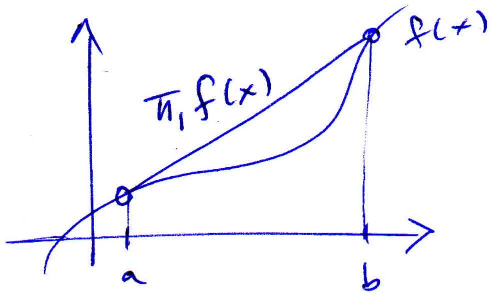
Polynomial interpolant $\pi_q f \in \mathcal{P}^q(a,b)$: defined

by $\pi_q f(\xi_i) = f(\xi_i)$ for all $i = 0, \ldots, q$.

$$\pi_q f(x) = f(\xi_0)\lambda_0(x) + \ldots + f(\xi_q)\lambda_q(x) = \sum_{i=0}^{q} f(\xi_i)\lambda_i(x)$$

Linear interpolant $\pi_1 f(x) = f(a)\dfrac{x-b}{a-b} + f(b)\dfrac{x-a}{b-a}$

choosing $\xi_0 = a$, $\xi_1 = b$.



Theorem 5.1 : Assume $f(x)$ has $q+1$ continuous derivatives in (a,b). Then for $a \le x \le b$:

$$|f(x) - \pi_q f(x)| \le \left| \frac{(x-\xi_0)\cdots(x-\xi_q)}{(q+1)!} \right| \max_{[a,b]} |D^{q+1} f|$$

Proof : $q=1$

(1) $\pi_1 f(x) = f(\xi_0)\lambda_0(x) + f(\xi_1)\lambda_1(x) = f(\xi_0)\dfrac{x-\xi_1}{\xi_0-\xi_1} + f(\xi_1)\dfrac{x-\xi_0}{\xi_1-\xi_0}$

Taylor's theorem for $x \in (\xi_0, \xi_1)$:

(2) $f(\xi_i) = f(x) + f'(x)(\xi_i - x) + \frac{1}{2} f''(\eta_i)(\xi_i - x)^2$

where η_i lies between x and ξ_i.