L1

To define $u(x)$ uniquely : boundary conditions

Ex. Homogeneous Dirichlet b.c. $u(0)=u(1)=0$
(temp. zero at endpoints)

$\Rightarrow$ Two-point boundary value problem

$$\begin{cases} -(au')' = f & \text{in } (0,1) \\ u(0)=u(1)=0 \end{cases}$$

Ex. Homogeneous Neumann b.c. $q(0) = -a(0)u'(0) = 0$
(insulating wire at $x=0$)

Non-homogeneous Dirichlet/Neumann b.c.
$u(0)=u_0$, $q(0)=g_0$, ... (prescribed temp/flux)

Robin boundary cond. at $x=1$:

$$a(1)u'(1) + \gamma(u(1)-u_1) = g_1$$

$\gamma \geq 0$ given boundary heat conductivity

$\gamma = 0 \Rightarrow$ Neuman ; $-a(1)u'(1) = -g_1$

$\gamma = \infty \Rightarrow$ Dirichlet; $u(1) = u_1$

$g_1 = 0 \Rightarrow$ Heat flux $-a(1)u'(1)$ prop. to temp. difference $u(1)-u_1$
(u_1 - outside temperature)