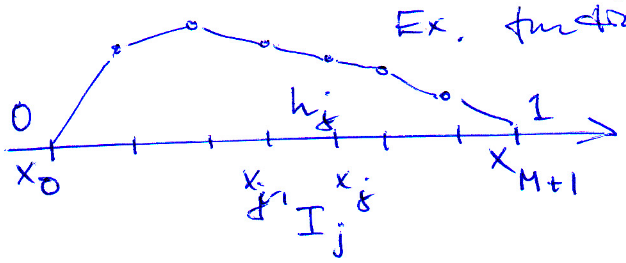
(L1) 

Find numerical approx. of D.E.

$$(*) \quad \begin{cases} -u'' = f & \text{in } (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

<u>Piecewise linear approximation</u>: on mesh $\mathcal{T}_h$

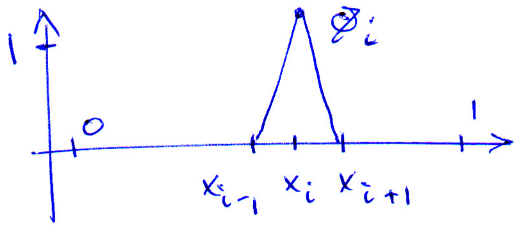



Divide $[0,1]$ into subintervals $I_j = (x_{j-1}, x_j)$ of length $h_j = x_j - x_{j-1}$ ; $\mathcal{T}_h$: $0 = x_0 < x_1 < \dots < x_{M+1} = 1$

$V_h = \{$set of all cont. p.w. lin. functions on $\mathcal{T}_h$ that are zero at $x=0$ and $x=1$.$\}$

$V_h$ finite dimensional vector space of dimension $M$ ($M$ degrees of freedom : nodes)

Basis of $V_h$ : hat functions $\{\varphi_j\}_{j=1}^{M}$




$$\varphi_i(x) = \begin{cases} 0 & x \notin [x_{i-1}, x_{i+1}] \\ \dfrac{x - x_{i-1}}{x_i - x_{i-1}} & x \in [x_{i-1}, x_i] \\ \dfrac{x - x_{i+1}}{x_i - x_{i+1}} & x \in [x_i, x_{i+1}] \end{cases}$$

Any function $v(x)$ in $V_h$ can be written

$$v(x) = \sum_{j=1}^{M} v(x_j)\,\varphi_j(x) = \sum_{j=1}^{M} \xi_j\,\varphi_j(x)$$