When computing FEM approximation U, set nodes on the boundary to $\mathcal{N}_b$ and interior nodes to $\mathcal{N}_h$:

$$U = \sum_{N_j \in \mathcal{N}_b} \xi_j \phi_j + \sum_{N_j \in \mathcal{N}_h} \xi_j \phi_j$$

where $\xi_j = g(N_j)$ for $N_j \in \mathcal{N}_b$.

$\Rightarrow$ The resulting discrete system is:

$$\sum_{N_j \in \mathcal{N}_h} \xi_j (\nabla\phi_j, \nabla\phi_i) = (f, \phi_i) - \sum_{N_j \in \mathcal{N}_b} g(N_j)(\nabla\phi_j, \nabla\phi_i)$$

for all $N_i \in \mathcal{N}_h$

Robin & Neumann b.c.

$$(D) \begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_1 \\ \partial_n u + \varkappa u = g & \text{on } \Gamma_2 \end{cases}$$

$\Gamma = \Gamma_1 \cup \Gamma_2$

$\varkappa \geq 0$

Test & trial space should satisfy Dirichlet boundary conditions:

$$V = \{ v : v = 0 \text{ on } \Gamma_1 \ \& \int_\Omega (|\nabla v|^2 + v^2)\, dx < \infty \}$$