

Differensmetod för randvärdesproblem DN1241, 1243, Fö 9
Icke-linjärt problem

$$y'' = \sqrt{x^2 + y^2 + (dy/dx)^2}$$

$$y(1) = 1; \quad y(2) = 3$$

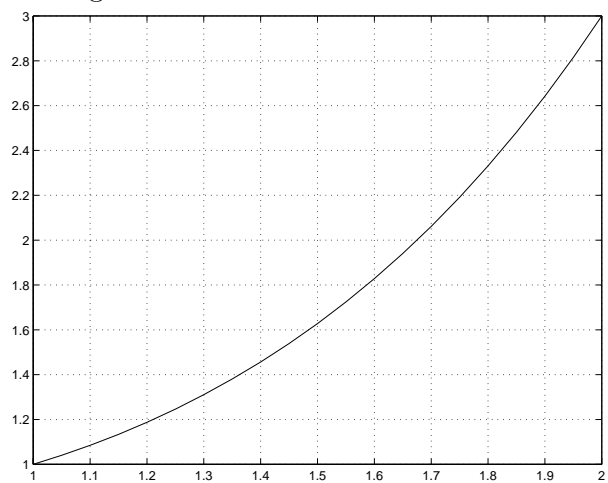
```
-----
function G=nonlinbd(z);
global x h yL yR
N=length(z); h2=h^2; hh=2*h;
y=[yL; z; yR];
G=( y(3:N+2)-2*y(2:N+1)+y(1:N) )/h2- ...
    sqrt(x.^2+y(2:N+1).^2 +( ( y(3:N+2)-y(1:N) )/hh ).^2);
-----
```

```
function jac=minjac(Fcn,z);
NR=length(z); F=feval(Fcn,z); jac=[]; stegtol=1.E-8;
for i=1:NR,
    z0=z;
    st=z0(i)*stegtol; if st==0, st=1.E-10; end
    z0(i)=z0(i)+st;
    jac=[jac ( feval(Fcn,z0)-F )/st];
end
-----
```

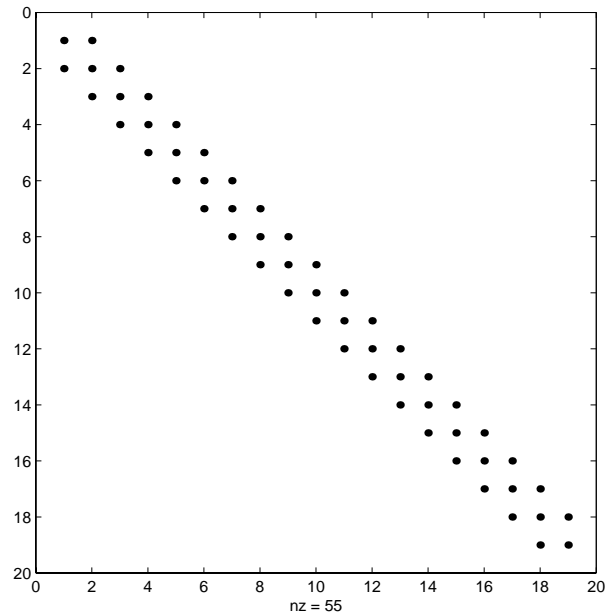
```
clear,clf,hold off
global x h yL yR
format short e, format compact
N=19; a=1; b=2; yL=1; yR=3;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
z=yL+2*x; %startgissning
dz=z;
while norm(dz,inf) >1.E-12*norm(z,inf),
    F=nonlinbd(z); J=minjac('nonlinbd',z);
    dz=-J\F; z=z+dz;
    disp(norm(dz,inf))
end
xp=[a;x;b]; yp=[yL;z;yR];
plot(xp,yp); grid; spy(J)
-----
```

```
Med stegtol=1.E-8;    stegtol=1.E-3;
    2.3372e+00        2.3198e+00
    3.9825e-02        5.6220e-02
    4.8788e-05        2.8812e-04
    1.0984e-10        9.8061e-07
    1.1979e-16        3.1840e-09
                        1.0278e-11
                        3.3213e-14
```

Figurer:
Lösningskurva.



Jakobianmatrisens struktur av icke-nollor.



Bandmatrismetod för linjärt randvärdesproblem

$$y'' = 10\sqrt{1 + e^{x^3}}$$
$$y(1) = 1; \quad y(2) = 3$$

```
clf,hold off
format short e, format compact
a=1; b=2; yL=1; yR=3;
N=19; Ns=5;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
A=sparse(N,N); %lagrar A glest;ger snabbare lösare A\F
for i=1:N,
    A(i,i)=-2;
    if i>1, A(i,i-1)=1; end
    if i<N, A(i,i+1)=1; end
end
F=h^2*10*sqrt(1+exp(x.^3));
F(1)=F(1)-yL; F(N)=F(N)-yR;
y=A\F;
xp=[a;x;b]; yp=[yL;y;yR];
plot(xp,yp);
xt=xp(1:Ns:N+2); yt=yp(1:Ns:N+2);
[xt yt]
```

```
-----
>> N=19; Ns=5;
    1.0000e+00    1.0000e+00
    1.2500e+00   -4.3671e+00
    1.5000e+00   -7.8801e+00
    1.7500e+00   -7.6680e+00
    2.0000e+00    3.0000e+00
```

```
>> N=39; Ns=10;
    1.0000e+00    1.0000e+00
    1.2500e+00   -4.3862e+00
    1.5000e+00   -7.9154e+00
    1.7500e+00   -7.7096e+00
    2.0000e+00    3.0000e+00
```

```
>> N=79; Ns=20;
    1.0000e+00    1.0000e+00
    1.2500e+00   -4.3909e+00
    1.5000e+00   -7.9243e+00
    1.7500e+00   -7.7201e+00
    2.0000e+00    3.0000e+00
```

Samma linjära randvärdesproblem med differensmetod, men utan matrisformulering. Jämför även med det icke linjära problemet!

```
-----
function G=lindb(z);
global x h yL yR
N=length(z); h2=h^2; hh=2*h;
y=[yL; z; yR];
G=( y(3:N+2)-2*y(2:N+1)+y(1:N) )/h2- ...
    10*sqrt(1+exp(x.^3));
-----
```

```
clear,clf,hold off
global x h yL yR
format short e, format compact
N=19;Ns=5; a=1; b=2; yL=1; yR=3;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
z=yL+2*x; %startgissning
dz=z;
while norm(dz,inf) >1.E-12*norm(z,inf),
    F=lindb(z); J=minjac('lindb',z);
    dz=-J\F; z=z+dz;
    disp(norm(dz,inf))
end
xp=[a;x;b]; yp=[yL;z;yR];
plot(xp,yp); grid;
xt=xp(1:Ns:N+2); yt=yp(1:Ns:N+2);
[xt yt]
```

```
-----
    1.2735e+01
    1.1866e-07
    2.8266e-15
-----
```

Tabellerna med utskrifter blir för övrigt desamma som för bandmatrismetoden!