Computation of element stiffness matrix

$$A^K_{ij} = \int_K \nabla\lambda_j \cdot \nabla\lambda_i \, dx \qquad i,j = 1,2,3$$

Quadrature: If exact evaluation of integrals is inefficient or impossible

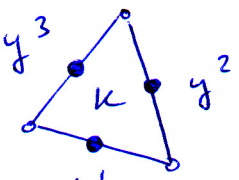
$$\int_K g(x)\,dx \approx \sum_{i=1}^{q} g(y^i)\,\omega_i$$

q = # quadrature points

y^i = quadrature points (nodes)

ω_i = quadrature weights

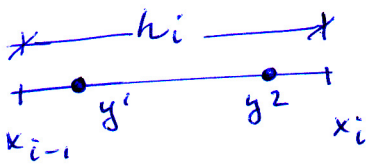
Ex: Midpoint quadrature



$q = 3$, y^i = edge midpoints

$\omega_i = \frac{|K|}{3}$, $|K|$ = area of K

Ex: 1D Gauss rule



$q = 2$, $\omega_i = \frac{h_i}{2}$

$$y^1 = \frac{x_i + x_{i-1}}{2} - \frac{\sqrt{3}}{6} h_i \,, \quad y^2 = \frac{x_i + x_{i-1}}{2} + \frac{\sqrt{3}}{6} h_i$$