First term: $\left| \sum_k \int_k h_k (f + \Delta U) h_k^{-1} (e - \tilde{\pi}_h e) \, dx \right|$

$\leq \| h R_1(U) \| \, \| h^{-1}(e - \tilde{\pi}_h e) \| \leq C_i \| h R_1(U) \| \, \| \nabla e \|$

with $R_1(U) = |f + \Delta U|$ on each $K \in \mathcal{T}_h$.

Second term: $\left| \sum_k \frac{1}{2} \int_{\partial k} h_k^{-1} [\partial_s U] (e - \tilde{\pi}_h e) h_k \, ds \right|$

$\leq \cdots$

$\Rightarrow$ A posteriori error estimate (Th 15.3)

There exist constant C_i depending only on $\mathcal{T}_h$ s.t.

$$\| \nabla_h - \nabla U \| \leq C_i \| h R(U) \|$$

with $R(U) = R_1(U) + R_2(U)$

$R_1(U) = |f + \Delta U|$ on $K \in \mathcal{T}_h$

$R_2(U) = \frac{1}{2} \max_{S \subset \partial K} h_K^{-1} |[\partial_s U]|$ on $K \in \mathcal{T}_h$

Further reading:

CDE [redacted] 5, 14, 2 (Interpolation)

CDE 8, 2 - 8.6, 15.2 - 15.3 (error estimation)