CTL Syntax

Let p range over a given set *Atoms* of atomic propositions.

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{A}\psi \mid \mathsf{E}\psi \qquad \text{(state formulas)}$$

$$\psi ::= \mathsf{X}\phi \mid \mathsf{G}\phi \mid \mathsf{F}\phi \mid \phi\mathsf{U}\phi \qquad \text{(path formulas)}$$

Models and Paths

These are defined exactly as for LTL (see handouts).

CTL Semantics

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model, and let $s \in S$ be a state.

$$\begin{aligned}
\mathcal{M}, s \models p &\overset{\text{def}}{\Leftrightarrow} p \in L(s) \\
\mathcal{M}, s \models \neg\phi &\overset{\text{def}}{\Leftrightarrow} \text{not } \mathcal{M}, s \models^{\mathcal{M}} \phi \\
\mathcal{M}, s \models \phi_1 \wedge \phi_2 &\overset{\text{def}}{\Leftrightarrow} \mathcal{M}, s \models^{\mathcal{M}} \phi_1 \text{ and } \mathcal{M}, s \models^{\mathcal{M}} \phi_2 \\
\mathcal{M}, s \models \mathsf{A}\psi &\overset{\text{def}}{\Leftrightarrow} \pi \models^{\mathcal{M}} \psi \text{ for all paths } \pi \text{ of } \mathcal{M} \text{ starting at } s \\
\mathcal{M}, s \models \mathsf{E}\psi &\overset{\text{def}}{\Leftrightarrow} \pi \models^{\mathcal{M}} \psi \text{ for some path } \pi \text{ of } \mathcal{M} \text{ starting at } s
\end{aligned}$$

$$\begin{aligned}
\pi \models^{\mathcal{M}} \mathsf{X}\phi &\overset{\text{def}}{\Leftrightarrow} \mathcal{M}, \pi(1) \models \phi \\
\pi \models^{\mathcal{M}} \mathsf{G}\phi &\overset{\text{def}}{\Leftrightarrow} \forall i \geq 0.\, \mathcal{M}, \pi(i) \models \phi \\
\pi \models^{\mathcal{M}} \mathsf{F}\phi &\overset{\text{def}}{\Leftrightarrow} \exists i \geq 0.\, \mathcal{M}, \pi(i) \models \phi \\
\pi \models^{\mathcal{M}} \phi_1\mathsf{U}\phi_2 &\overset{\text{def}}{\Leftrightarrow} \exists i \geq 0.\, (\mathcal{M}, \pi(i) \models \phi_2 \ \wedge\ \forall j < i.\, \mathcal{M}, \pi(j) \models \phi_1)
\end{aligned}$$