Lecture 2

Poisson's equation in $\mathbb{R}^2$ with homogeneous Dirichlet boundary conditions :

$$(D) \quad \begin{cases} -\Delta u(x) = f(x) & x \in \Omega \\ u(x) = 0 & x \in \Gamma \end{cases}$$

Ω bounded domain in $\mathbb{R}^2$ with boundary Γ.

$$\left[-\Delta u = -\frac{\partial^2 u}{\partial x_1} - \frac{\partial^2 u}{\partial x_2} \quad , \text{ with } x = (x_1, x_2) \right.$$

$\left. \text{"Laplacian of } u(x)\text{"} \right]$

Variational (Weak) formulation :

$$\boxed{\begin{array}{l} \text{Find } u \in V \text{ such that} \\ (W) \qquad (\nabla u, \nabla v) = (f, v) \qquad \forall v \in V \end{array}}$$

where $(w, v) = \int_\Omega w v \, dx$, $(\nabla w, \nabla v) = \int_\Omega \nabla w \cdot \nabla v \, dx$

and $V = \left\{ v : \int_\Omega (|\nabla v|^2 + v^2) \, dx < \infty \text{ and } v = 0 \text{ on } \Gamma \right\}$

$L_2 = \left\{ v : \int_\Omega v^2 dx < \infty \right\}$

If $u, v \in V$ then $(\nabla u, \nabla v)$ is well defined

If $v \in V$ and $f \in L_2$ then (f, v) — " —