

②

Multiply $-\Delta u = f$ by $v \in V$ and use Green's formula (partial integration in \mathbb{R}^d , $d > 1$)

$$\int_{\Omega} f v \, dx = - \int_{\Omega} \Delta u v \, dx = - \int_{\Gamma} \partial_n u v \, ds + \int_{\Omega} \nabla u \cdot \nabla v \, dx = \int_{\Omega} \nabla u \cdot \nabla v \, dx$$

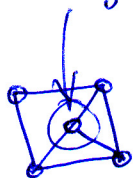
(since $v \in V \Rightarrow v = 0$ on Γ)



$$\left[\begin{array}{l} \text{Normal derivative } \partial_n u = \nabla u \cdot n = \frac{\partial u}{\partial x_1} n_1 + \frac{\partial u}{\partial x_2} n_2 \\ \text{with } n = (n_1, n_2) \text{ outward unit normal} \end{array} \right]$$

\Rightarrow (D) and (W) have the same solution
(if f continuous, see Ch. 21)

A triangular mesh of Ω (with a polygonal Γ)
 $\mathcal{T}_h = \{K\}$ is a sub-division of Ω into a non-overlapping set of triangles/elements/cells K , constructed such that no vertex of one triangle lies on the edge of another triangle:
no "hanging nodes"



$\mathcal{N}_h = \{N\}$ nodes/vertices of \mathcal{T}_h
 $\mathcal{S}_h = \{S\}$ edges of \mathcal{T}_h