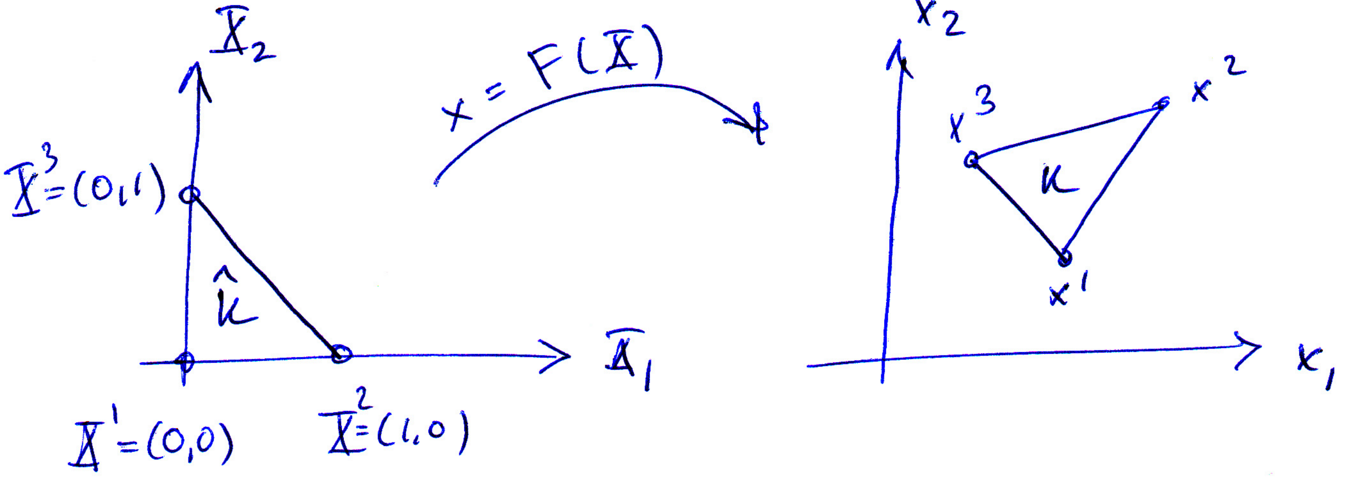
It is common practice to map the triangle K to a reference triangle $\hat{K}$ (reference element), and formulate the quadrature rule in terms of reference coordinates.



<u>Affine mapping</u>: $x = F(X) = x^1\,\Phi_1(X) + x^2\,\Phi_2(X) + x^3\,\Phi_3(X)$

With Φ_1, Φ_2, Φ_3 nodal basis functions for $\hat{K}$:

$$\Phi_1(X) = 1 - X_1 - X_2, \quad \Phi_2(X) = X_1, \quad \Phi_3(X) = X_2$$

$$\nabla_X \Phi_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \nabla_X \Phi_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \nabla_X \Phi_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

For the element basis functions of K: $\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1(x) = \Phi_1(X), \quad \lambda_2(x) = \Phi_2(X), \quad \lambda_3(x) = \Phi_3(X)$$

The <u>Jacobian</u> $J = F'(X)$ of the mapping:

$$J = F'(X) = \begin{bmatrix} x^1_1 \\ x^1_2 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix} + \begin{bmatrix} x^2_1 \\ x^2_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} x^3_1 \\ x^3_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(J constant!)

$$\nabla_x \lambda_i(x) = J^{-1} \nabla_X \Phi_i(X) \quad \left(\text{Solve } J(\nabla_x \lambda_i) = \nabla_X \Phi_i\right)$$