## LTL Syntax

Let $p$ range over a given set *Atoms* of atomic propositions.

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{X}\phi \mid \mathsf{G}\phi \mid \mathsf{F}\phi \mid \phi\mathsf{U}\phi$$

## Models and Paths

A *model* is a tuple $\mathcal{M} = (S, \rightarrow, L)$ where:

(i) $S$ is a set of states,

(ii) $\rightarrow \subseteq S \times S$ is a transition relation,

(iii) $L : S \rightarrow 2^{Atoms}$ is a labelling function.

A *path* of $\mathcal{M}$ is an infinite sequence of states $\pi = s_0 s_1 s_2 s_3 \ldots$ such that $s_i \rightarrow s_{i+1}$ for all $i \geq 0$. Given such a path, we denote by $\pi(i)$ the $i$-th element $s_i$ of $\pi$, and we denote by $\pi^i$ the $i$-th suffix $s_i s_{i+1} s_{i+2} s_{i+3}$ of $\pi$.

## LTL Semantics

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model, and let $\pi$ be a path of $\mathcal{M}$.

$$\begin{aligned}
\pi \models^{\mathcal{M}} p &\overset{\text{def}}{\Leftrightarrow} p \in L(\pi(0)) \\
\pi \models^{\mathcal{M}} \neg\phi &\overset{\text{def}}{\Leftrightarrow} \text{not } \pi \models^{\mathcal{M}} \phi \\
\pi \models^{\mathcal{M}} \phi \wedge \psi &\overset{\text{def}}{\Leftrightarrow} \pi \models^{\mathcal{M}} \phi \text{ and } \pi \models^{\mathcal{M}} \psi \\
\pi \models^{\mathcal{M}} \mathsf{X}\phi &\overset{\text{def}}{\Leftrightarrow} \pi^1 \models^{\mathcal{M}} \phi \\
\pi \models^{\mathcal{M}} \mathsf{G}\phi &\overset{\text{def}}{\Leftrightarrow} \forall i \geq 0.\, \pi^i \models^{\mathcal{M}} \phi \\
\pi \models^{\mathcal{M}} \mathsf{F}\phi &\overset{\text{def}}{\Leftrightarrow} \exists i \geq 0.\, \pi^i \models^{\mathcal{M}} \phi \\
\pi \models^{\mathcal{M}} \phi\mathsf{U}\psi &\overset{\text{def}}{\Leftrightarrow} \exists i \geq 0.\, (\pi^i \models^{\mathcal{M}} \psi \wedge \forall j < i.\, \pi^j \models^{\mathcal{M}} \phi)
\end{aligned}$$

Model $\mathcal{M}$ at state $s \in S$ satisfies formula $\phi$, denoted $\mathcal{M}, s \models \phi$, if all paths $\pi$ of $\mathcal{M}$ starting at $s$ satisfy $\phi$.