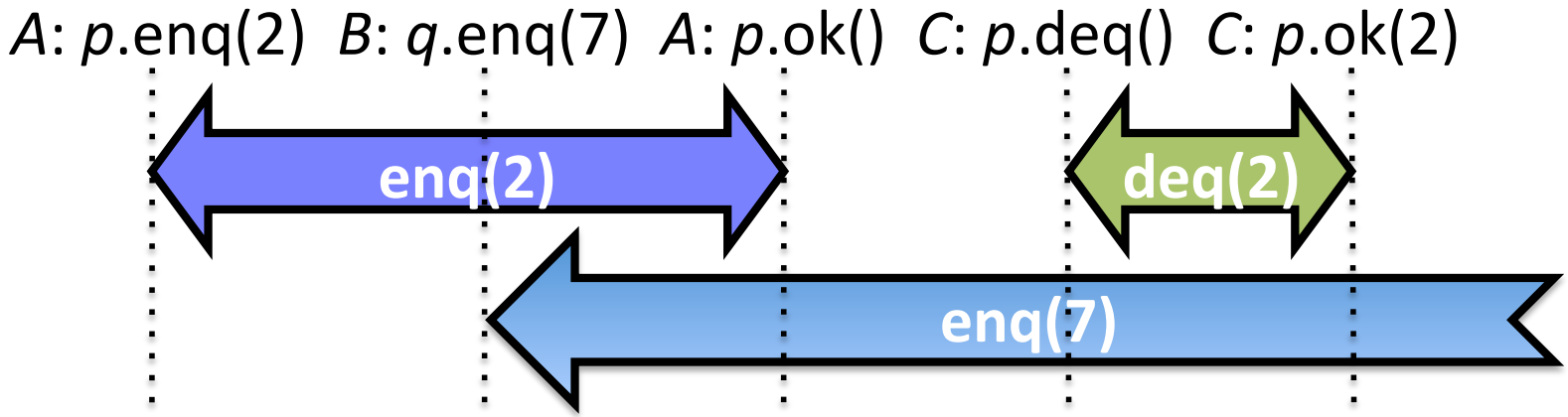
Extension of trace *H*:

- *H* followed by matching responses to unmatched calls in *H*

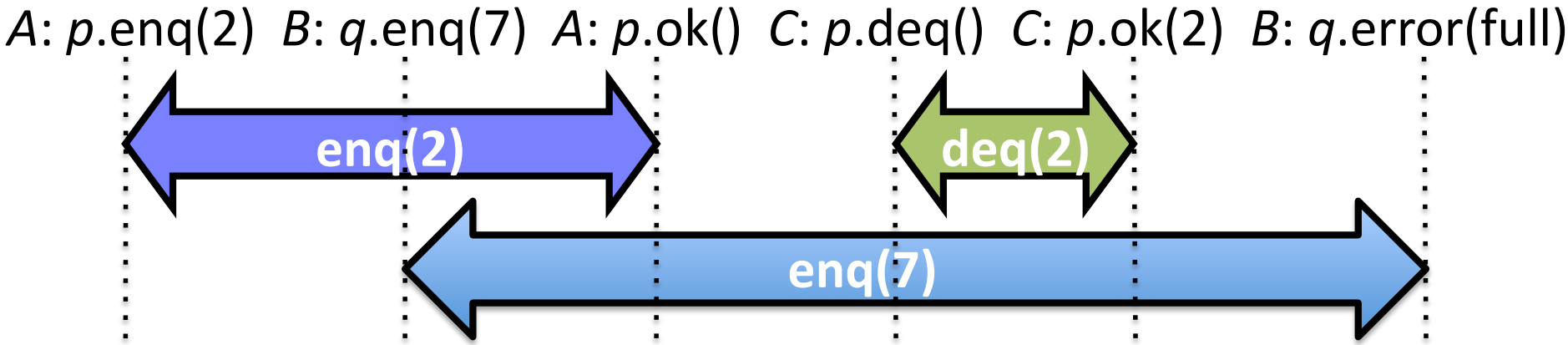
Complete(*H*):

- Subsequence of *H* with unmatched calls removed

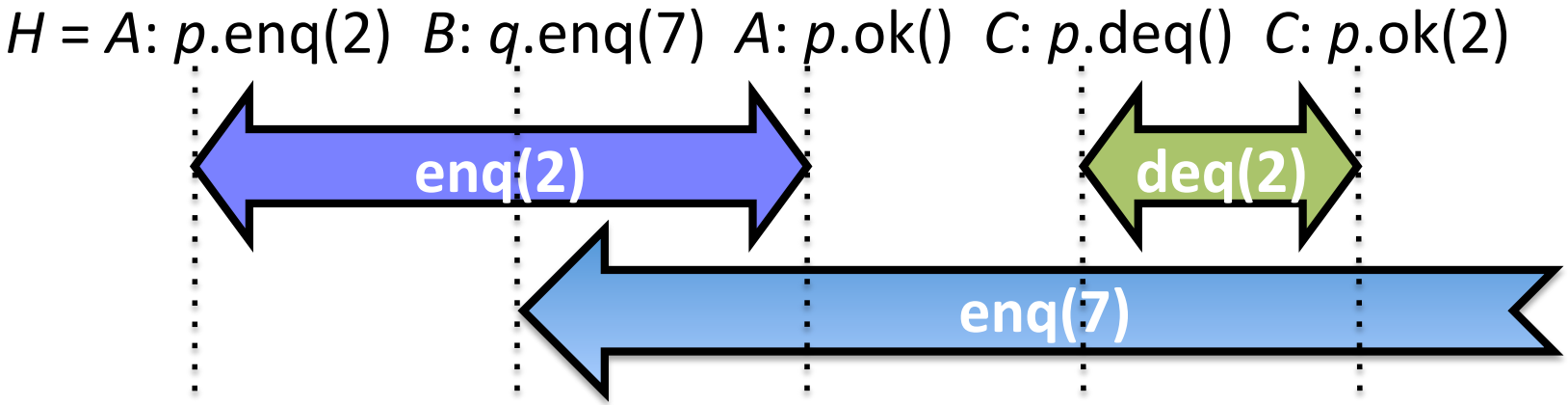
Example - Extension



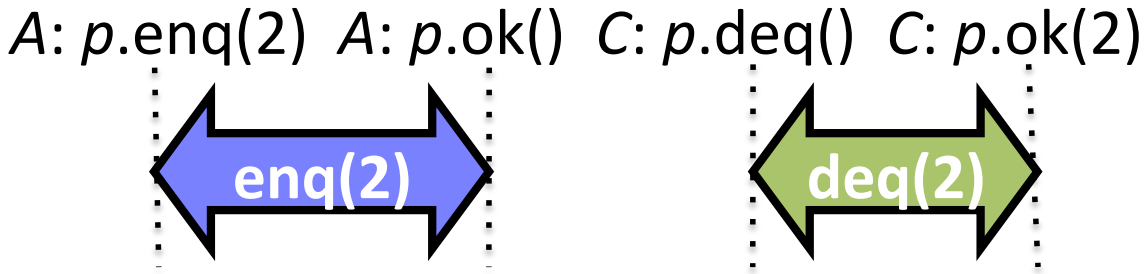
Extension:



Example - Completion



complete(*H*):



Subtraces and Equivalence

Thread subtrace $H|A$:

- Subsequence of H with all events not from A removed

$H = A$: p.enq(2) B: q.enq(7) C: p.deq() A: p.ok() C: p.ok(2)

$H|A = A$: p.enq(2) B: q.enq(7) C: p.deq() A: p.ok() C: p.ok(2)

Object subtrace $H|x$:

- Do. with all events not on x removed:

$H|p = A$: p.enq(2) B: q.enq(7) C: p.deq() A: p.ok() C: p.ok(2)

Sequentiality

Sequential trace:

- First event is call. All calls immediately followed by matching response

  A: *p*.enq(2) *A*: *p*.ok() *C*: *p*.deq() *C*: *p*.ok(2)

Claim: Thread subtraces are always sequential

  – In the absence of recursion

Claim: Object subtraces may not be sequential. Why?

Sequential Specifications

A sequential specification is some way of telling whether a

- Single-thread, single-object history
- Is legal

For example:

- Pre and post-conditions
- But plenty of other techniques exist

For a sequential multi-object trace *H*:

- *H* is *legal* iff
- *H*|*x* is in the sequential specification of *x*, for all *x*

Equivalence

Traces H_1 and H_2 are *equivalent* if $H_1|A = H_2|A$ for all A

Equivalence: Threads see the same subtraces in both

H_1 = A: p.enq(2) A: p.ok() C: p.deq() C: p.ok(2)

H_2 = A: p.enq(2) C: p.deq() A: p.ok() C: p.ok(2)

Condition:

- $H_1 \mid A = H_2 \mid A$
- $H_1 \mid B = H_2 \mid B$

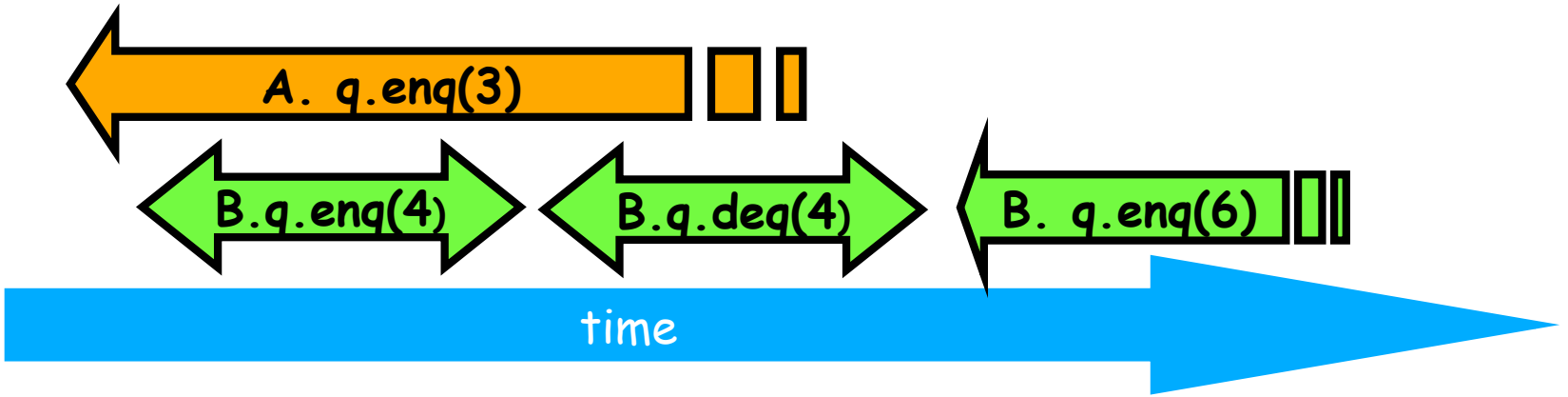
Linearizability

Definition: Trace *H* is linearizable if

- There is a legal sequential multi-object trace *S*
- *H* has an extension *H'*
  - (So pending calls can get response)
- *complete*(*H'*) is equivalent to *S*
  - (Not all pending calls need get a response)
- If m_0 -> m_1 in *H* then m_0 -> m_1 in *S*
  - Method calls that are sequential in *H* must appear in the right order
  - Recall the interval ordering -> from last lecture

Example

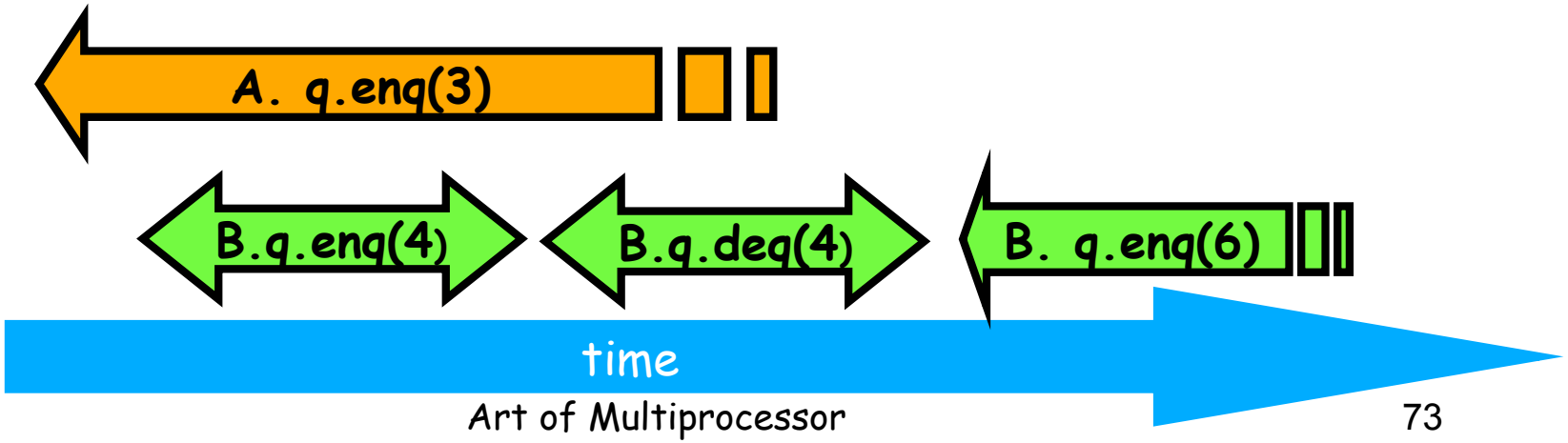
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



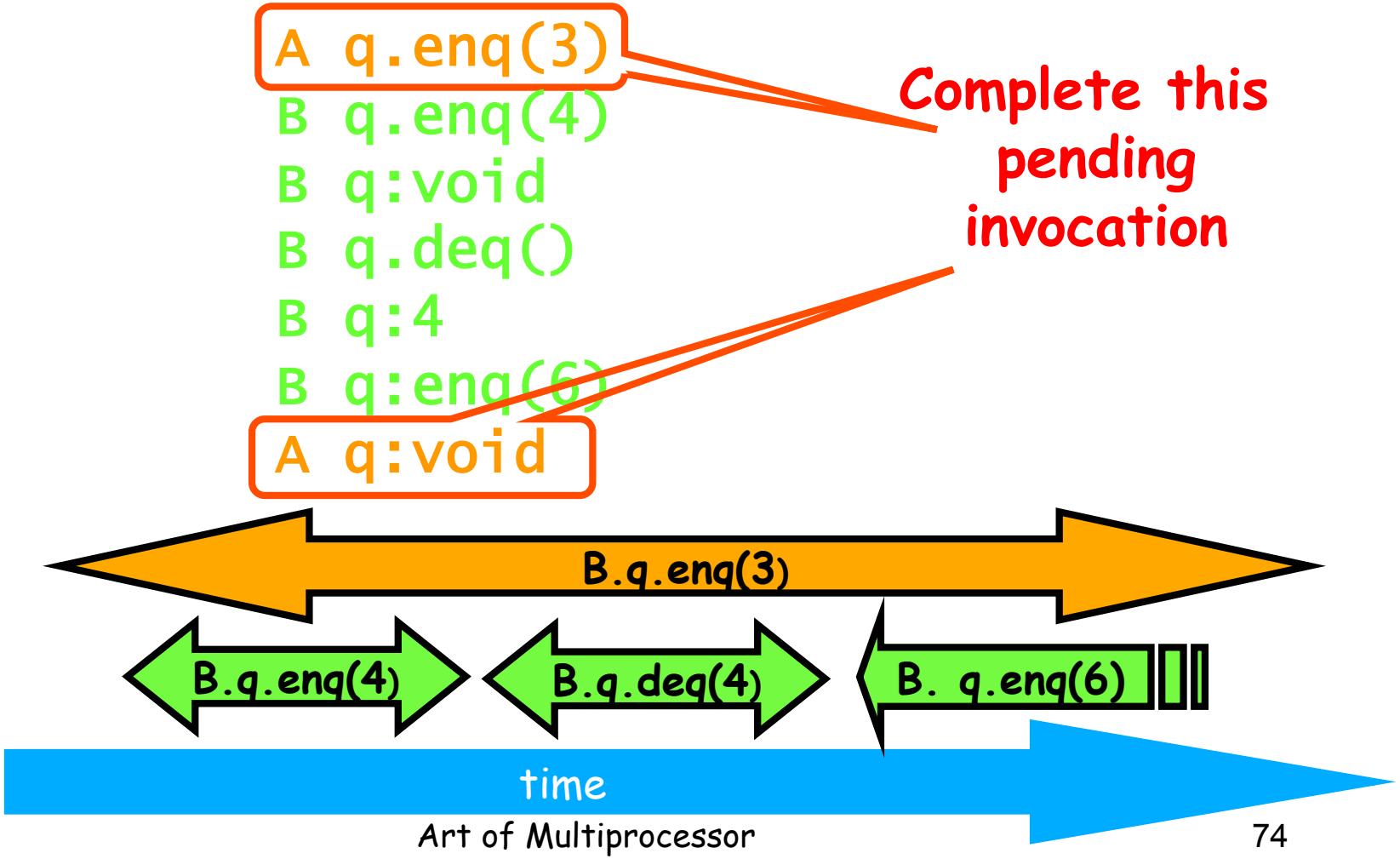
Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```

Complete this pending invocation



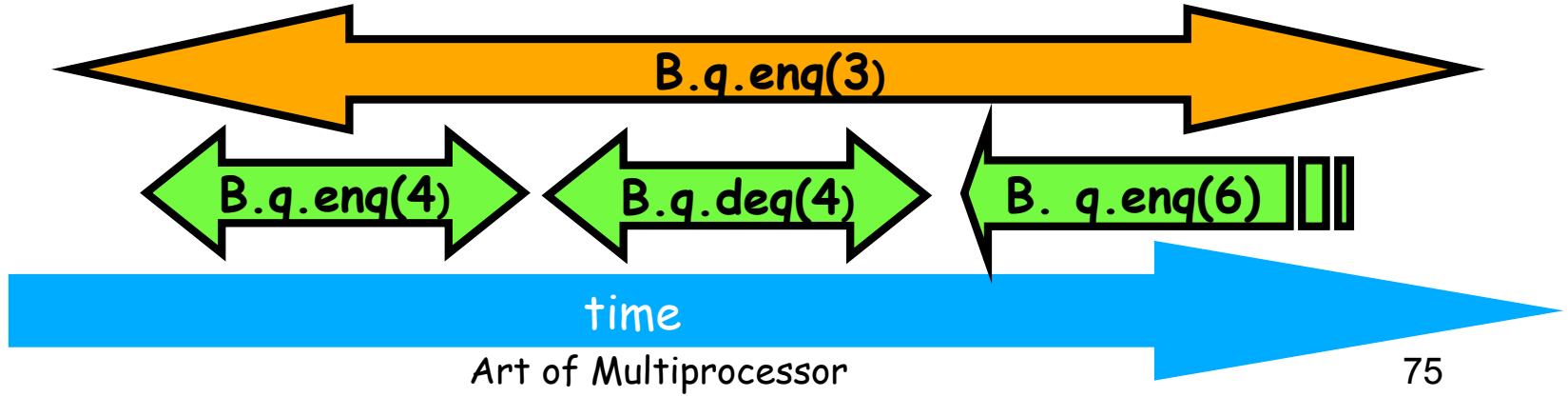
Example


A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
Complete this pending invocation
B.q.enq(3)
B.q.enq(4)
B.q.deq(4)
B. q.enq(6)
time

Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```

discard this one

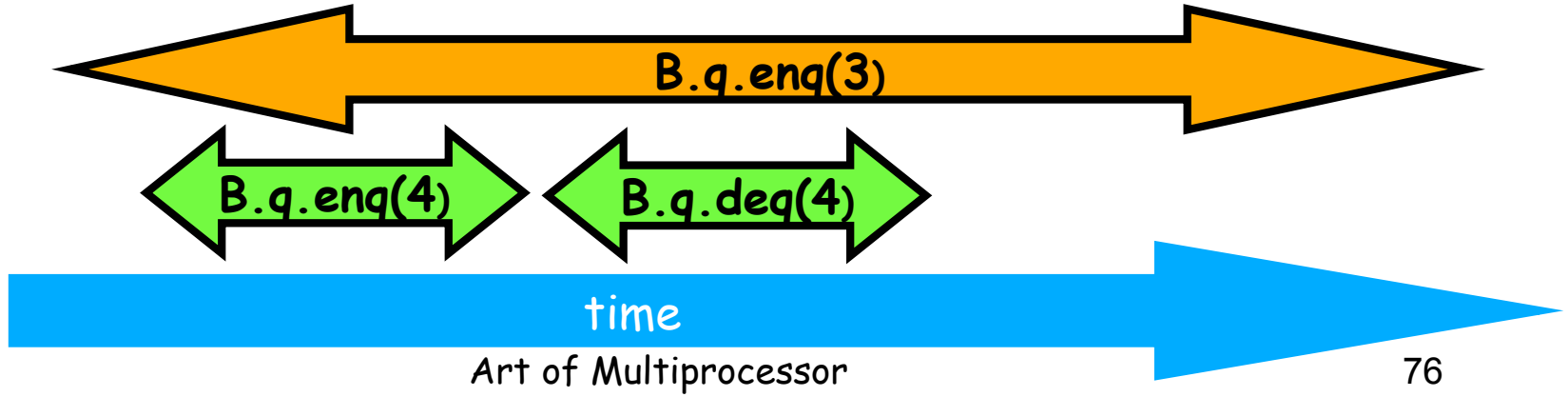


Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4

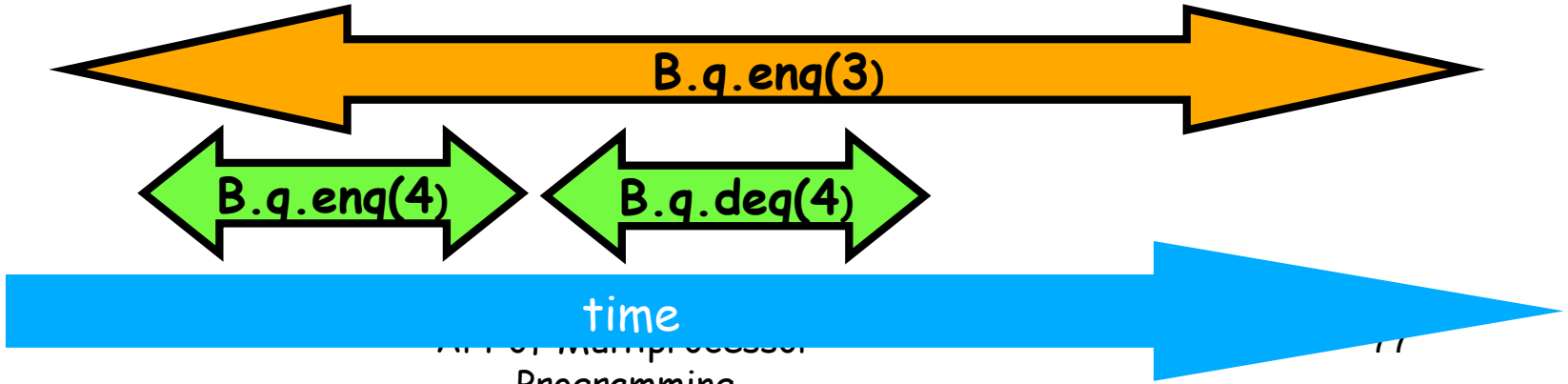
discard this one

A q:void



Example

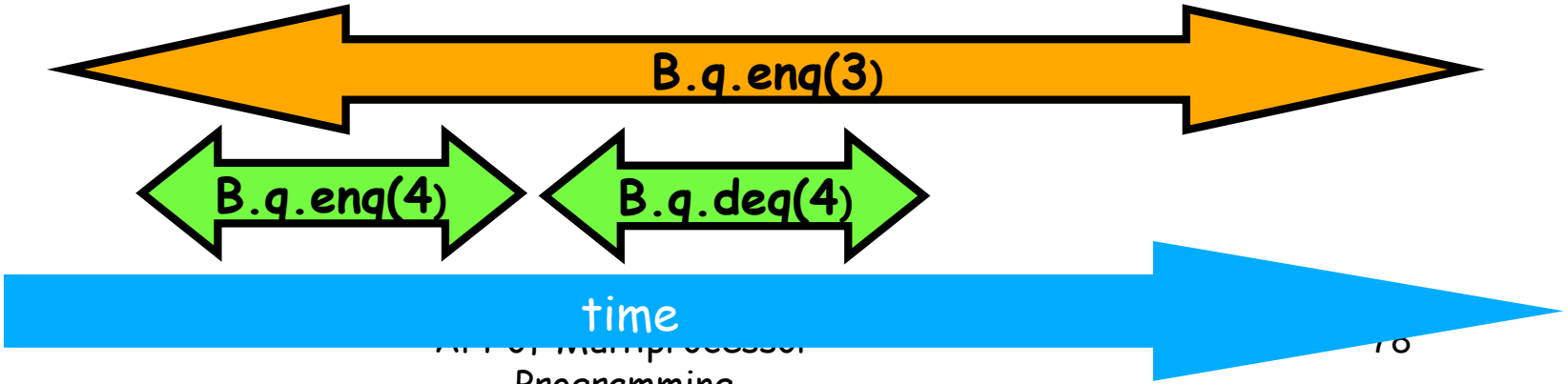
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```

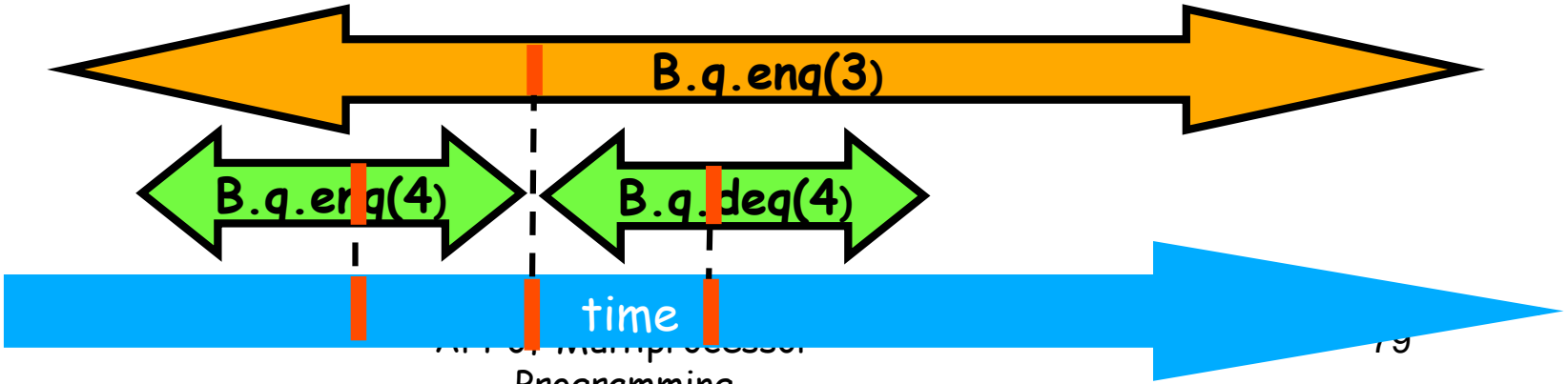


Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

Equivalent sequential history

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```



Composability

Theorem: *H* is linearizable iff for each object *x*, *H* | *x* is linearizable

Proof: See Herlihy-Shavit

This is important!

- Linearizable objects => linearizable systems
- Otherwise might have to impose extra scheduler or locks to ensure linearizability at system level

Proving Linearizability

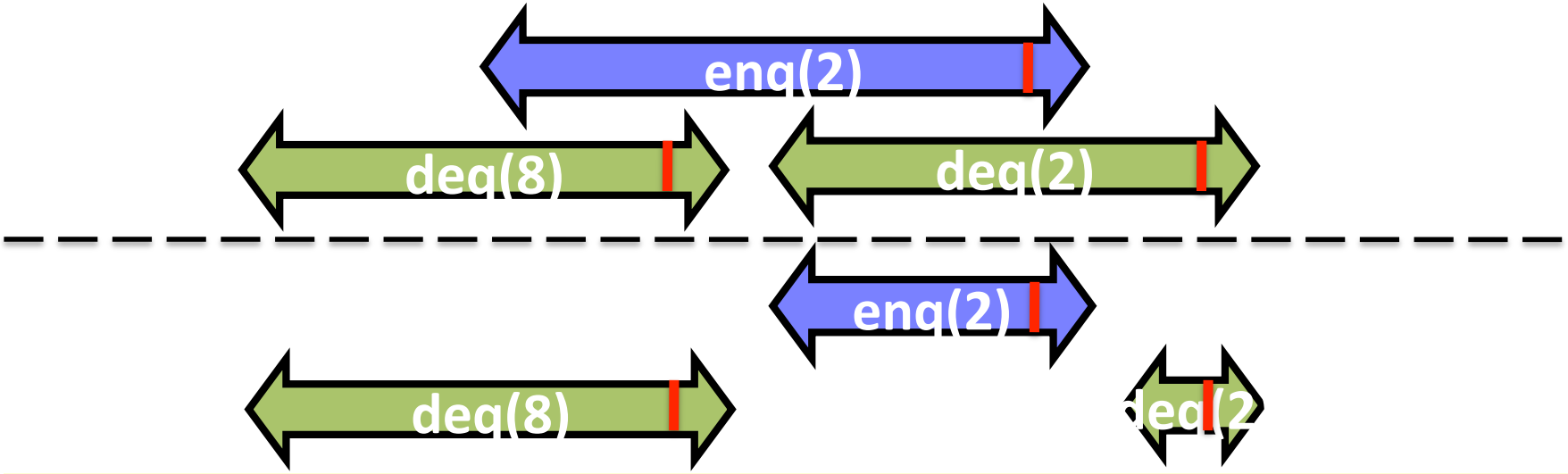
Recall: Trace *H* is linearizable if

- There is a legal sequential multi-object trace *S*
- *H* has an extension *H'*
- *complete*(*H'*) is equivalent to *S*
- If m_0 -> m_1 in *H* then m_0 -> m_1 in *S*

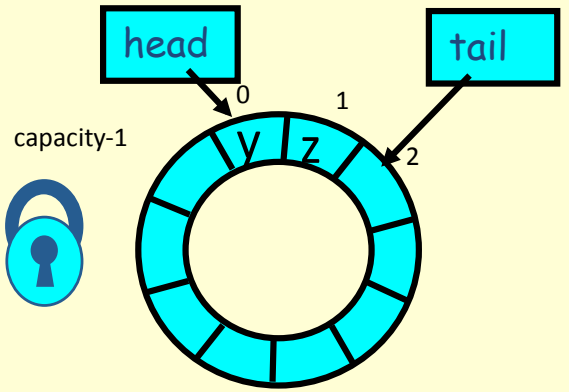
Strategy:

- Consider each object in turn – ok by composability
- Identify a single linearization point in each method
  - (Not always possible, alas)
- Show that all method events can be "shifted" to the linearization point

Proving Linearizability – with Locking




```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();  }  }
```



Linearization point

The Lock-free Case

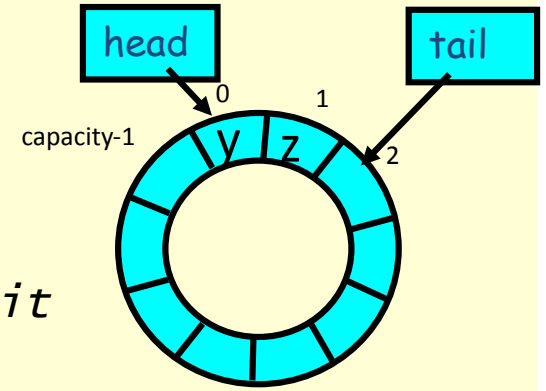
Linearization points where tail/head updated

Remember: Only one producer and one consumer!

```
public class LockFreeQueue {

  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];

  public void enq(Item x) {
    while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     while (tail == head);      // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```



Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don’t leave home without it

Alternative: Sequential Consistency

Definition: Trace *H* is sequentially consistent if

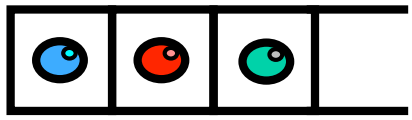
- There is a legal sequential multi-object trace *S*
- *H* has an extension *H'*
- *complete*(*H'*) is equivalent to *S*
- ~~If m_0 -> m_1 in *H* then m_0 -> m_1 in *S*~~

Not necessary to preserve precedence order between threads

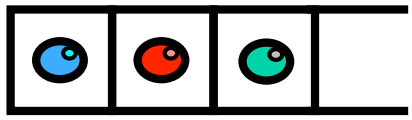
But program order must be preserved !

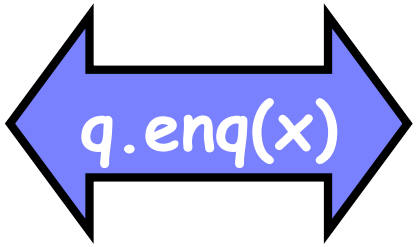
Program order: Per thread precedence order - equivalence

Example: Sequential Consistency

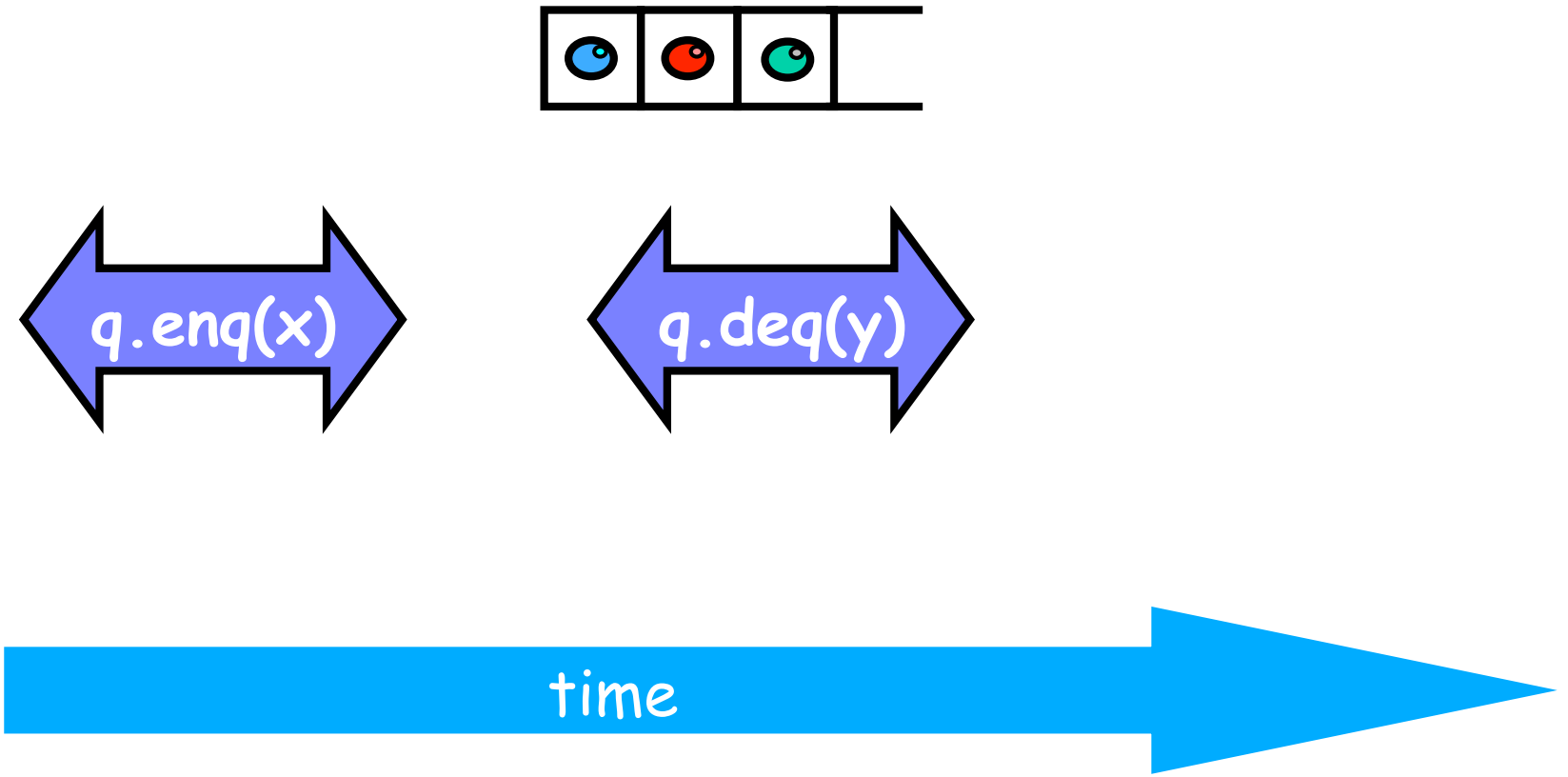


Example: Sequential Consistency

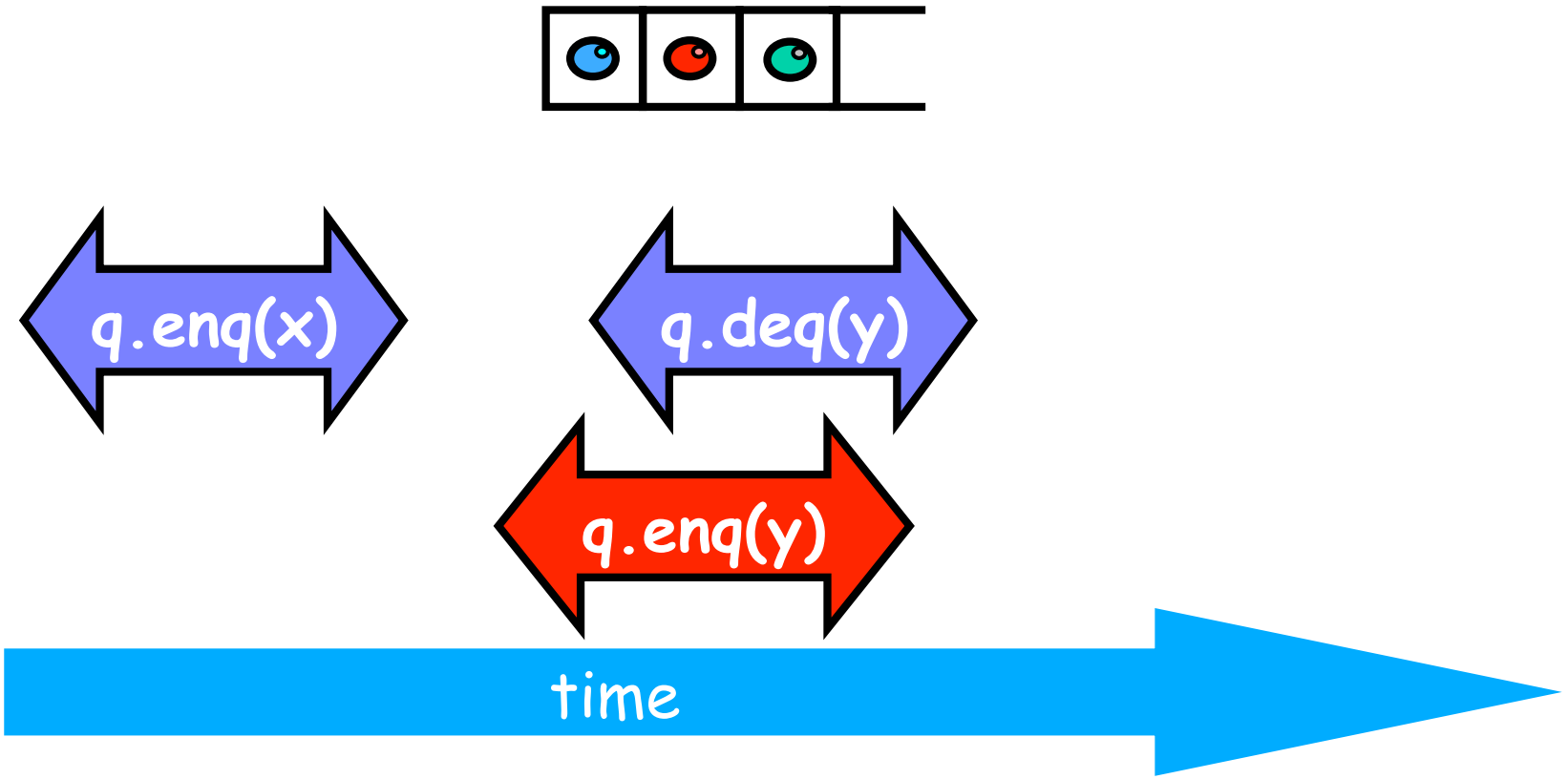




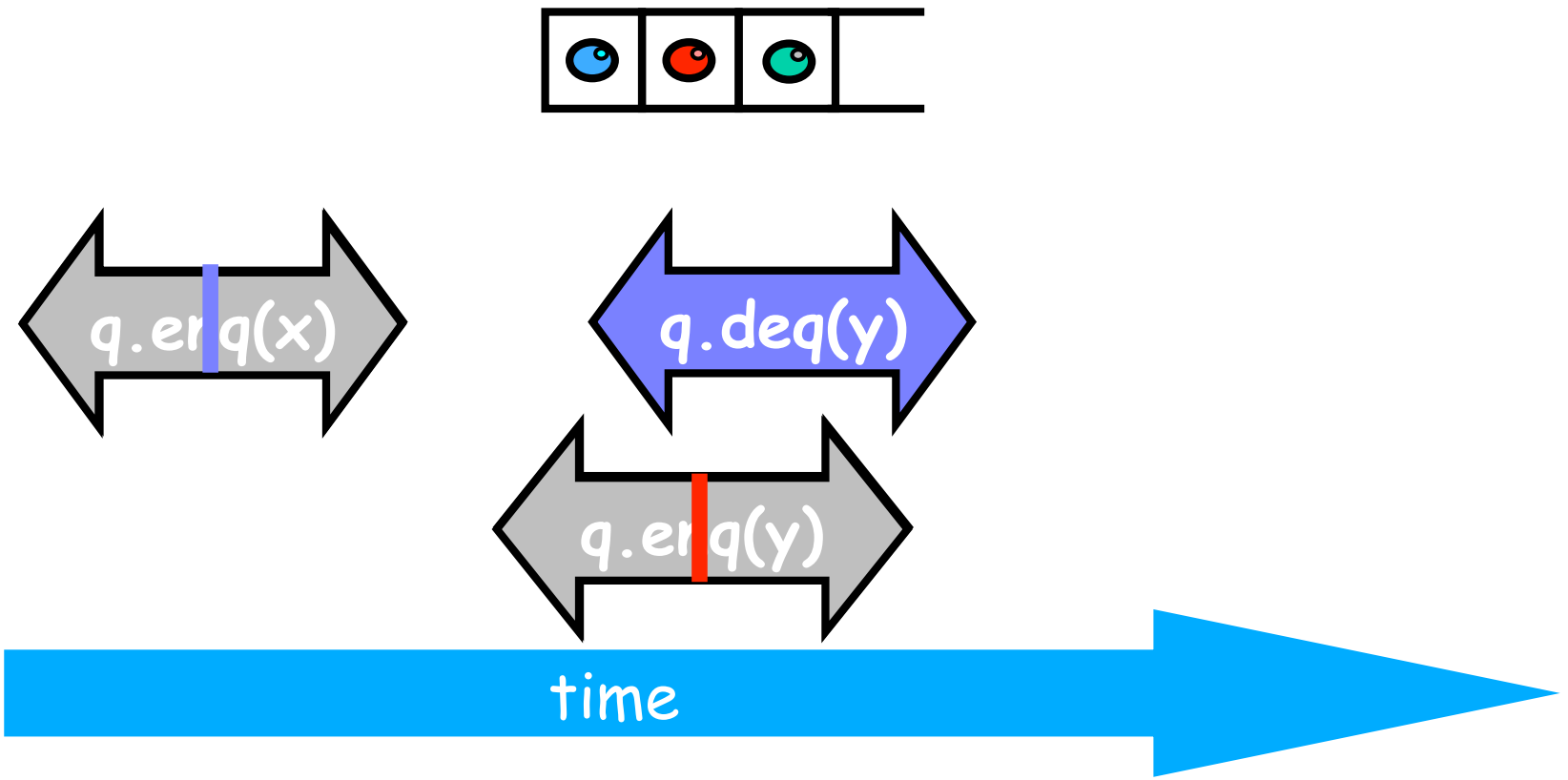
Example: Sequential Consistency



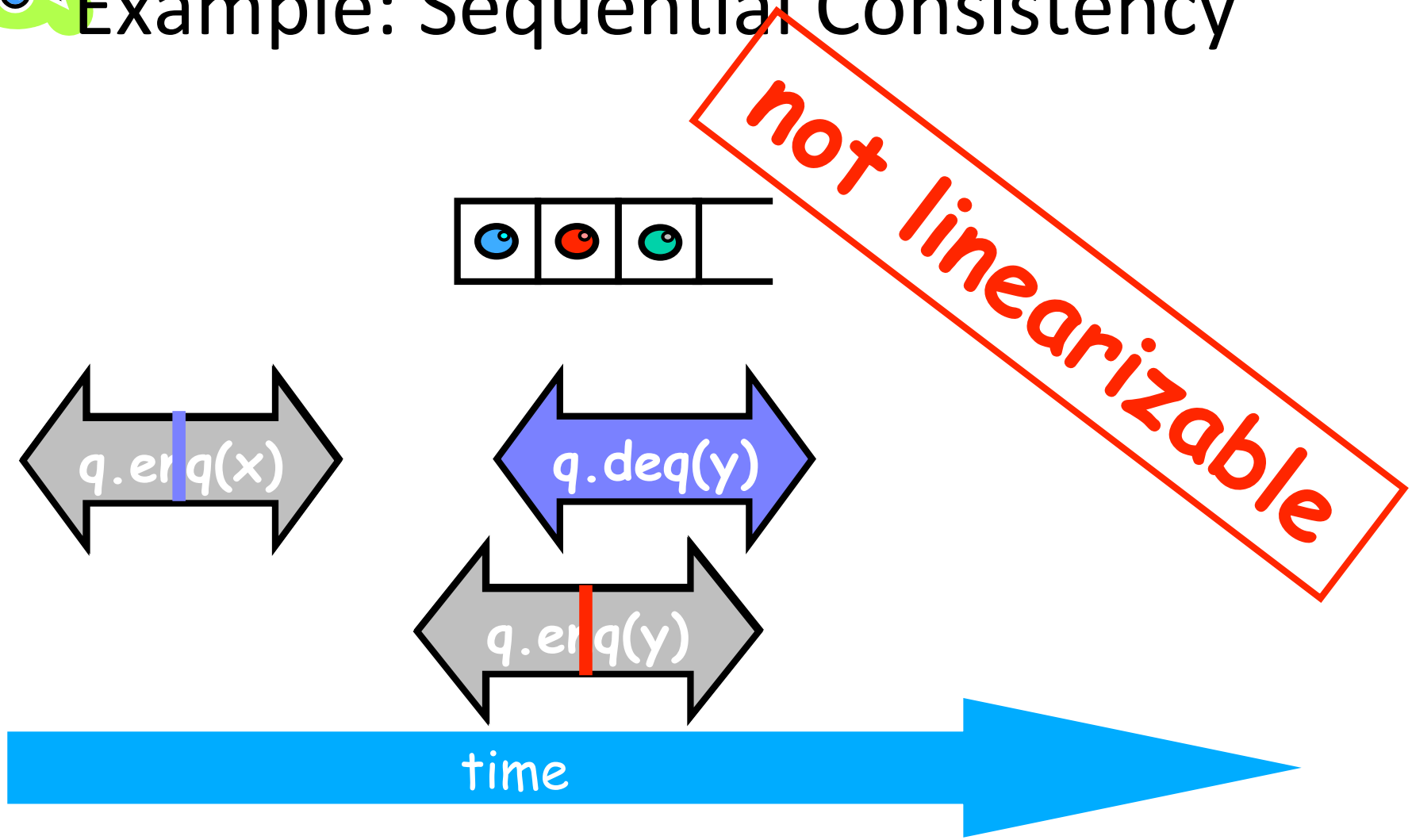
Example: Sequential Consistency



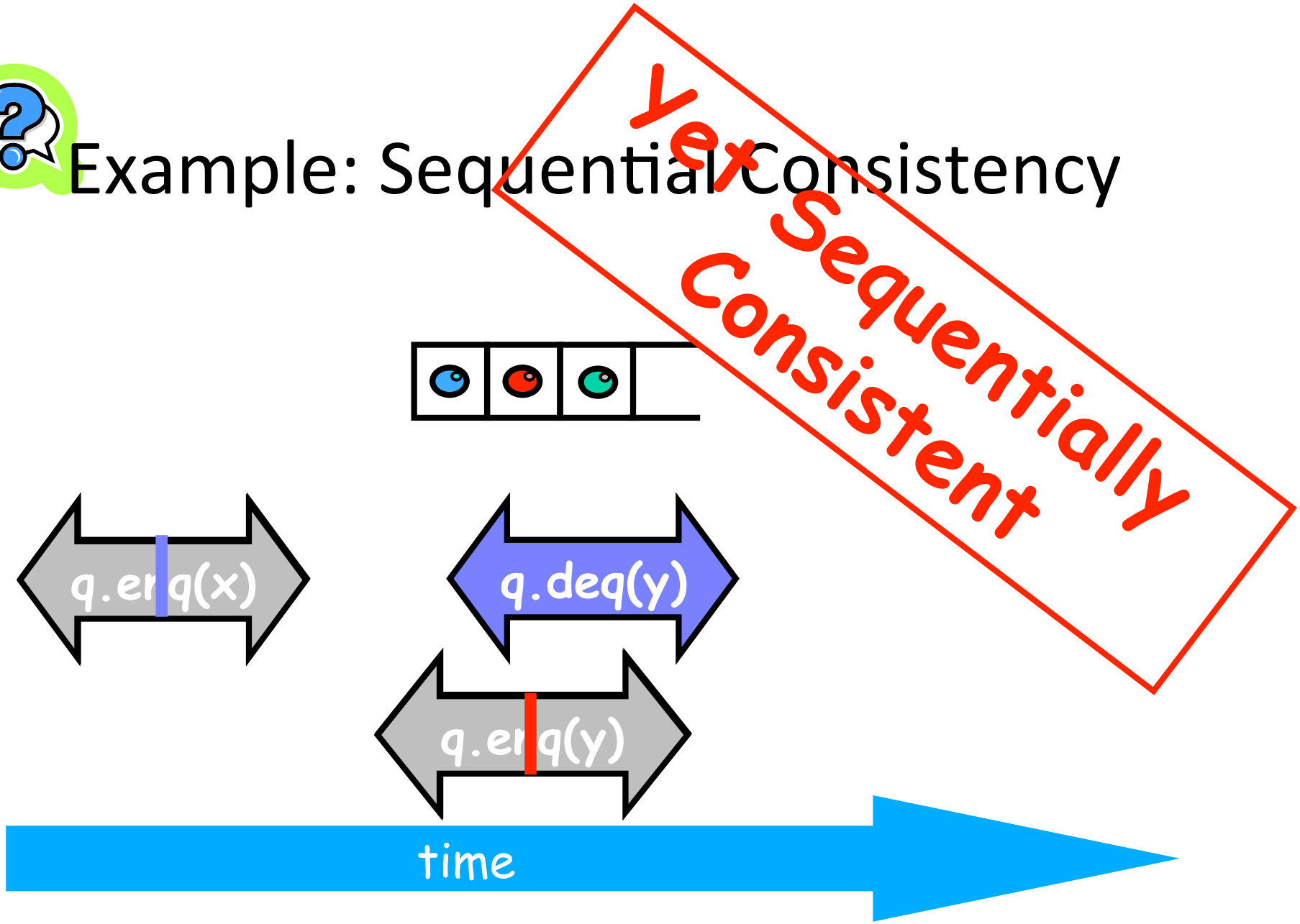
Example: Sequential Consistency



Example: Sequential Consistency



Example: Sequential Consistency

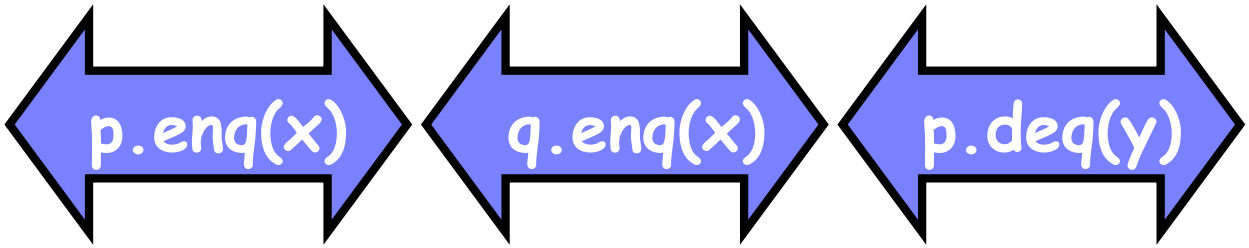


Theorem

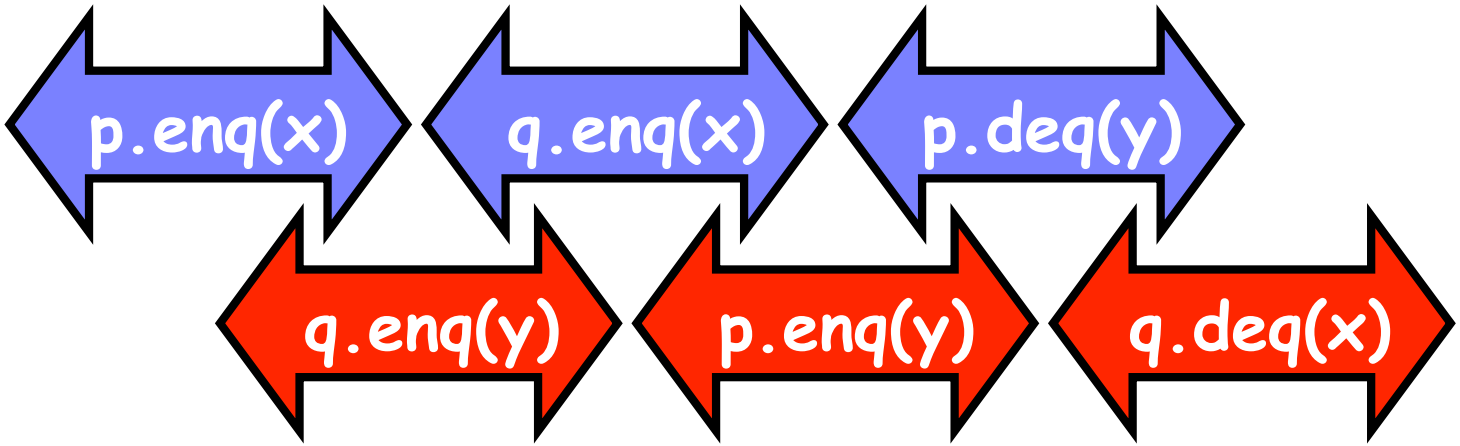
Sequential Consistency is not a local property

(and thus we lose composability…)

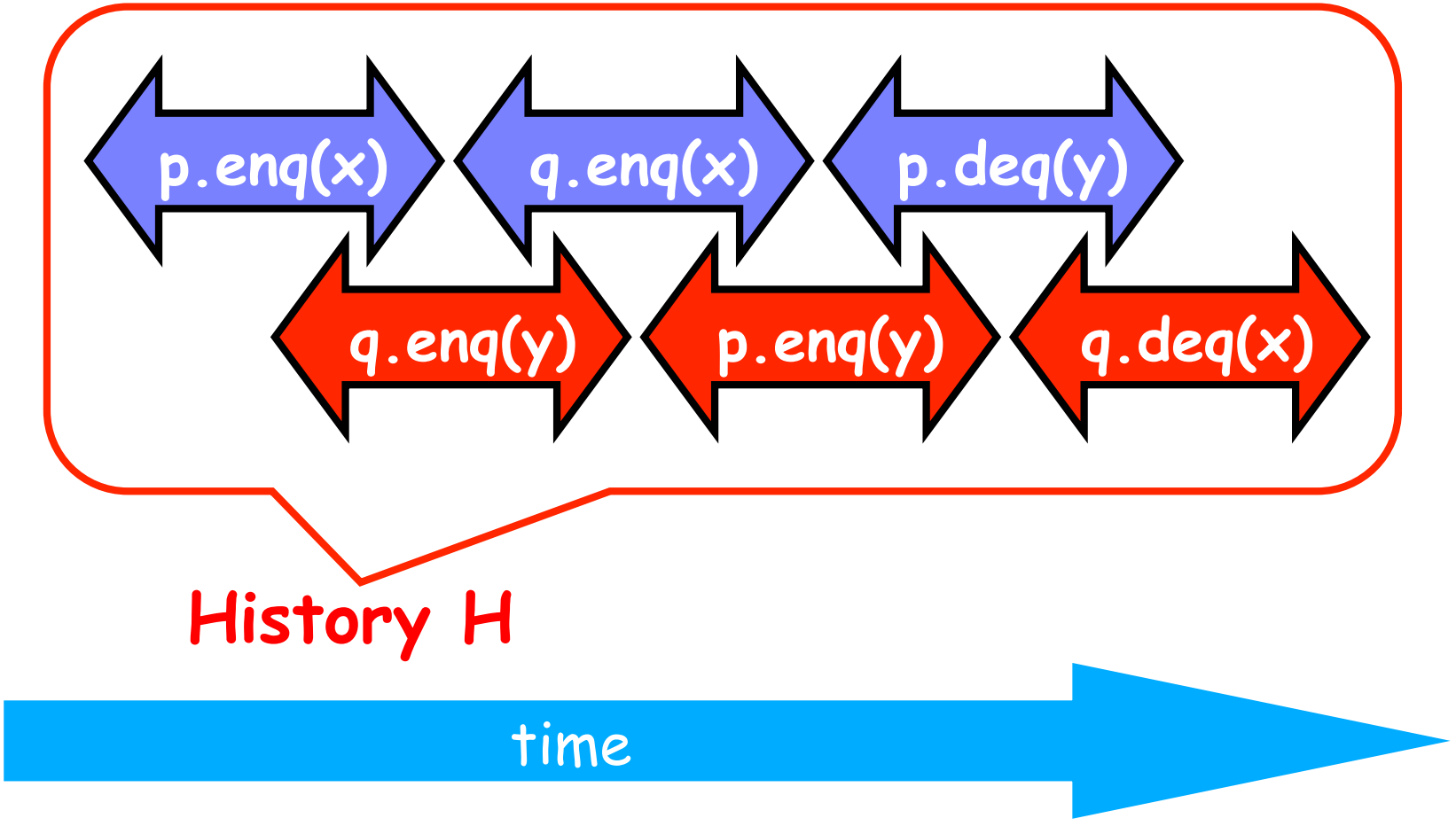
FIFO Queue Example



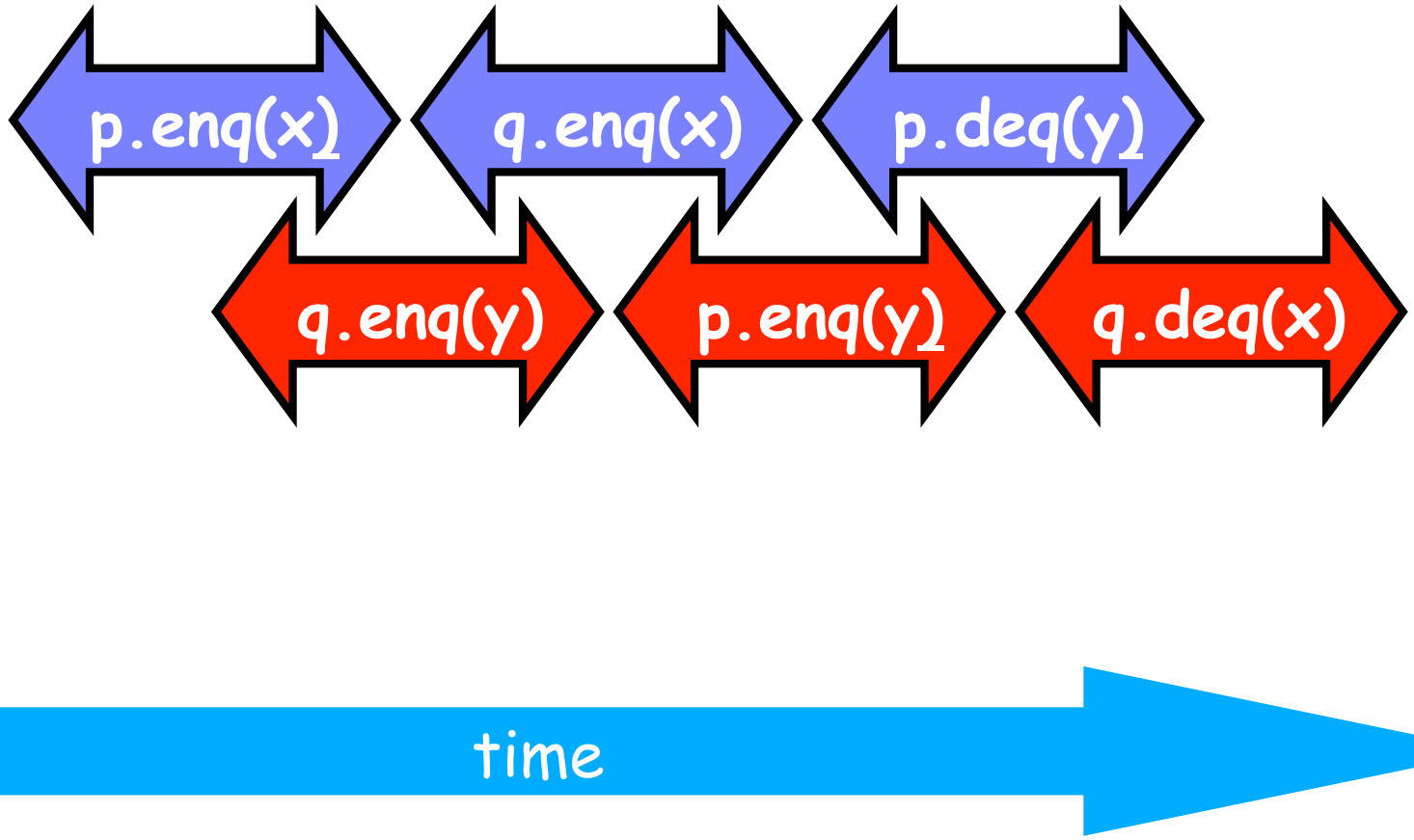
FIFO Queue Example



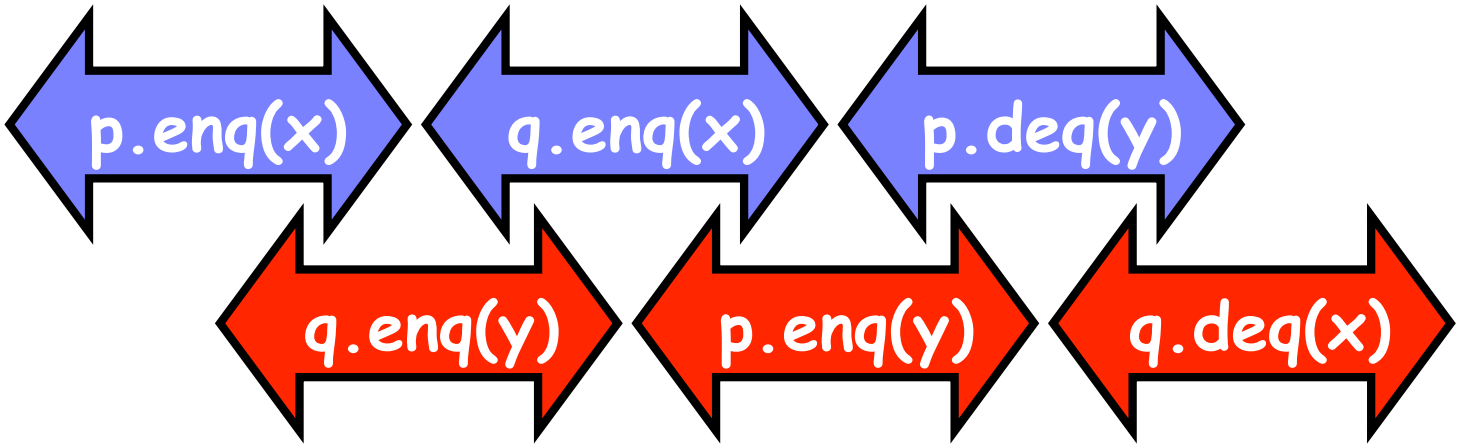
FIFO Queue Example



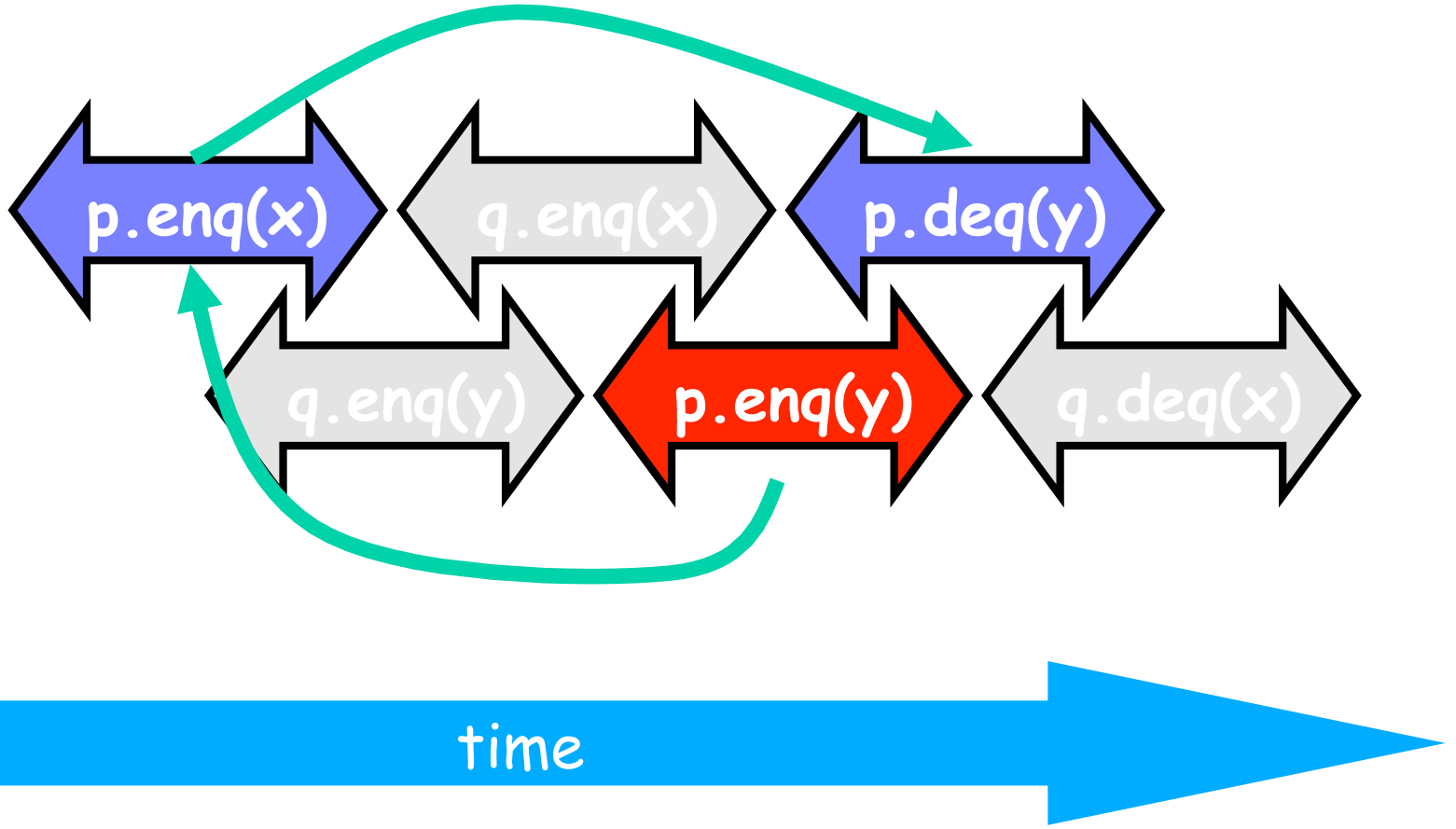
H|p Sequentially Consistent



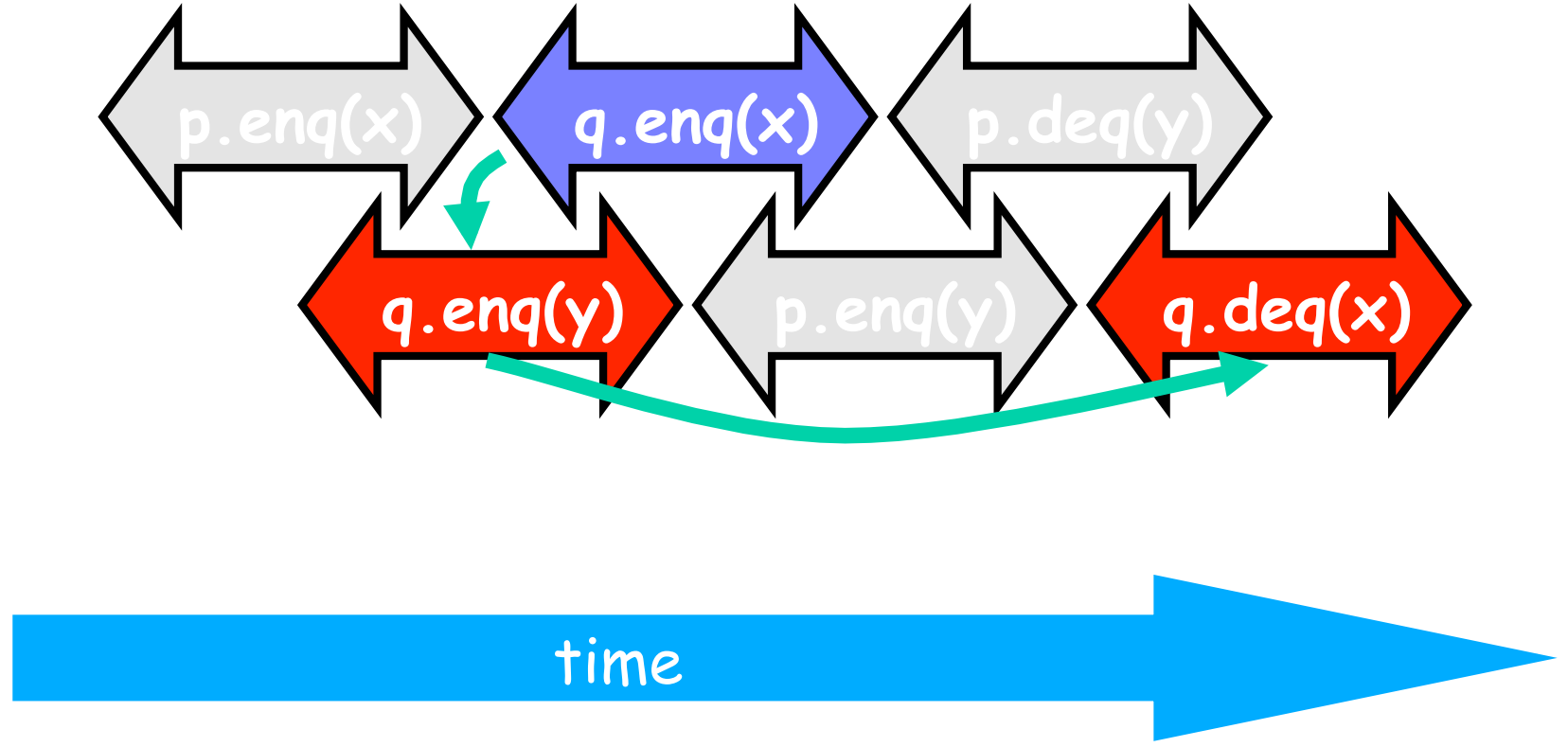
H|q Sequentially Consistent



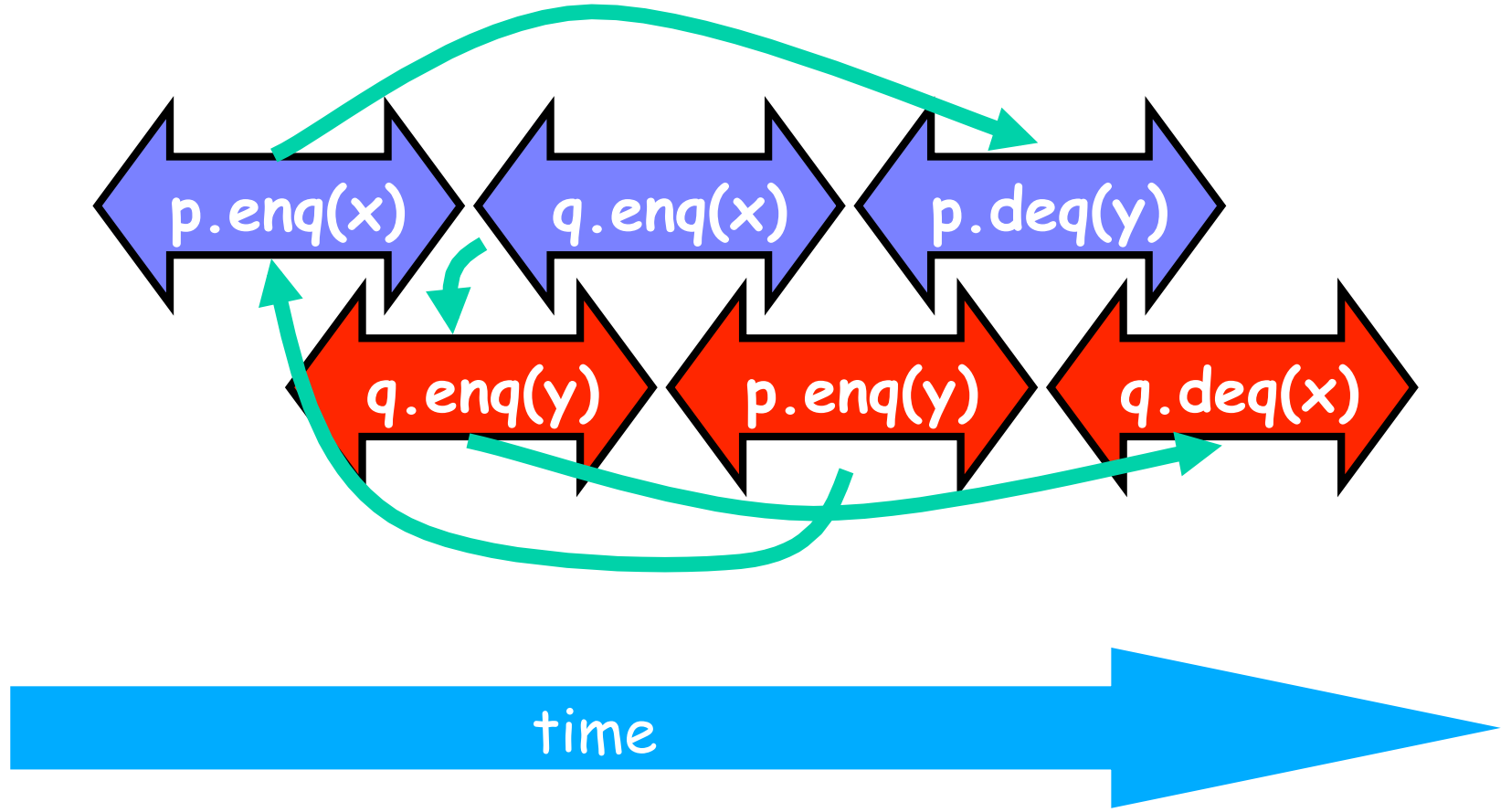
Ordering imposed by p



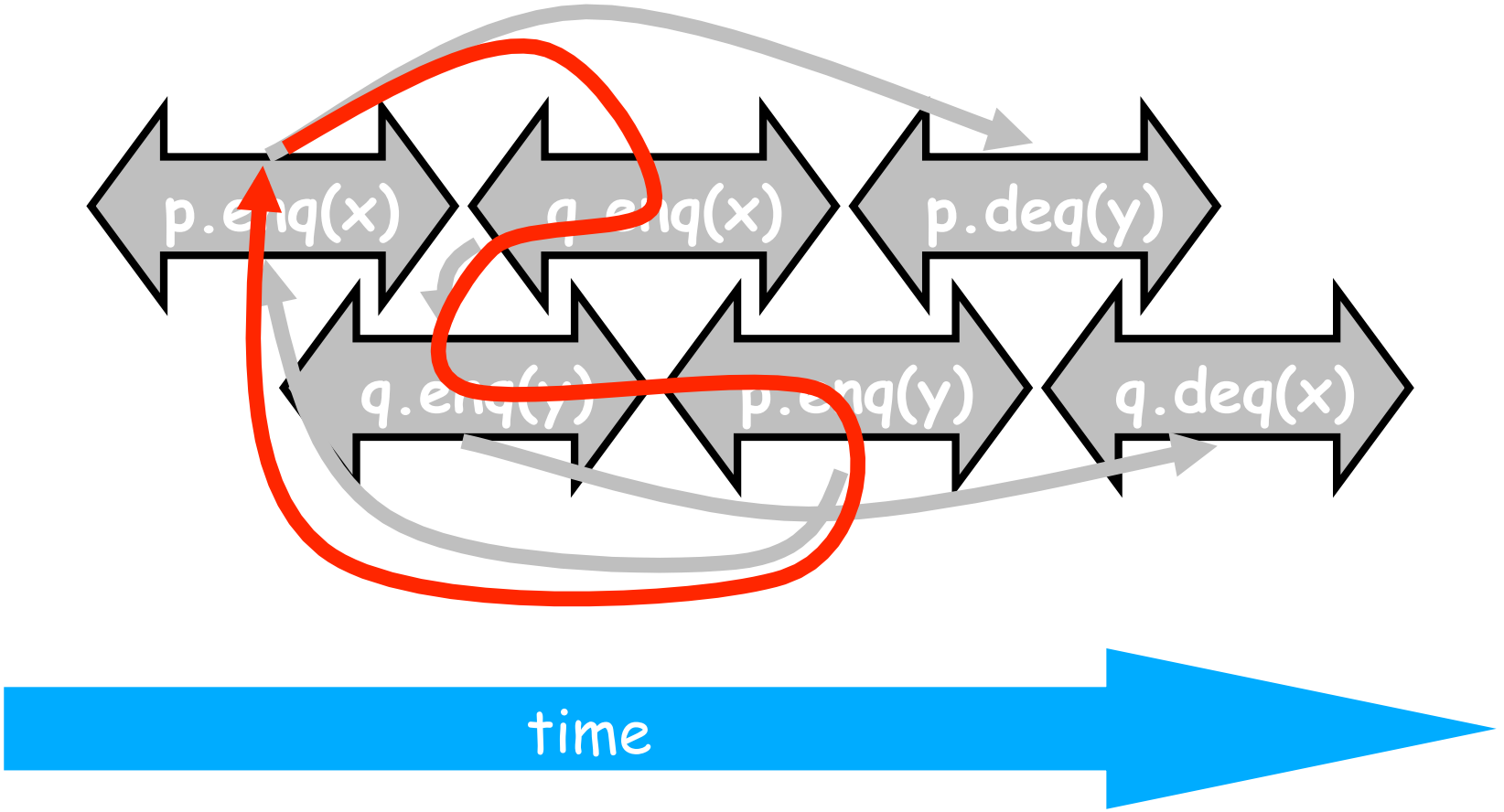
Ordering imposed by q



Ordering imposed by both



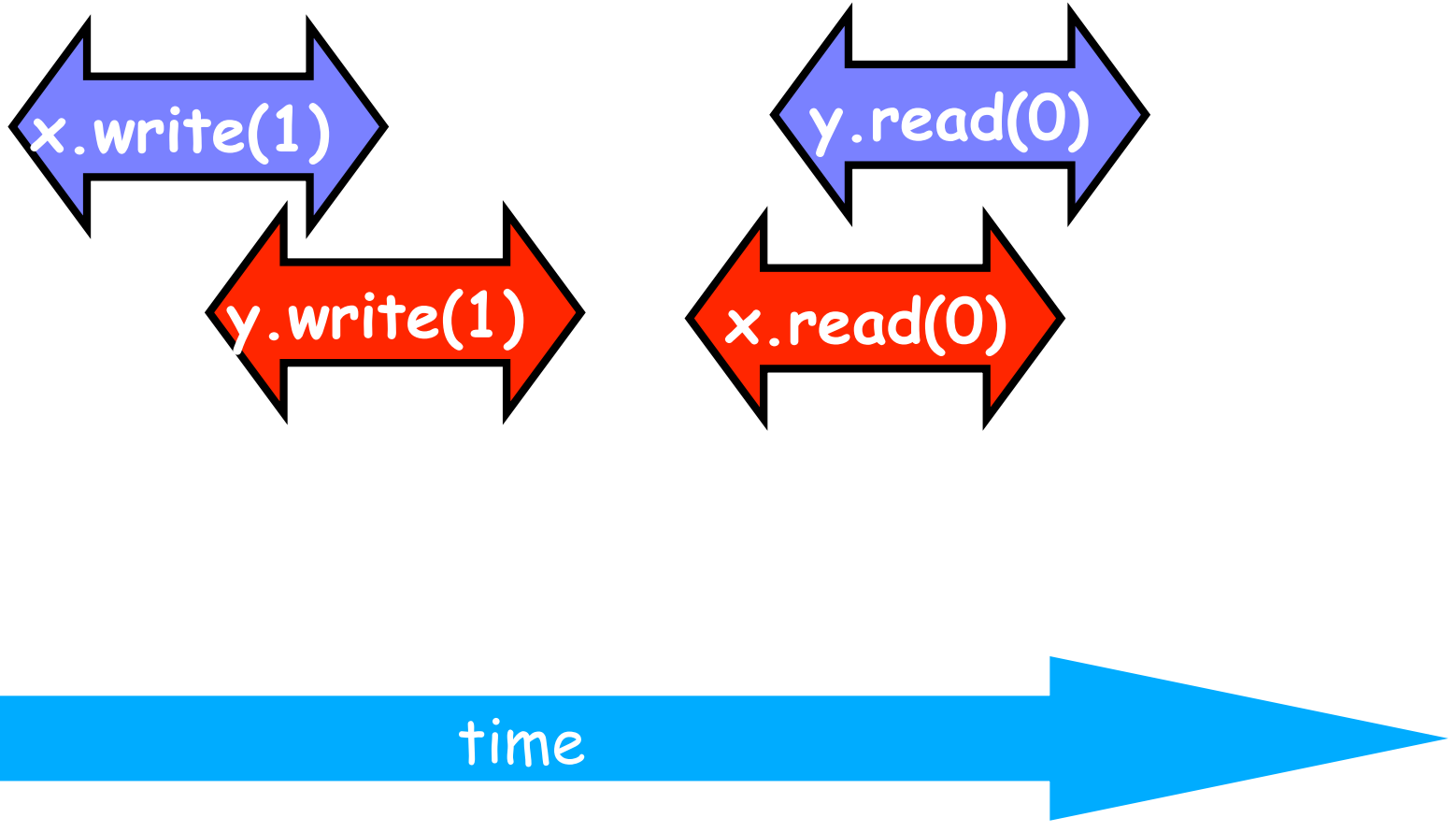
Combining orders



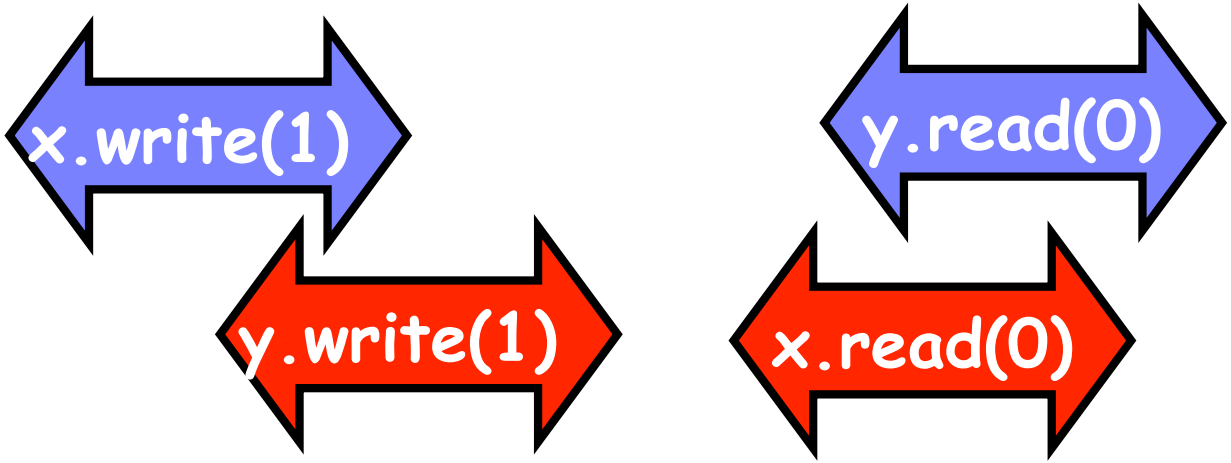
Fact

- Most hardware architectures don't support sequential consistency
- Because they think it's too strong
- Here's another story …

Example



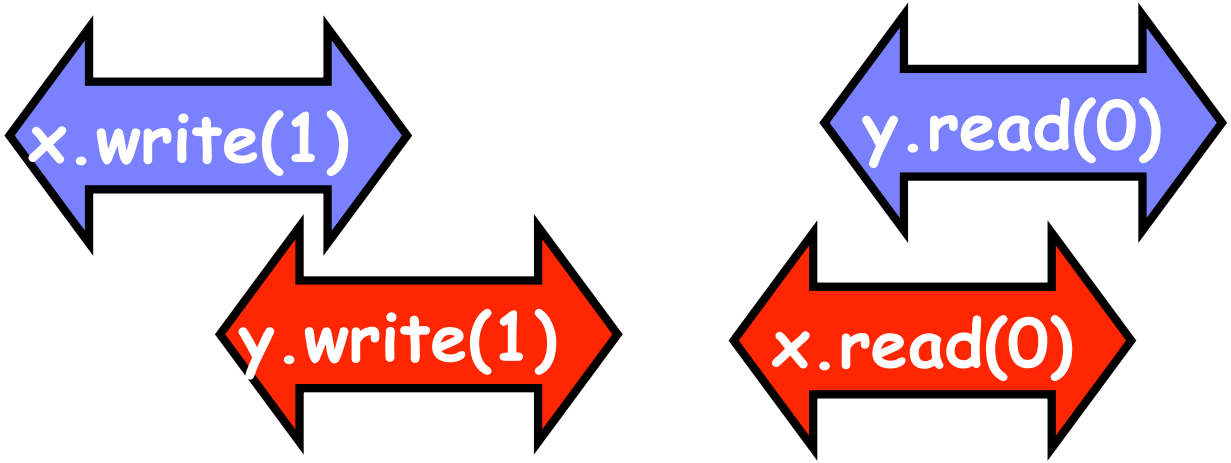
Example



Each thread's view is sequentially consistent

– It went first

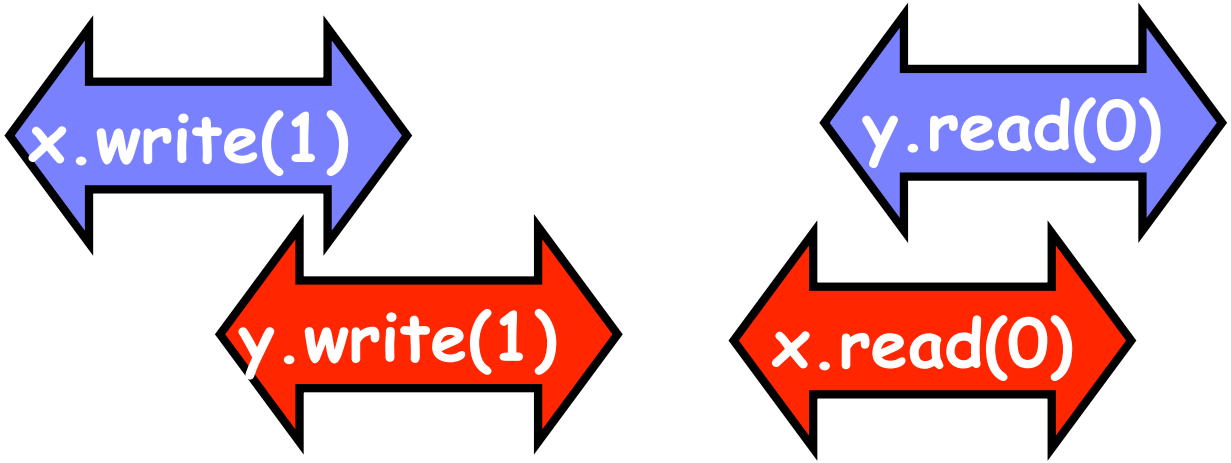
Example



Entire history isn't sequentially consistent

- Can't both go first
- Highly counterintuitive

Example



But what if each thread executes on its own core?

Read and writes address cores own cache

Writes and reads to main memory buffered

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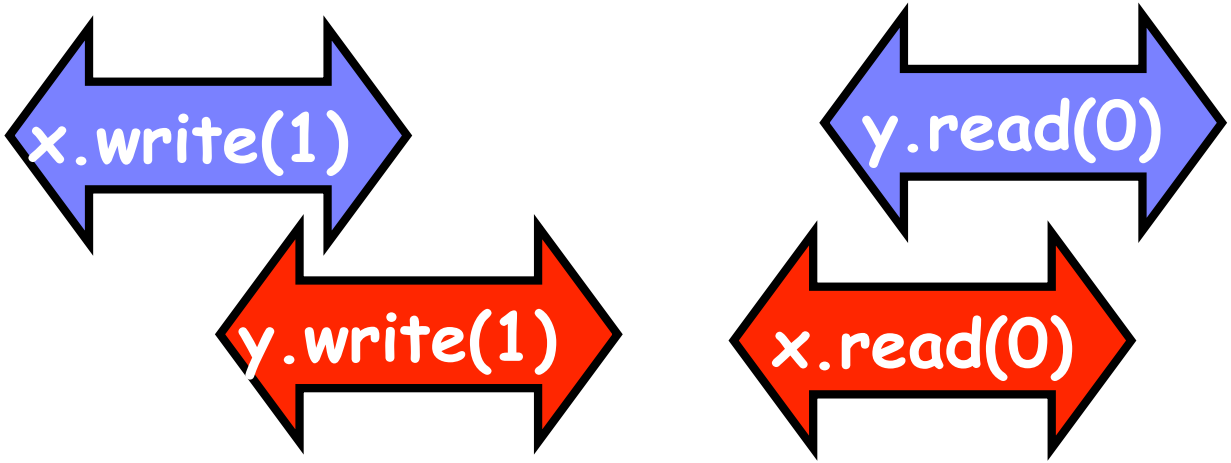

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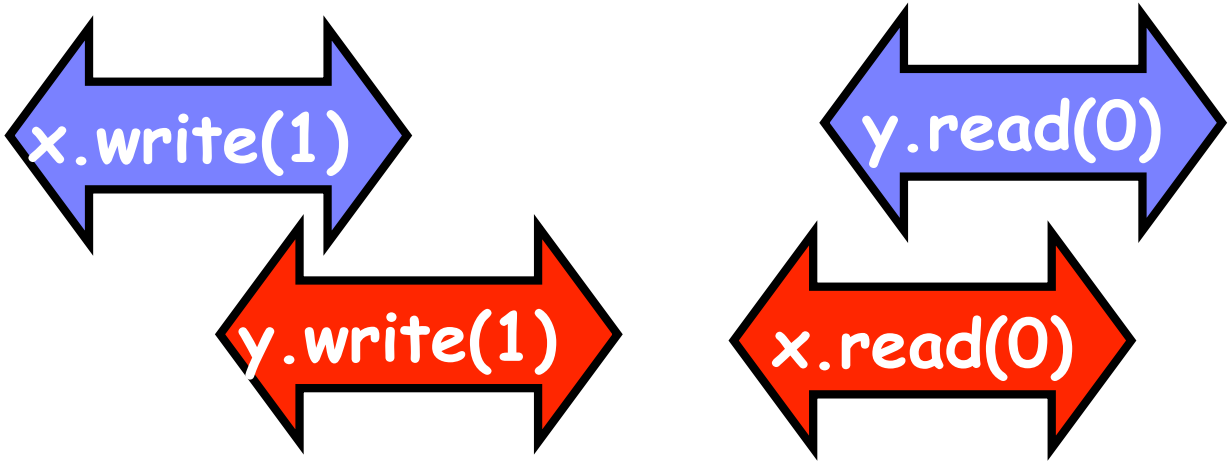

Is this behavior wrong?

– We can argue either way …

The Hardware View

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if fiction of sequentiality
  - Violated by default
  - Honored by explicit request

Memory Hierarchy

- On modern multiprocessors, processors do not read and write directly to memory
- Memory accesses are very slow compared to processor speeds
- Instead, each processor reads and writes directly to a cache
- To read a memory location, load data into cache.
- To write a memory location, update cached copy,
  - Lazily write cached data back to memory

While Writing to Memory

- A processor can execute hundreds, or even thousands of instructions
- Why delay on every memory write?
- Instead, write back in parallel with rest of the program
- Processors delay writing to memory
- Until reads have been issued
- Otherwise get severe performance degradation
- If you need to synchronize, say so

Who Knew You Wanted to Synchronize?

- Writing to memory = mailing a letter
- Vast majority of reads & writes
- Not for synchronization
- No need to idle waiting for post office
- If you want to synchronize
  - Announce it explicitly
  - Pay for it only when you need it

Explicit Synchronization

- Memory barrier instruction
  - Flush unwritten caches
  - Bring caches up to date
- Compilers often do this for you
  - Entering and leaving critical sections
- Expensive
- In Java, can ask compiler to keep a variable up-to-date with volatile keyword
- Also inhibits reordering, removing from loops, & other "optimizations"

Real-World Hardware Memory

- Weaker than sequential consistency
- But you can get sequential consistency at a price
- OK for experts, tricky stuff
  - assembly language, device drivers, etc.
- Linearizability more appropriate for high-level software

Progress Conditions

- *Deadlock-free:* some thread trying to acquire the lock eventually succeeds.
- *Starvation-free:* every thread trying to acquire the lock eventually succeeds.
- *Lock-free:* some thread calling a method eventually returns.
- *Wait-free:* every thread calling a method eventually returns.

	Non-Blocking	**Blocking**
Everyone makes progress	**Wait-free**	**Starvation-free**
Someone makes progress	**Lock-free**	**Deadlock-free**

Summary

- Concurrent access to objects need thought
- Linearizability:
  - Useful high-level model
  - Local and composable
- Locks: Easy way to obtain linearizability
  - But expensive
  - Deadlock freedom and starvation freedom can be obtained
- Can do better without locks
  - Trickier, but can get better progress properties
- For better performance yet need to use weaker memory models
  - Even trickier