Homework 2, DN2230

Due November 16, 2012

If a correct solution is handed in before the deadline (i.e. November 16) two bonus points will be awarded to the final written exam. If a solution that is handed in before that date is not correct, it has to be redone, but the second time without yielding bonus points for the exam.

Solutions must be clearly written, and easy to follow. If not, they will not generate bonus points, and must be redone.

Together with the written solution, I want you to send me your code (for exercise 1 and 2) by e-mail.

1. Implement the full multigrid algorithm. You may do this by extending your multigrid program from homework 1. By section 3.2.2 in "Multigrid", by Trottenberg et al. it should be possible to choose the ingredients in the algorithm such that the FMG error

   $$\|u_h^{FMG} - u_h\|$$

   is of the same (or higher) order in the discretization step length h as the discretization error

   $$\|u - u_h\|.$$

   Here u is the PDE solution, e.g. the solution to Poisson's equation in one dimension, $-u'' = f$ in $(0, 1)$, u_h is the exact solution to the discretized problem with step length h, and u_h^{FMG} is the corresponding full multigrid solution.

   Verify numerically that you can choose the ingredients in the FMG algorithm (interpolation, MG iterations per level etc.) such that the FMG error is of the same (or higher) order in h as the discretization error.

2. Adapt the Power Iteration method so that it is possible to find the second largest (in magnitude) eigenvalue of a symmetric matrix, and the corresponding eigenvector. Give a Matlab program that uses your algorithm to compute these for the matrix

   $$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 9 \end{pmatrix}.$$

3. Exercise 7.3 in "Numerical Linear Algebra" (NLA).

4. What is the computational work in flops to perform a QR factorization of a symmetric tridiagonal matrix $A \in \mathbb{R}^{m \times m}$? We are primarily interested in the order of the method; hence, the task here is to determine an optimal number α, such that the computational work is less than or equal to a constant times m^{α}.

Good luck!