Course 2D1453, 2006-07

Advanced Formal Methods

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KTH/CSC

Prerequisites

- Undergraduate logic and discrete maths
- CS literacy
- Some functional programming experience useful
  - The theorem prover Isabelle is programming in SML
  - Some SML programming may be needed for course projects
- Semantics and formal methods advisable

Course Structure

- Lectures
  - Initial six scheduled, more when needed
- Hand-in assignments
- Course project
  - Formalize a theory and prove some theorems about it in Isabelle
- Presentation at final workshop
  - Course projects
  - Accompanied by written report
- Final take home exam
  - Details to be determined
- Reading
  - Slides, web, references on course page

Requirements

- Hand-in assignments
  How?
- Course project presentation and report
- Take home exam
  How?
- Course grade determined by exam
  Agreed?
- Graduate students: By agreement

Course Committee - Kursnämnd

NN1:

NN2:

NN3:

Practicalities

Course web

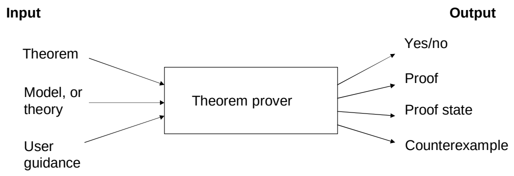
http://www.csc.kth.se/utbildning/kth/kurser/2D1453/aform07/

Essential – updated without warning

Registration:
Please sign up with
Name
Program and year
Personnummer
Email contact

Special wishes or interests?

What is a Theorem Prover?



Automated theorem prover:
- Read first order logic sentence, crunch, answer yes or no or fail to terminate, maybe produce counterexample

Interactive theorem prover:
- Formalize problem in theory, guide theorem prover in proof search, automate when possible, output is proof (or failure)

What is a Theorem?

Theorem: A formalizable statement which is provable on the basis of explicit, formalizable assumptions

Pythagoras theorem: In a right triangle with sides *A, B, C* where *C* is hypotenuse, $C^2 = A^2+B^2$
- Theorem in the theory of geometry

Fundamental theorem of arithmetic: A whole number bigger than 1 can be uniquely represented as a product of primes
- Theorem in the theory of arithmetic

What is a Theorem?

Theorem: The program "x:=n ; while x > 0 do x=x-1 od" terminates
- Theorem in the theory of while program execution

Some fictive theorem of Java bytecode verification: After passing the Java bytecode verifier (version x.y.z, this and that implementation) programs written in the Java Virtual Machine language are guaranteed to be type safe
- Theorem in the theory of JVM classfile execution

Formalized Theorems

- Theorems are stated in a formal logic
  - Self-contained
  - No hidden assumptions
- Many different logics are possible
  - Propositional logic, first order logic, higher order logic, type theory, linear logic, temporal logic, epistemic logic,...
- Not mathematical theorems
  - Theorems in math are informal
  - Mathematicians are happy with informal statements and proofs

Formalized Proofs

- Proofs are formal objects, subject to manipulation
- Not mathematical proofs
- Proofs in math are
  - Informal
  - Validated by "peer review"
    Same role as code inspection in software engineering
  - Meant to convey a message – how the proof works
  - Formal details are too cumbersome

So Why Bother?

- The problem itself rather than the maths is interesting
  Want to know e.g.:
  - Does program P deadlock?
  - Is programming language L type safe?
  - Does API A guarantee release of keys only to properly authorized users?
- Proofs may be huge, boring and repetitive, and not likely to be examined by peers
- Formalizing gives a chance to leave the mechanics to the machine
  - Proof manipulation and proof recognition
- We can carry on with the interesting bits:
  - Formalization and proof search

Automated or Interactive proof?

The two are obviously related, and yet not

Automated theorem proving:
- Use: Posing questions small/easy enough to be tractable
- Technology: Algorithms and semi-algorithms

Interactive theorem proving:
- Use: Formal modelling and proof search
- Technology: Proof representation and manipulation

But of course the two are tightly related
- Pointless to do algorithmic work by hand

This course: Mainly interactive theorem proving
- At least initially

Some History

1929: M. Presburger shows that linear arithmetic is decidable

60's: Field of automated theorem proving starts
- SAT – boolean satisfiability solving
- Resolution (Robinson, 1965)
- Lots of enthusiasm

70's: Reality sinks in
- Complexity theory, hard problems
- Difficult to prove "interesting" theorems

70's – present: Many theorem proving systems built
- Otter, Boyer-Moore, NuPrl, isabelle, Coq, PVS, ESC/Java and simplify,...

In Maths

1976: Appel and Haken proves four colour conjecture

Splits proof into about 1500 cases examined by computer plus manual part

First use of programs to solve open problem in math
- Highly controversial at the time

Since then other open problems in math have been settled

2004: Werner and Gonthier formalizes and proves four colour conjecture in CoQ
- Eliminates need to trust Appel and Haken's program
- Instead need to trust CoQ higher order dependent type theory and its kernel implementation

Current Situation

- Software issues gain importance
  - Internet – ease of downloading executable code, ease of attacks
  - Java etc. – code mobility
  - Increased public awareness of computer security issues
- New interest in software verification
  - Automated and interactive program verification
  - Protocols
  - Language generics: Compilers, type systems, bytecode verifiers
- But the decidability and complexity bounds remain ...

What We'll Do in the Course

- Theoretical underpinnings
  - Lambda calculus
  - Type systems
  - Proof systems, natural deduction
  - Some theorem proving
  - Some decision procedures, probably
- Isabelle
  - Getting you started
  - Some Isabelle specifics
  - Assignments mix pen and paper + Isabelle
- Projects
  - Formalize some theory and prove things about it
  - Security protocols, a machine model, a type system

Isabelle

- Generic proof assistant
- Developed by Larry Paulson at Cambridge and Tobias Nipkow at Munich
  - Lots of other contributors
- Main instantiations are HOL and ZF
- URL: `isabelle.in.tum.de`
- Several layers:
  - Proof General: User interface
  - HOL, ZF: Object logics
  - Isabelle: Generic proof assistant
  - Standard ML: Programming language
  - All layers can be accessed

Homework

- Look up the course page for papers by Hoare, Moore, Demillo et al.
- Visit the Isabelle site, download and install if needed
- Browse the documentation
- Familiarize yourself with the tool. Look through the preview at the overview page.
- Start reading the Isabelle tutorial, work through sections 2.1 and 2.2 to do a first example.