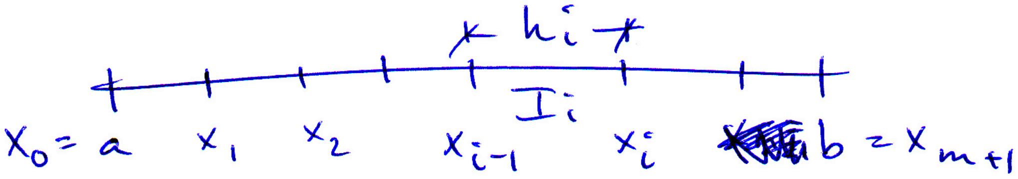
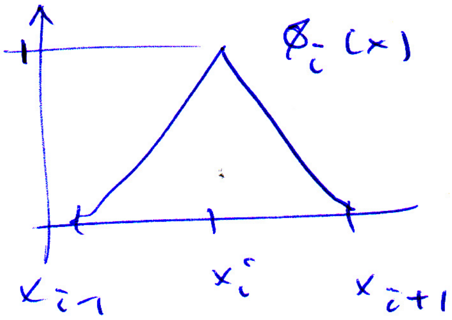
Piecewise linear functions on a mesh $\tau_h = \{I_i\}$

$$V_h = V_h^{(1)} = \{ v \text{ cont} : v|_{I_i} \in \mathcal{P}^q(I_i),\ i = 1, \dots, m+1 \}$$



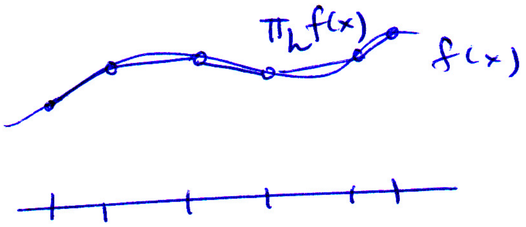
$$v \in V_h \Rightarrow v(x) = \sum_{i=0}^{m+1} v(x_i)\,\varphi_i(x)$$

$\varphi_i(x)$ hat functions



Continuous piecewise linear interpolant $\pi_h f \in V_h$

def. by $\pi_h f(x) = f(x_{i-1}) \dfrac{x - x_i}{x_{i-1} - x_i} + f(x_i) \dfrac{x - x_{i-1}}{x_i - x_{i-1}}$



for $x_{i-1} \le x \le x_i$

& $i = 1, \dots, m+1$

Theorem 5.4: For $p = 1, 2, \infty$ there are constants C_i such that

$$\|f - \pi_h f\|_{L_p(a,b)} \le C_i \|h^2 f''\|_{L_p(a,b)}$$

$$\|f - \pi_h f\|_{L_p(a,b)} \le C_i \|h f'\|_{L_p(a,b)}$$

$$\|f' - (\pi_h f)'\|_{L_p(a,b)} \le C_i \|h f''\|_{L_p(a,b)}$$