Some Improved hints for problemset 1 in 2D1255:

Read 6.3 and 6.7 in Demmel,

1. Consider first the corresponding problem with coefficients=1. Then the straightforward discretization yields a system to be solved in each timestep of the form

$$\alpha Q - TQ - QT = P$$

Here Q is the unknown matrix, P is known from the previous timestep and

$$T = \begin{pmatrix} -1 & 1 & & & \\ 1 & -2 & \ddots & & \\ & \ddots & \ddots & 1 & \\ & & 1 & -2 & 1 \\ & & & 1 & -1 \end{pmatrix}$$

The matrix T has eigenvectors and eigenvalues

$$z_j = s_j \begin{pmatrix} \cos(\frac{j\pi}{2N}) \\ \cos(\frac{3j\pi}{2N}) \\ \vdots \\ \cos(\frac{j(2N-1)\pi}{2N}) \end{pmatrix}, \; s_0 = \sqrt{\frac{1}{N}}, \; s_j = \sqrt{\frac{2}{N}}, j = 1,..N-1,$$

$$\lambda_j = -2 + 2\cos(\frac{j\pi}{N}), \quad j = 0,1,...N-1.$$

Let Z be the *NxN* matrix with the eigenvectors as columns,

$$Z = \begin{pmatrix} z_o & \cdots & z_{N-1} \end{pmatrix}, \quad Z^{-1} = Z^T$$

Introducing $Q=ZW$ leads to N tridiagonal systems. Note that computing ZW must be done by using the Fast Cosine Transform (DCT in Matlab). Otherwise the method is not fast.

2. To get the Fast Cosine Transform to work it may be usefull to first apply it to a vector where the answer is known.

3. Introducing the variable coefficient in the equation yields

$$\alpha Q - TQK - QT = P$$

where K is a diagonal matrix of the same size as Q and T. By introducing $Q=ZW$ solving this system is reduced to solving N tridiagonal systems for the rows of W.